One body decoherence: fluctuations, recurrences, and the statistics of the quantum Zeno suppression due to erratic driving

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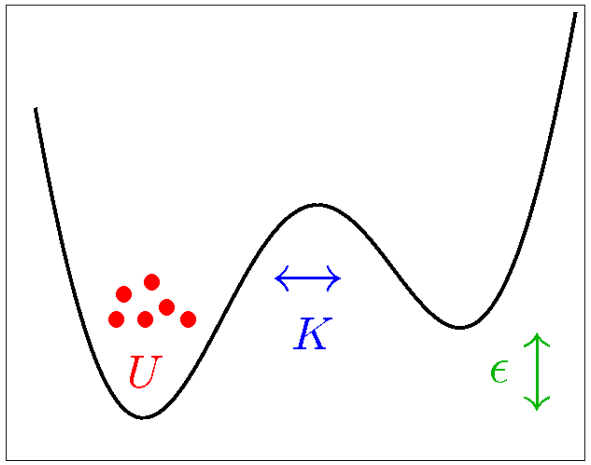


$ISF, $BSF, $DIP

[1] Dynamics & fluctuations (PRL 2009)

[2] Dynamics & fluctuations (PRA 2009)

[3] Dynamics & fluctuations (PRA 2010)

[4] Sweep operation, Landau-Zener (PRL 2009)

[5] Periodic driving, Chaos, Kapitza (PRL 2010)

[6] Erratic vs Noisy driving, Zeno (PRA 2012)

The Bose-Hubbard Hamiltonian (BHH) for a dimer

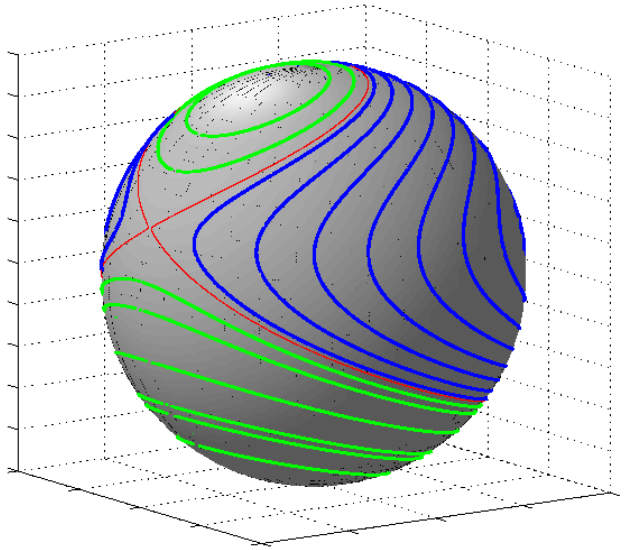
$$\mathcal{H} \;=\; \sum_{i=1,2}\left[\mathcal{E}_i\hat{n}_i + \frac{U}{2}\hat{n}_i(\hat{n}_i-1)\right] - \frac{K}{2}(\hat{a}_2^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_2)$$

K = hopping
U = interaction
$\mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1$ = bias

N particles in a double well is like spin $j = N/2$ system

$$u \equiv \frac{NU}{K}, \qquad \varepsilon \equiv \frac{\mathcal{E}}{K}$$

$$\mathcal{H} \;=\; -\mathcal{E}\hat{J}_z + U\hat{J}_z^2 \;-\; K\hat{J}_x$$



Similar to the Josephson Hamiltonian

$$\mathcal{H}(\hat{n},\varphi) \;=\; U(\hat{n}-\epsilon)^2 - \frac{1}{2}KN\,\cos(\varphi)$$

$\hat{n} \;=\; \hat{J}_z \;=\;$ occupation difference

$\varphi \;=\;$ conjugate phase

Assuming $u>1$ and $|\varepsilon| < \varepsilon_c$
Sea, Islands, Separatrix

Rabi regime:	$u < 1$	(no islands)
Josephson regime:	$1 < u < N^2$	(sea, islands, separatrix)
Fock regime:	$u > N^2$	(empty sea)

$$\varepsilon_c \;=\; \left(u^{2/3}-1\right)^{3/2}$$

WKB quantization (Josephson regime)

$$h = \text{Planck cell area in steradians} = \frac{4\pi}{N+1}$$

$$A(E_\nu) = \left(\frac{1}{2}+\nu\right)h \qquad \nu = 0, 1, 2, 3, \ldots$$

$$\omega(E) \equiv \frac{dE}{d\nu} = \left[\frac{1}{h}A'(E)\right]^{-1}$$

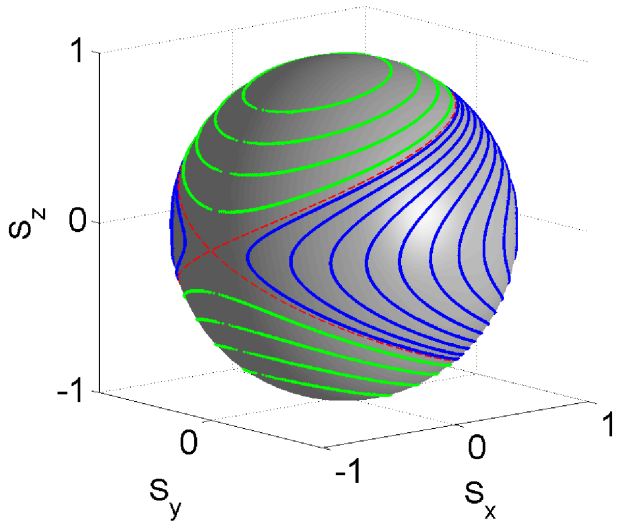
$$\omega_K \approx K = \text{Rabi Frequency}$$

