Energy absorption and the conductance of small mesoscopic rings

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$ISF, $GIF, $DIP, $BSF

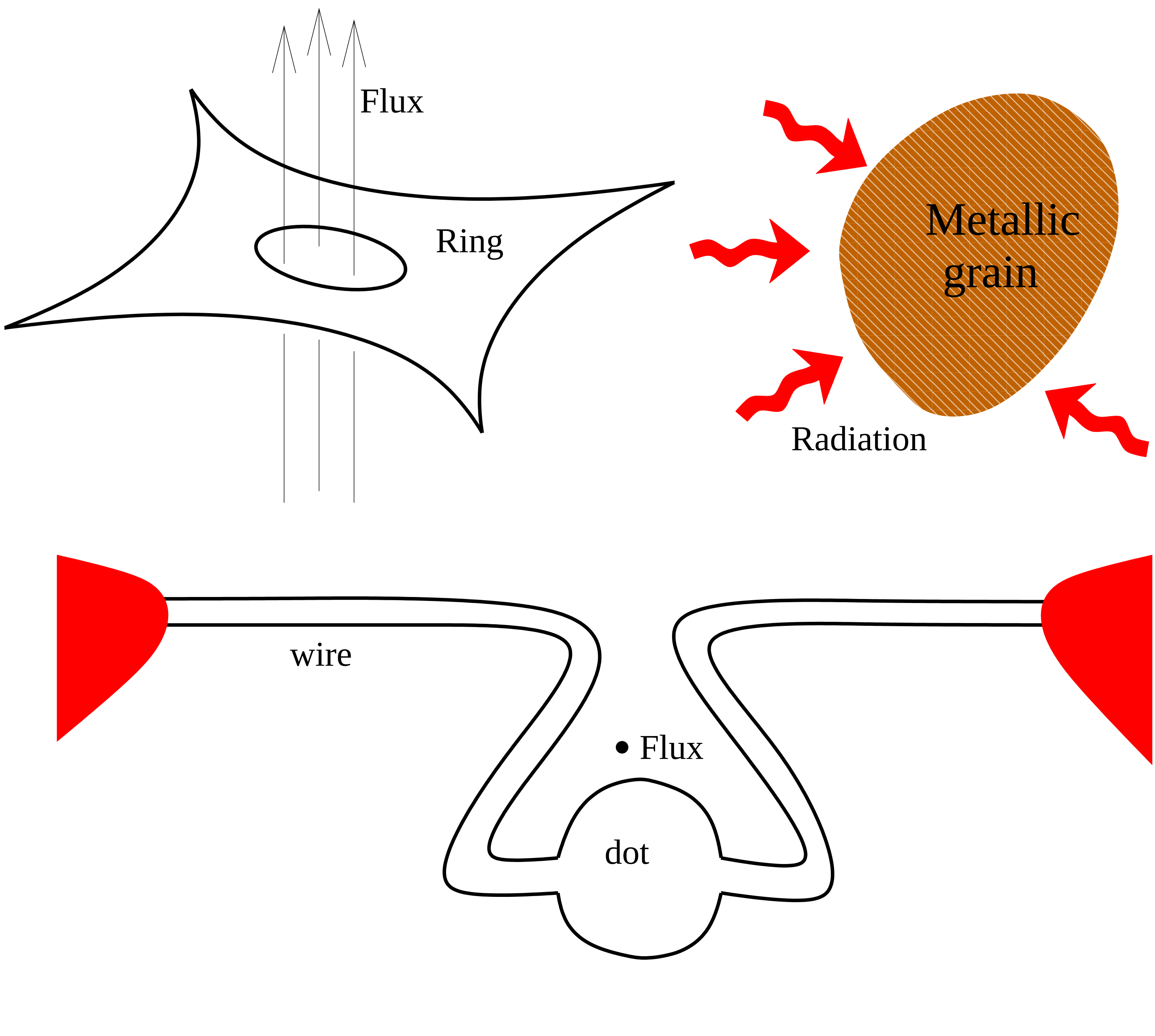
Driven Systems

Non interacting "spinless" electrons in a ring.

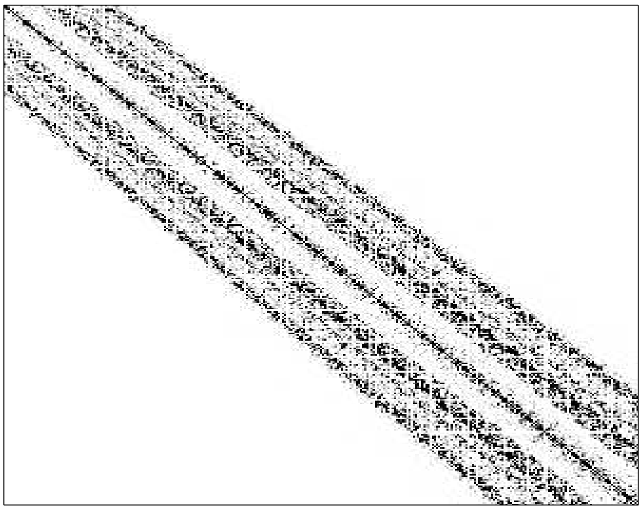
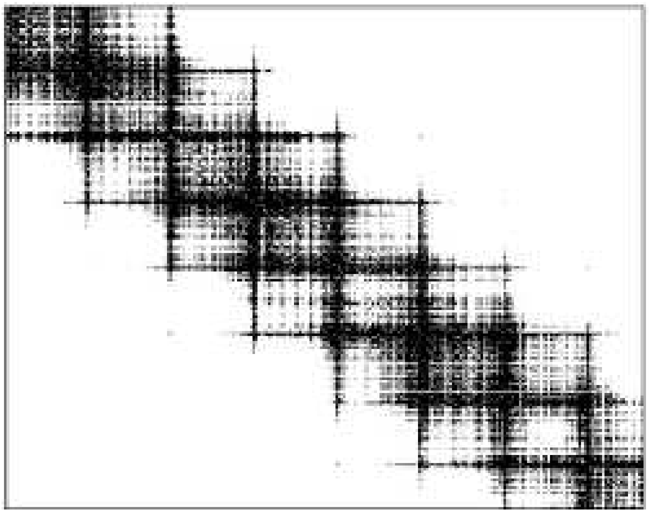
$$\mathcal{H}(Q, P; \Phi(t))$$

$$-\dot{\Phi} \quad = \quad \text{electro motive force}$$

$$G\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$



Linear Response Theory (LRT)

$$H = \{E_n\} - \Phi(t)\{\mathcal{I}_{nm}\}$$

$$\boldsymbol{G} = \pi\hbar \sum_{n,m} |\mathcal{I}_{mn}|^2 \ \delta_T(E_n - E_F)\ \delta_\Gamma(E_m - E_n)$$

$$\boldsymbol{G} \ = \ \pi\hbar(\varrho(E_F))^2 \ \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

applies if

EMF driven transitions $\ll$ relaxation

otherwise

connected sequences of transitions are essential.

leading to

Semi Linear Response Theory (SLRT)