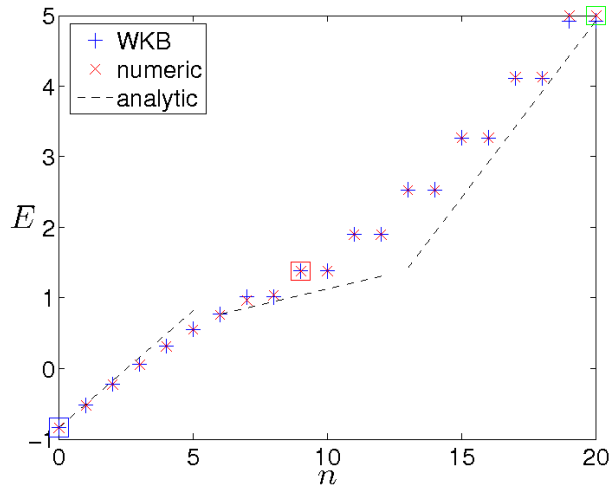
$$\omega_J \approx \sqrt{NUK} = \text{Josephson Frequency}$$

$$\omega_+ \approx NU = \text{Island Frequency}$$

$$\omega_x \approx \left[\log\left(\frac{N^2}{u}\right)\right]^{-1} \omega_J$$

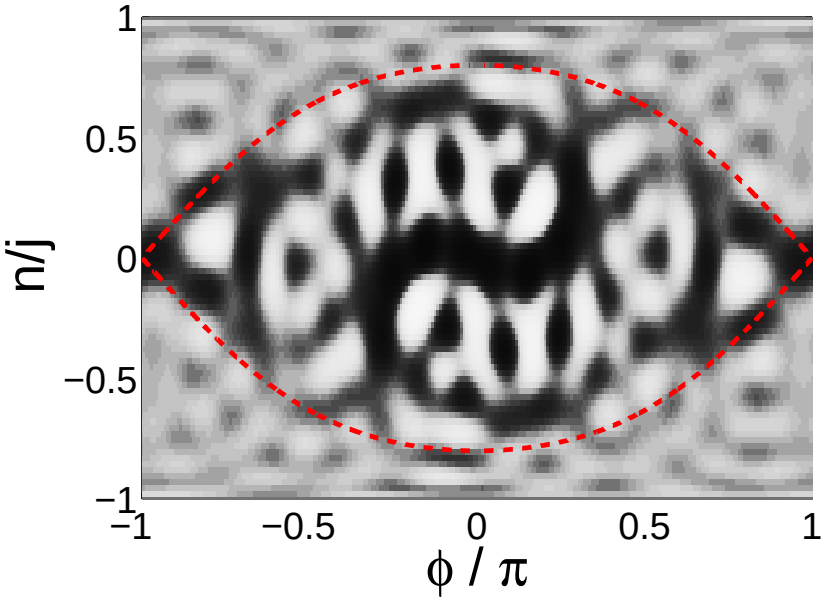
Eigenstates $|E_\nu\rangle$ are like strips along contour lines of $\mathcal{H}$.

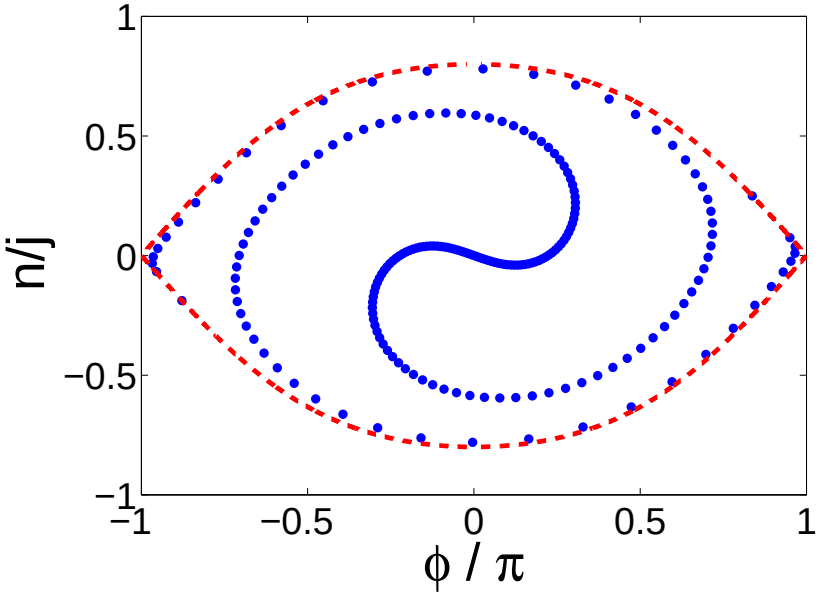




Wavepacket dynamics

MeanField theory (GPE) = classical evolution of a point in phase space
SemiClassical theory = classical evolution of a distribution in phase space
Quantum theory = recurrences, fluctuations (WKB is very good)

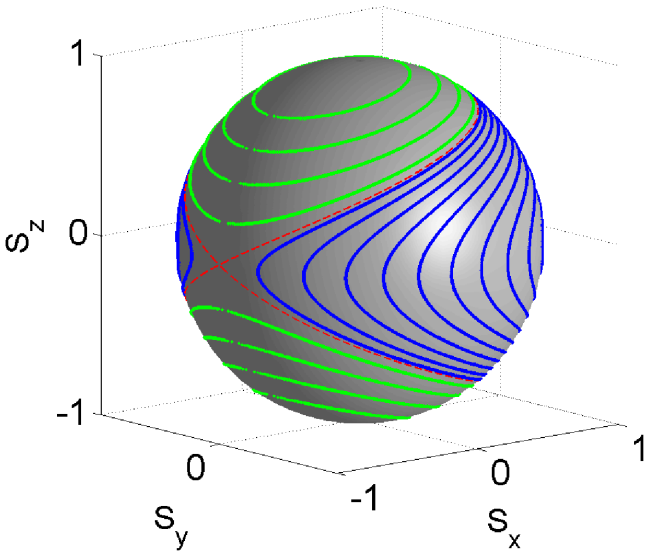


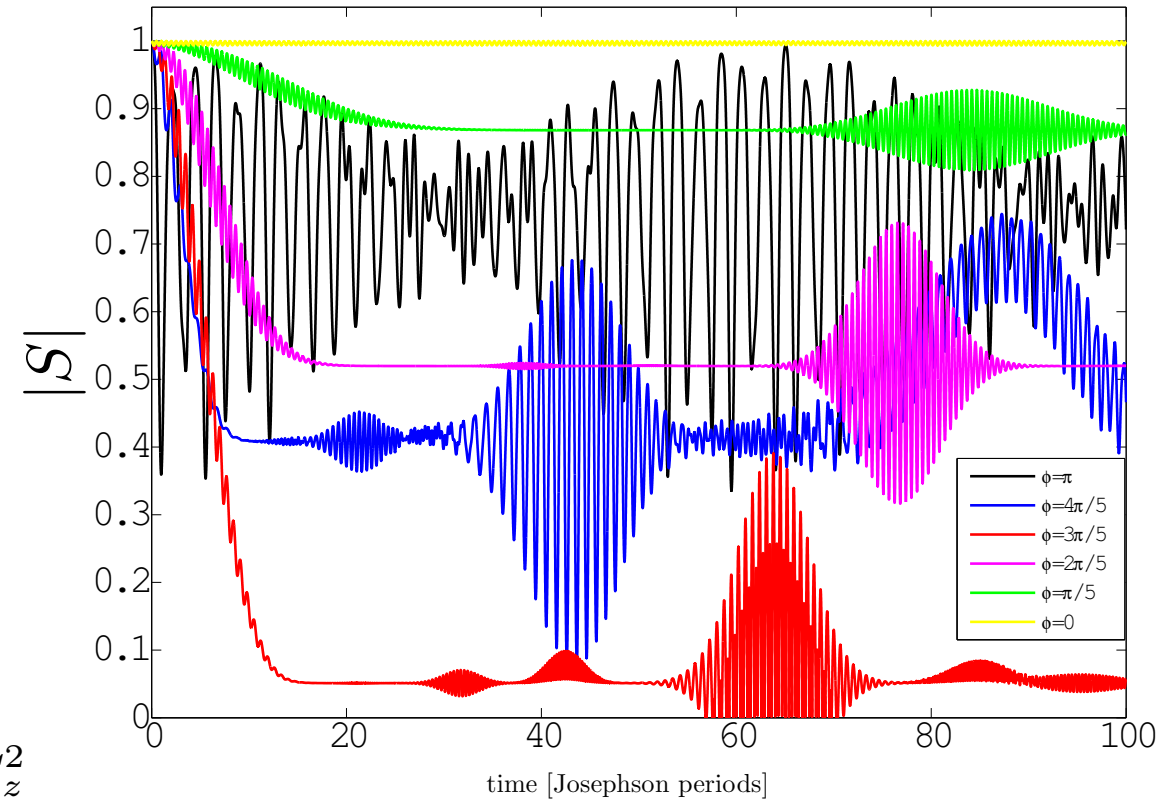


Any operator $\hat{A}$ can be presented by the phase-space function $A_{\mathrm{W}}(\Omega)$

$$\left\langle \hat{A} \right\rangle \quad = \quad \mathrm{trace}[\hat{\rho}\ \hat{A}] \quad = \quad \int \frac{d\Omega}{h} \rho_{\mathrm{W}}(\Omega) A_{\mathrm{W}}(\Omega)$$