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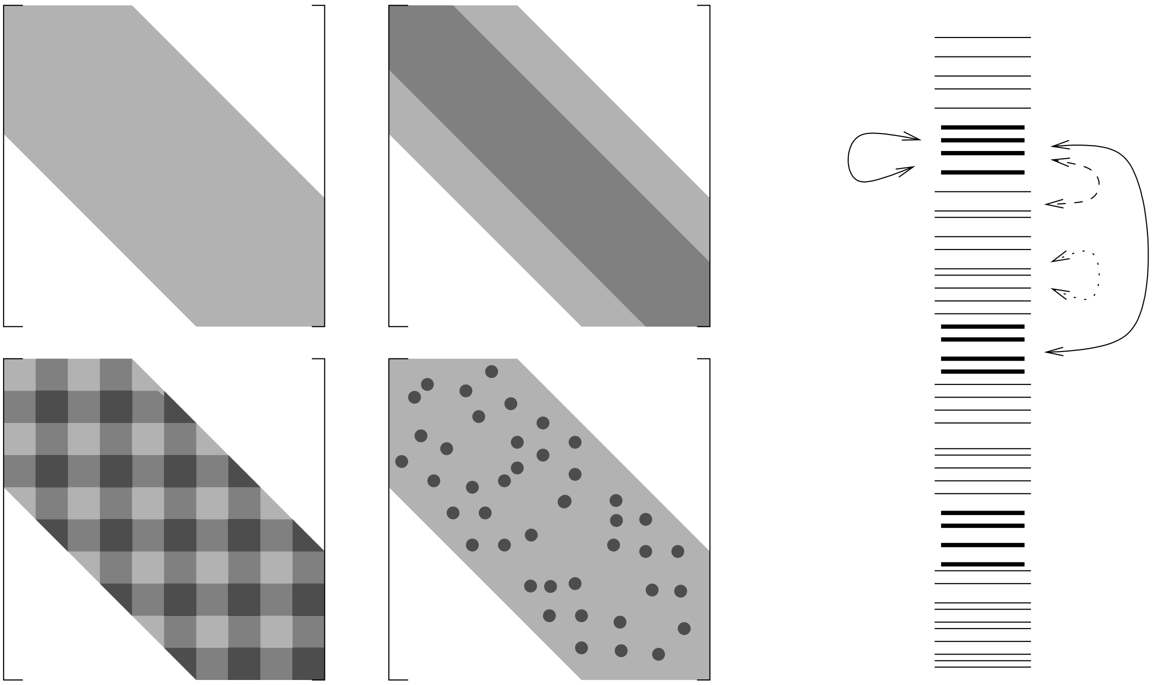


$$H = \{E_n\} - \Phi(t)\{\mathcal{I}_{nm}\}$$

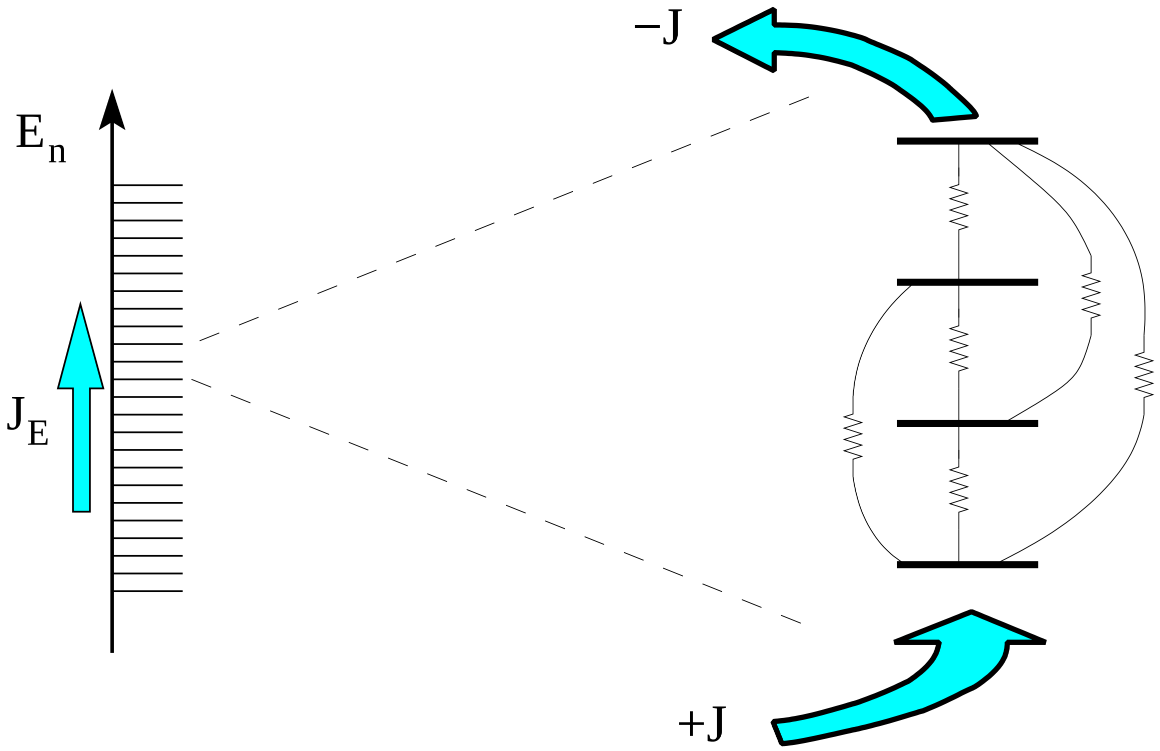
$$\frac{dp_n}{dt} = -\sum_m w_{nm}(p_n - p_m)$$

$$w_{nm} \quad = \quad \text{const} \times \mathrm{g}_{nm} \times \text{EMF}^2$$

Scaled transition rates:

$$\mathrm{g}_{nm} = 2\varrho_{\text{F}}^{-3}\ \frac{|\mathcal{I}_{nm}|^2}{(E_n{-}E_m)^2}\ \delta_\Gamma(E_n{-}E_m)$$

Semi Linear Response Theory (cont.)



$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3} \; \frac{|\mathcal{I}_{nm}|^2}{(E_n - E_m)^2} \; \delta_{\Gamma}(E_n - E_m)$$

The SLRT analog of the Kubo formula:

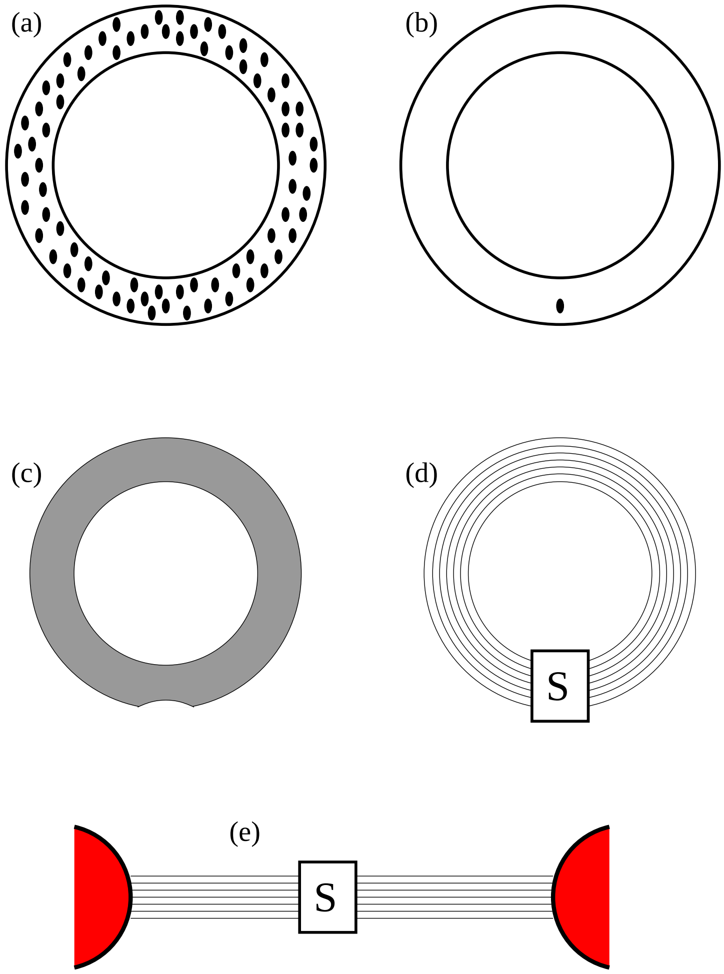
$$\boldsymbol{G} \;\; = \;\; \pi\hbar(\varrho(E_F))^2 \; \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

where

$\langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \equiv$ inverse resistivity of the network

$$\langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle_{\mathrm{harmonic}} \ll \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \ll \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle_{\mathrm{algebraic}}$$

Conductance of mesoscopic rings



Naive expectation (assuming $\Gamma > \Delta$):

$$G = \frac{e^2}{2\pi\hbar}\mathcal{M}\frac{\ell}{L} + \mathcal{O}\left(\frac{\Delta}{\Gamma}\right)$$