Recurrences and fluctuations





$$\vec{S} = \langle \vec{J} \rangle / (N/2) = (S_x, S_y, S_z)$$

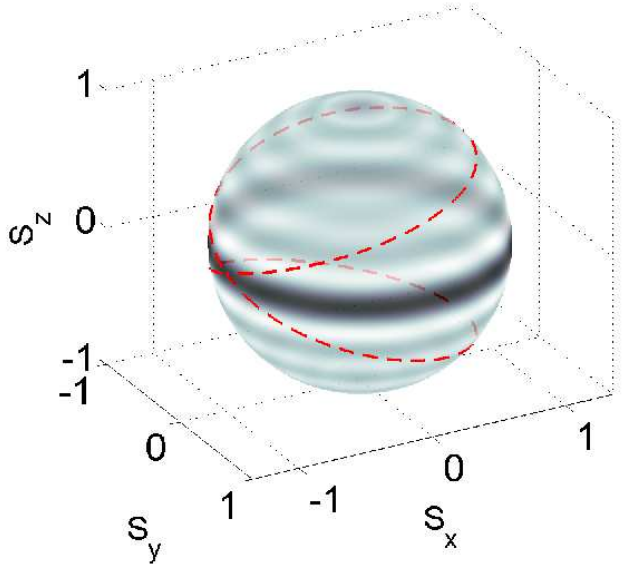
$$\text{OccupationDifference} = (N/2)\ S_z$$

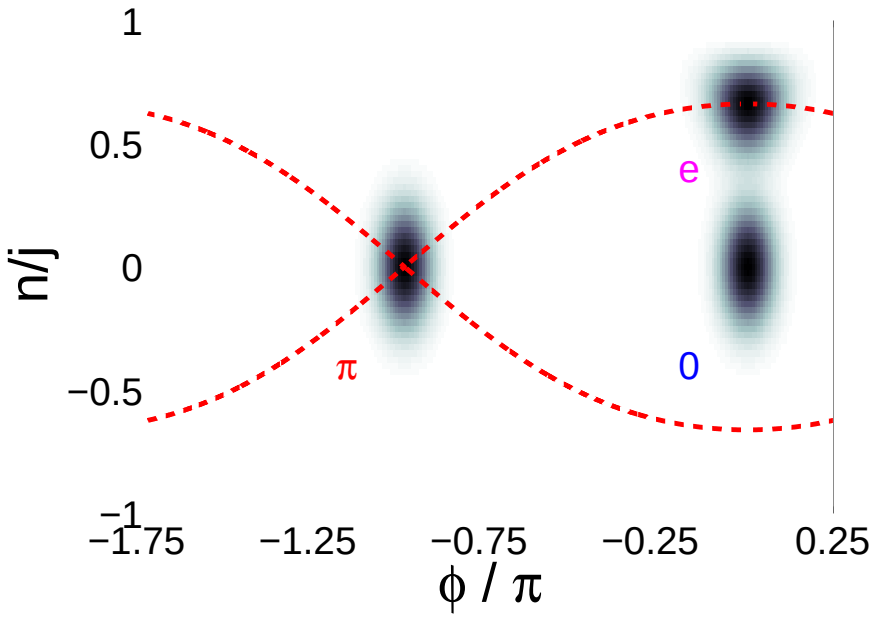
$$\text{OneBodyCoherence} = S_x^2 + S_y^2 + S_z^2$$

$$\text{FringeVisibility} = \left[S_x^2 + S_y^2\right]^{1/2}$$

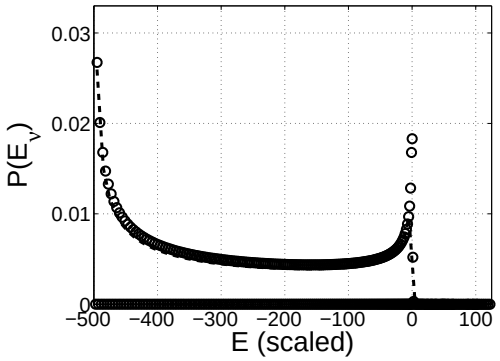
Spectral analysis of the fluctuations: dependence on u and on N, various preparations.

The preparations, and their LDOS $P(E)$



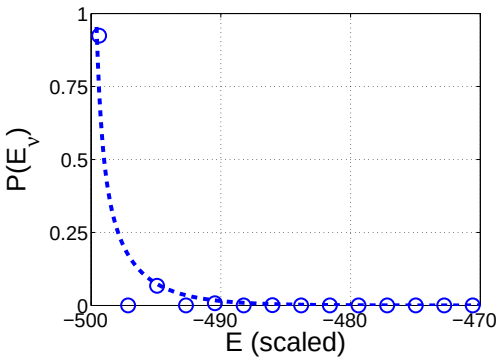


TwinFock preparation



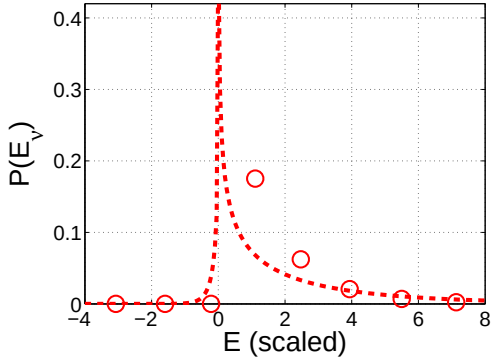
$$\sim \left[1-\left(\frac{2E}{NK}\right)^2\right]^{-1/2}$$

Zero preparation



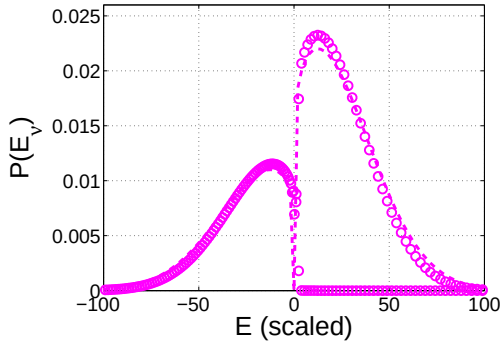
$$\sim \text{BesselI}\left[\frac{E-E_-}{NU}\right]$$