L = perimeter of the ring

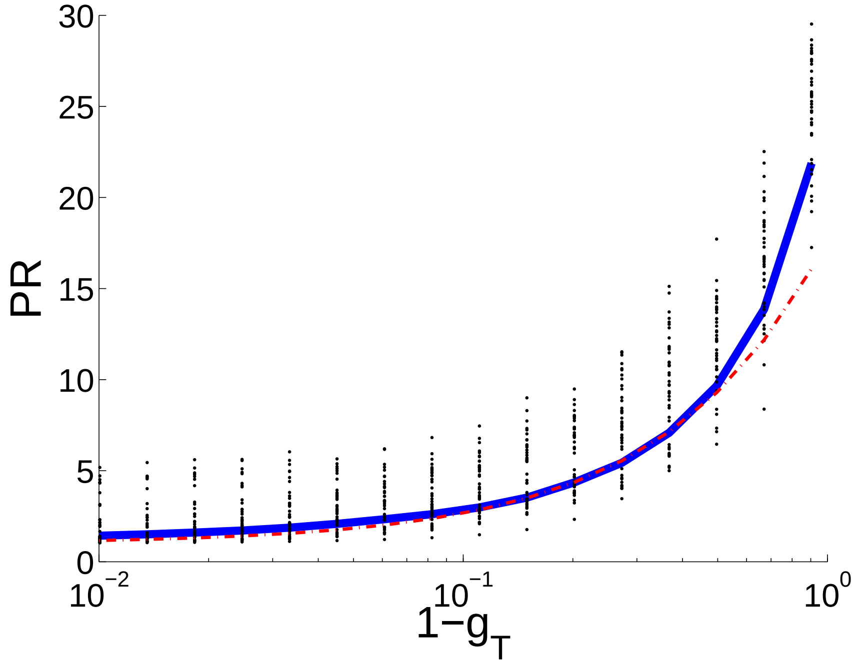
ℓ = mean free path

(!) Diffusive ring: $\ell \ll L$

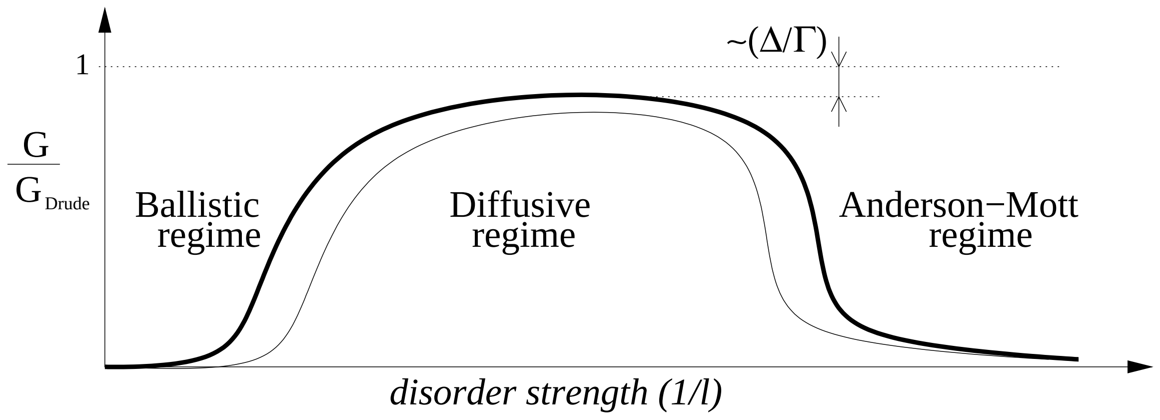
??? Ballistic ring: $\ell \gg L$

Conductance versus disorder

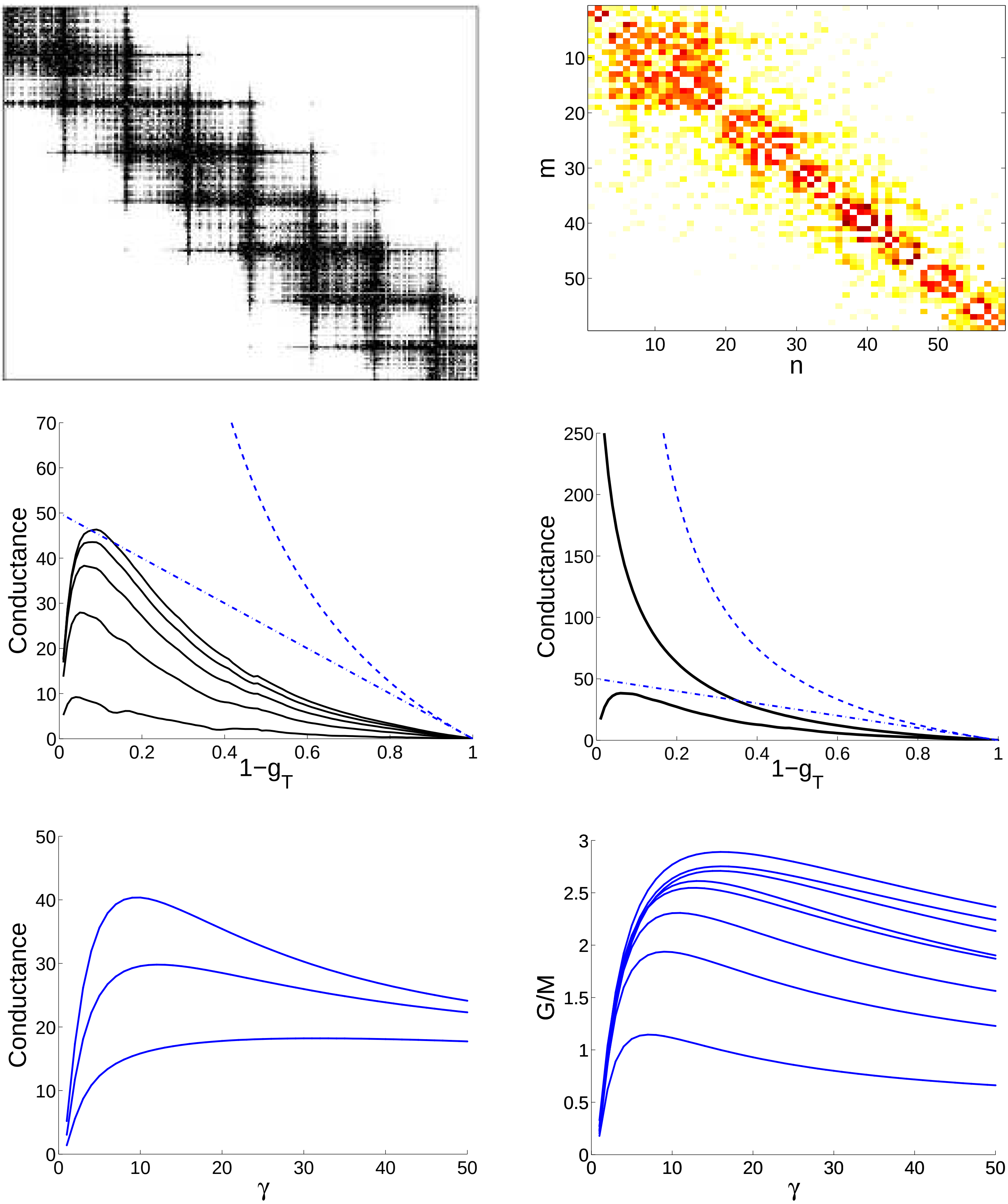
- Weak disorder (ballistic rings):
  Wavefunctions are localized in mode space.
  [See numerical results]



- Strong disorder (Anderson localization):
  Wavefunctions are localized in real space.
  [SLRT $\rightsquigarrow$ VRH]



Numerical results for a ballistic ring



LRT, SLRT and beyond

$$-\dot{\Phi} \quad = \quad \text{electro motive force}$$

$$G\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$

Semi linear response theory

[1] D. Cohen, T. Kottos and H. Schanz,
"Rate of energy absorption by a closed ballistic ring",
(JPA 2006)

[2] S. Bandopadhyay, Y. Etzioni and D. Cohen,
"The conductance of a multi-mode ballistic ring: beyond Landauer and Kubo",
(EPL 2006)

[3] M. Wilkinson, B. Mehlig, D. Cohen,
"Semilinear response",
(EPL 2006)