Pi preparation



$$\sim \text{BesselK}\left[\frac{E-E_x}{NU}\right]$$

Edge preparation



$$\sim \exp\left[-\frac{1}{\mathcal{N}}\left(\frac{E-E_x}{\omega_J}\right)^2\right]$$

The participation number M

$$M \equiv \left[\sum_{\nu} \mathrm{P}(E_\nu)^2\right]^{-1} = \text{number of participating levels in the LDOS}$$

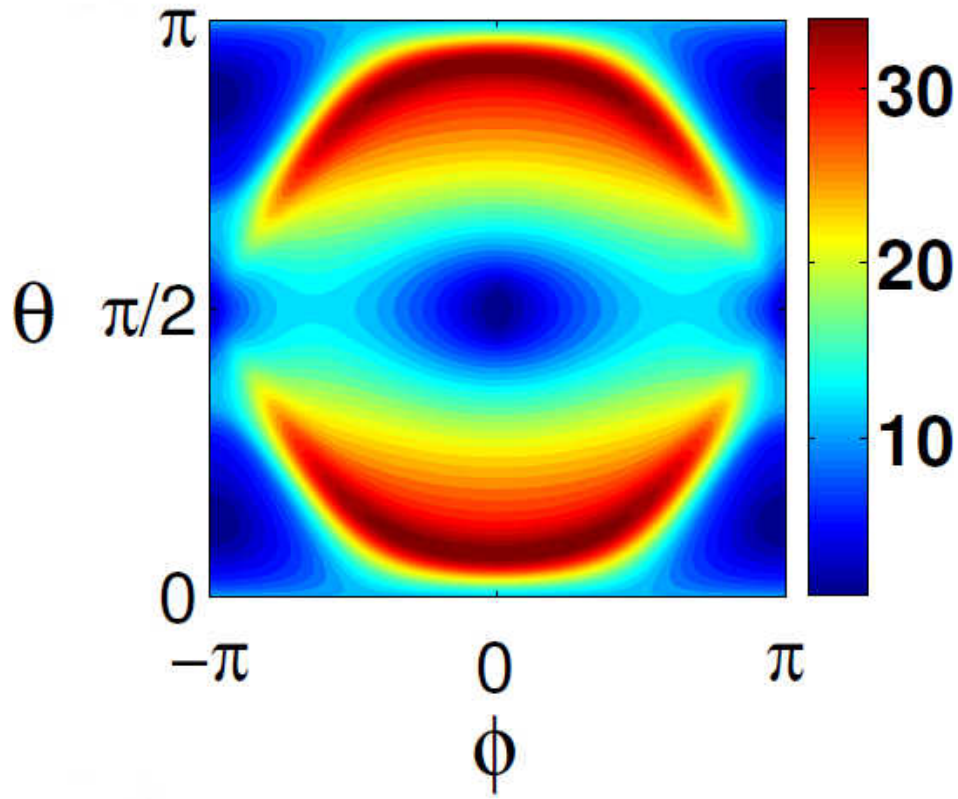
In the semiclassical analysis there is scaling with respect to $(u/N)^{1/2}$ which is [the width of the wavepacket] / [the width of the separatix]