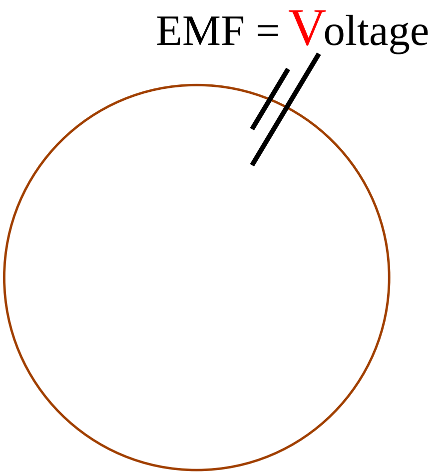
[4] D. Cohen, *"From the Kubo formula to variable range hopping"*
(PRB 2007)

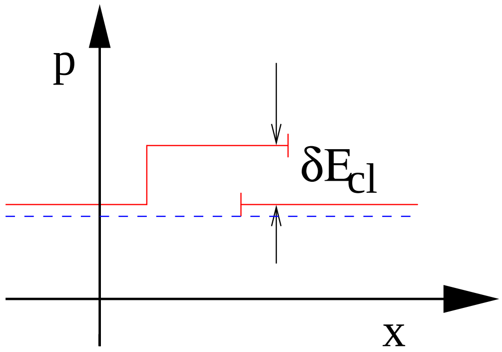
Beyond (semi) linear response theory

[5] D. Cohen and T. Kottos,
"Non-perturbative response of Driven Chaotic Mesoscopic Systems"
(PRL 2000)

[6] A. Stotland and D. Cohen,
"Diffractive energy spreading and its semiclassical limit"
(JPA 2006)

Beyond (semi) linear response





$$\delta E_{\mathrm{cl}} = eV_{\mathrm{EMF}}$$

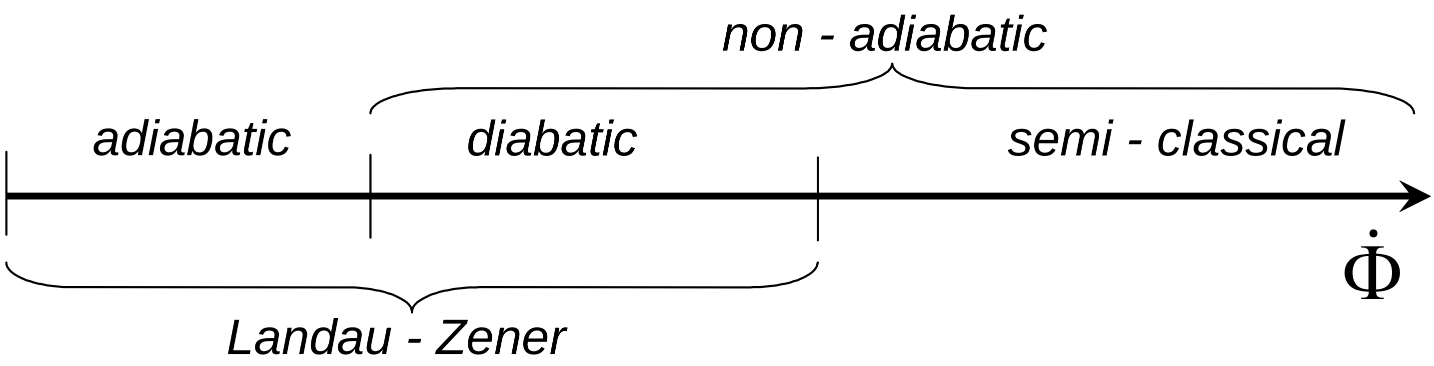
$A(x;t) = \Phi(t)\delta(x - x_0)$ = the vector potential.