$$M[\text{generic}] \approx \sqrt{N}$$

$$M[\text{Edge}] \approx \left[\log\left(\frac{N}{u}\right)\right]\sqrt{N}$$

$$M[\text{Pi}] \approx \left[\log\left(\frac{N}{u}\right)\right]\sqrt{u}$$

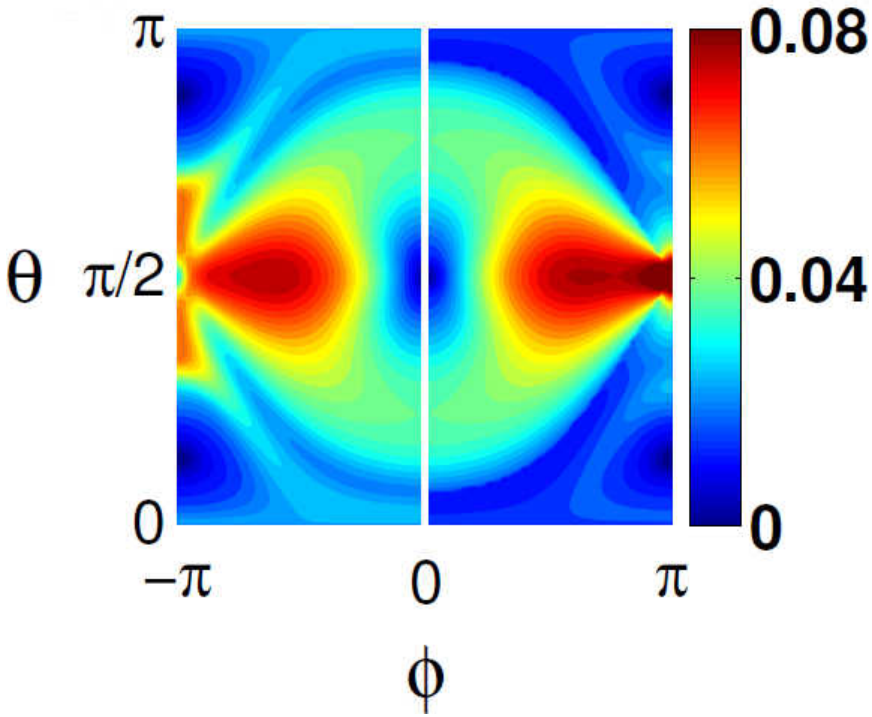
$$M[\text{Zero}] \approx \sqrt{u}$$



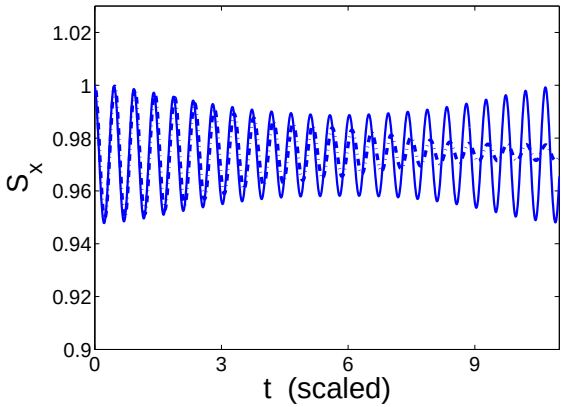
Analysis

* Spectral content: $\omega \sim \omega_{\rm osc}$
* Fluctuations: $\mathrm{Var}[S(t)]$

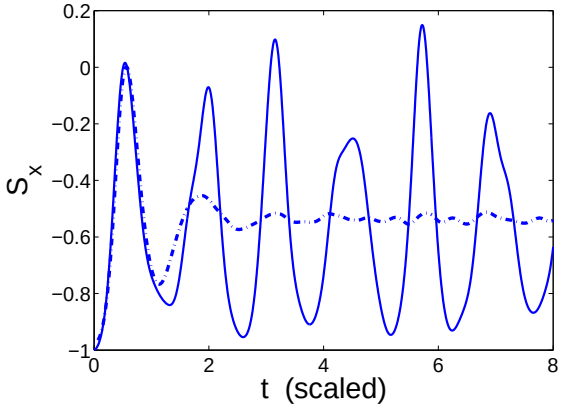
$$\mathrm{Var}[S] = \frac{1}{M}\int \tilde{C}_{\rm cl}(\omega)d\omega$$



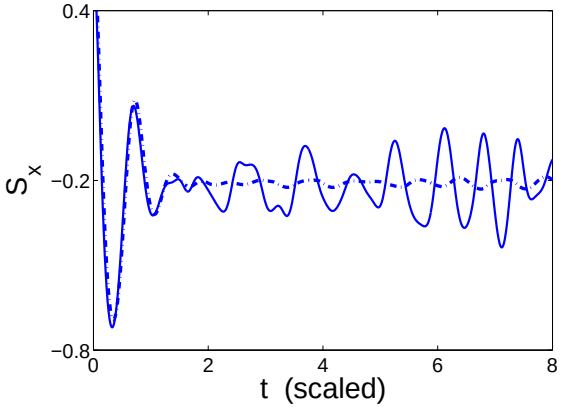
Zero prep



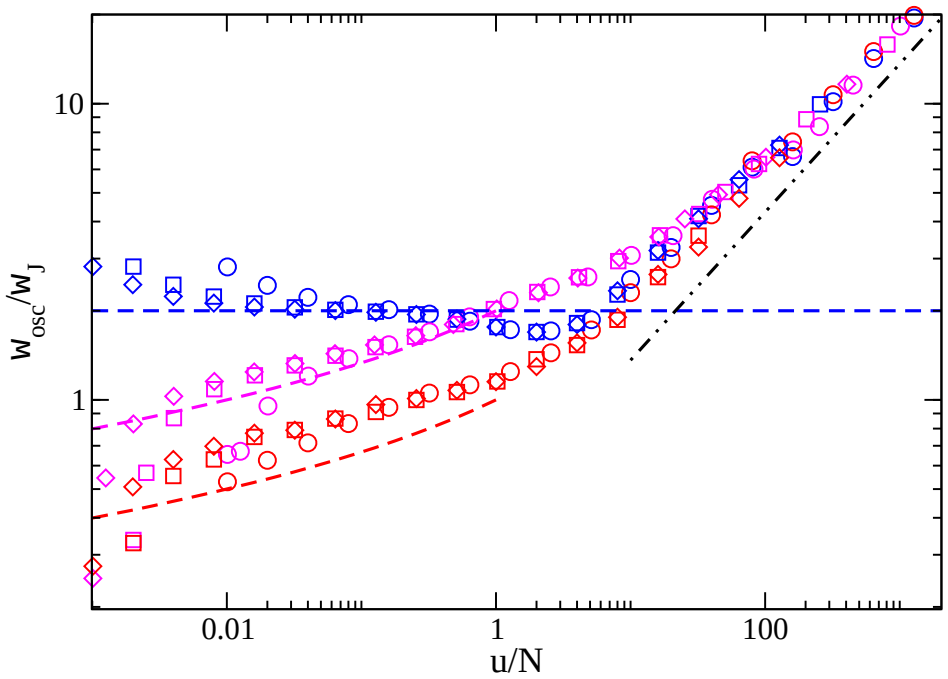
Pi prep



Edge prep



The spectral content of S_x



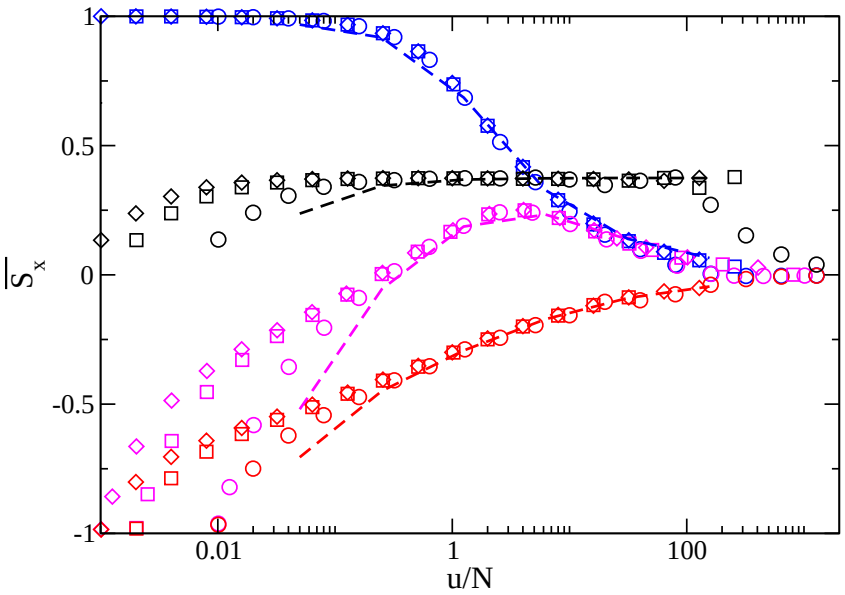
$$\omega_{\rm osc} \approx 2\omega_J \qquad \text{[Zero]}$$

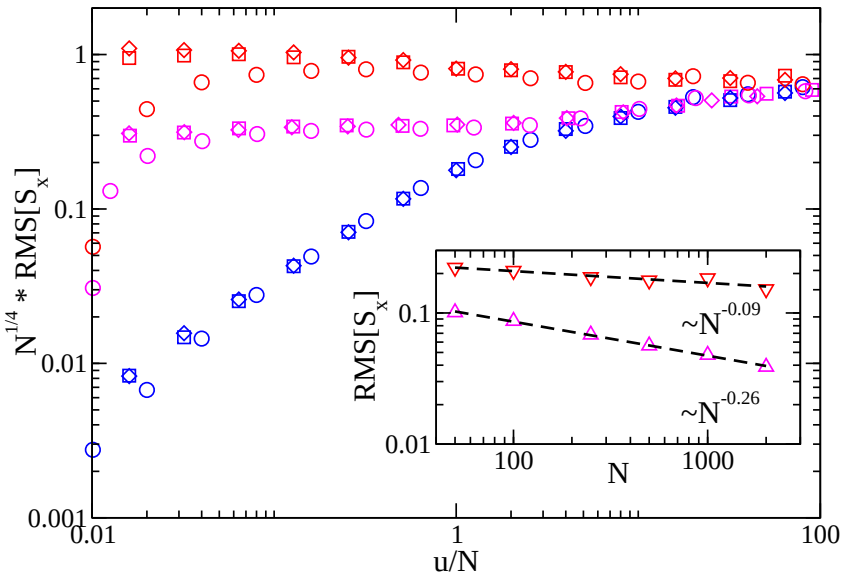
$$\omega_{\rm osc} \approx 1\times\left[\log\left(\frac{N}{u}\right)\right]^{-1} 2\omega_J \qquad \text{[Pi]}$$

$$\omega_{\rm osc} \approx 2\times\left[\log\left(\frac{N}{u}\right)\right]^{-1} 2\omega_J \qquad \text{[Edge]}$$

$$\omega_{\rm osc} \approx \left(\frac{u}{N}\right)^{1/2} 2\omega_J \qquad [u \gg N]$$

Fluctuations of S_x





$N = 100$ ○, $N = 500$ □, $N = 1000$ ◇.

Naive expectation: phase spreading diminishes coherence.
In the Fock regime $\langle S_x \rangle_\infty \approx 0$ [Leggett's "phase diffusion"]
In the Josephson regime $\langle S_x \rangle_\infty$ is determined by u/N.

$$\overline{S_x} \approx 1/3 \qquad \text{[TwinFock]}$$

$$\overline{S_x} \approx \exp[-(u/N)] \qquad \text{[Zero]}$$

$$\overline{S_x} \approx -1 - 4/\log\left[\tfrac{1}{32}(u/N)\right] \qquad \text{[Pi]}$$

$$\text{RMS}\left[\langle A \rangle_t\right] = \left[\frac{1}{M}\int \tilde{C}_{\text{cl}}(\omega)d\omega\right]^{1/2}$$

$$\text{RMS}\left[S_x(t)\right] \sim N^{-1/4} \qquad \text{[Edge]}$$

$$\text{RMS}\left[S_x(t)\right] \sim (\log(N))^{-1/2} \qquad \text{[Pi]}$$

TwinFock: Self induced coherence leading to $\overline{S_x} \approx 1/3$.
Zero: Coherence maintained if $u/N < 1$ (phase locking).
Pi: Fluctuations are suppressed by u.
Edge: Fluctuations are suppressed by N (classical limit).