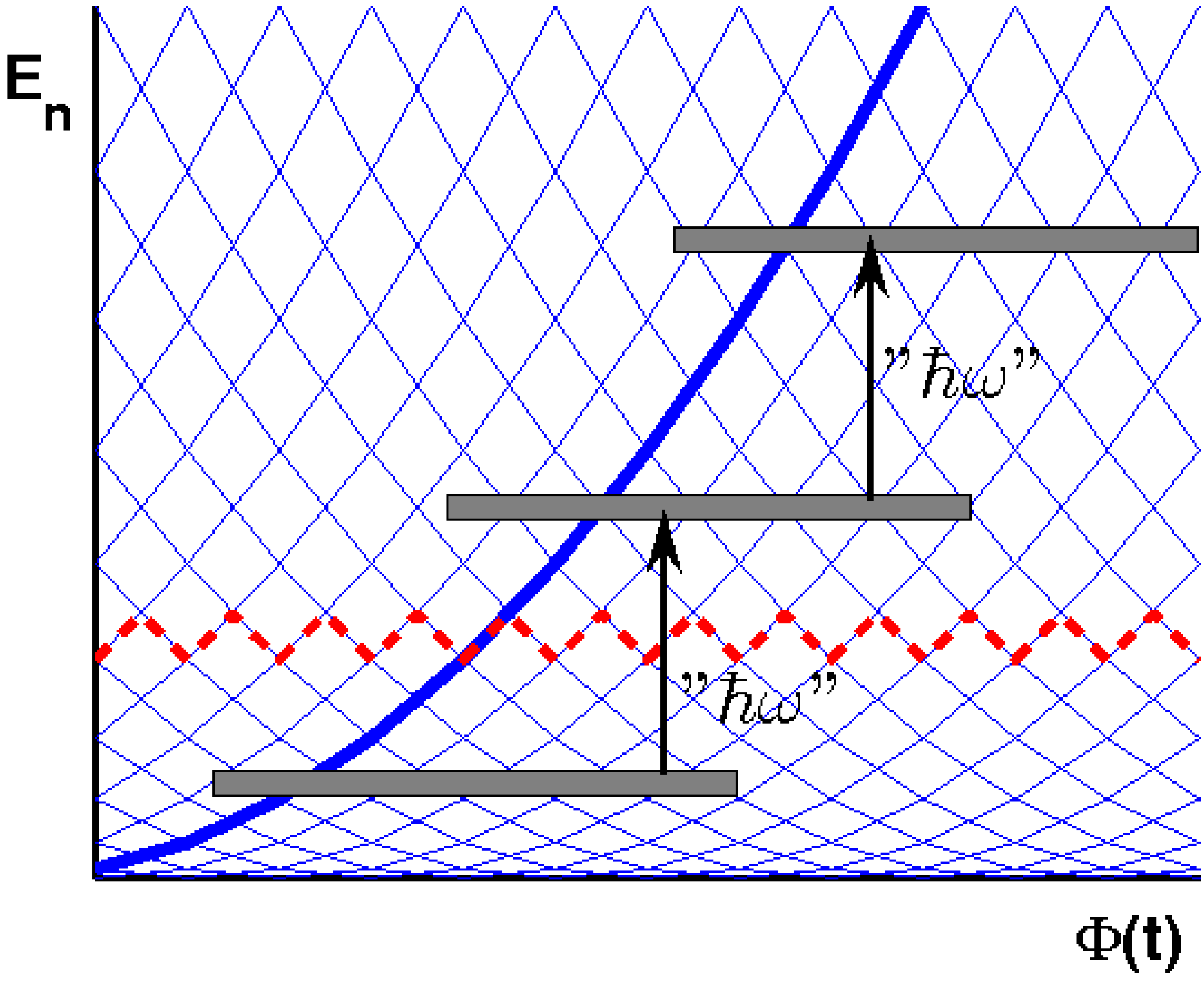
$\mathcal{E}(x) = -\frac{1}{c}\dot{\Phi}\delta(x - x_0)$ = the electric field.

semiclassical regime: $\delta E_{\mathrm{cl}} \gg \Delta$

Note: $\delta E_{\mathrm{cl}} \gg \Delta \quad \Longleftrightarrow \quad V_{\mathrm{EMF}} \gg \frac{\hbar v_{\mathrm{E}}}{L}$

Beyond (semi) linear response: regimes





Beyond (semi) linear response: other effects

(ordered by degree of relevance):

$$H = \{E_n\} - \Phi(t)\{\mathcal{I}_{nm}\}$$

Beyond FGR:

Silva and Kravtsov

[cond-mat/0611083]

Non-perturbative response:

Cohen and Kottos

[PRL 2000]

Dynamical localization:

Basko, Skvortsov and Kravtsov

[PRL 2003]