Periodic / Erratic / Noisy driving

$$\mathcal{H} \;=\; U\hat{J}_z^2 \;-\; (K + f(t))\hat{J}_x$$

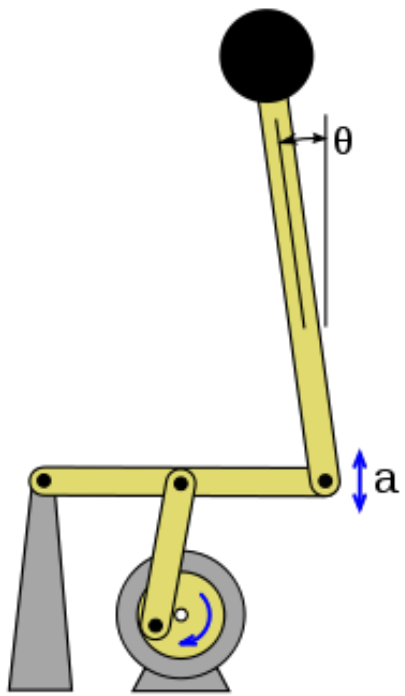
Periodic driving: We distinguish between nearly resonant driving ($\rightsquigarrow$ Chaos), and high frequency driving ($\rightsquigarrow$ Kapitza).

$$f(t) \;\propto\; \sin(\Omega t)$$

Erratic driving: deterministic but fluctuating $f(t)$. An experimentalist can repeat the experiment with the same realization, or produce other realizations, as desired.

Noisy driving: arise due to the interaction with a stationary source ("high temperature bath"). The realizations of $f(t)$ are not under experimental control. Nature is doing the averaging. ($\rightsquigarrow$ Zeno).

$$\overline{f(t)f(t')} \;=\; 2D\delta(t-t')$$



Kapitza pendulum

Kapitza physics in spherical phase-space

$$\frac{d\rho}{dt} = i[\mathcal{H} + f(t)W,\ \rho]$$

$$f(t) = \sin(\Omega t)$$

3 iterations and averaging over a cycle $\rightsquigarrow$

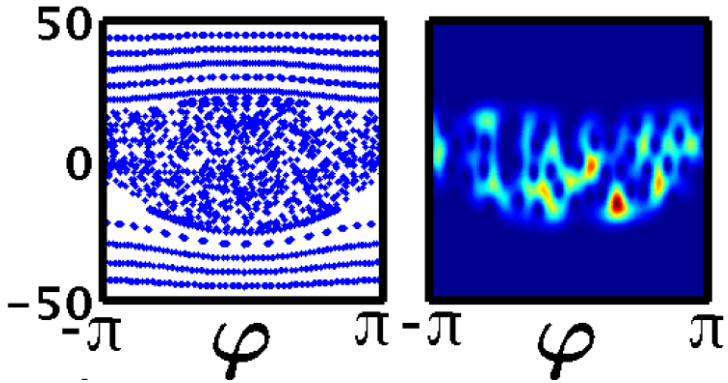
$$\frac{d\rho}{dt} = i[\mathcal{H} + V^{\rm eff},\ \rho]$$

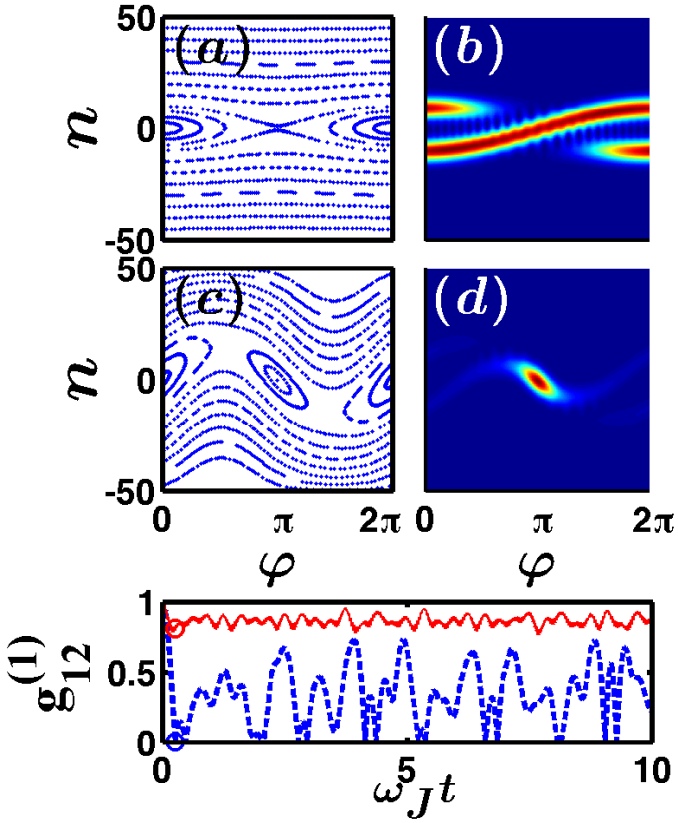
$$V^{\rm eff} = -\frac{1}{4\Omega^2}\ [W,[W,\mathcal{H}]]$$

Here:

$$W \propto J_x = (N/2)\sin\theta\ \cos\varphi$$

$$\rightsquigarrow \quad V^{\rm eff} \propto J_y^2 \sim \sin^2\varphi$$



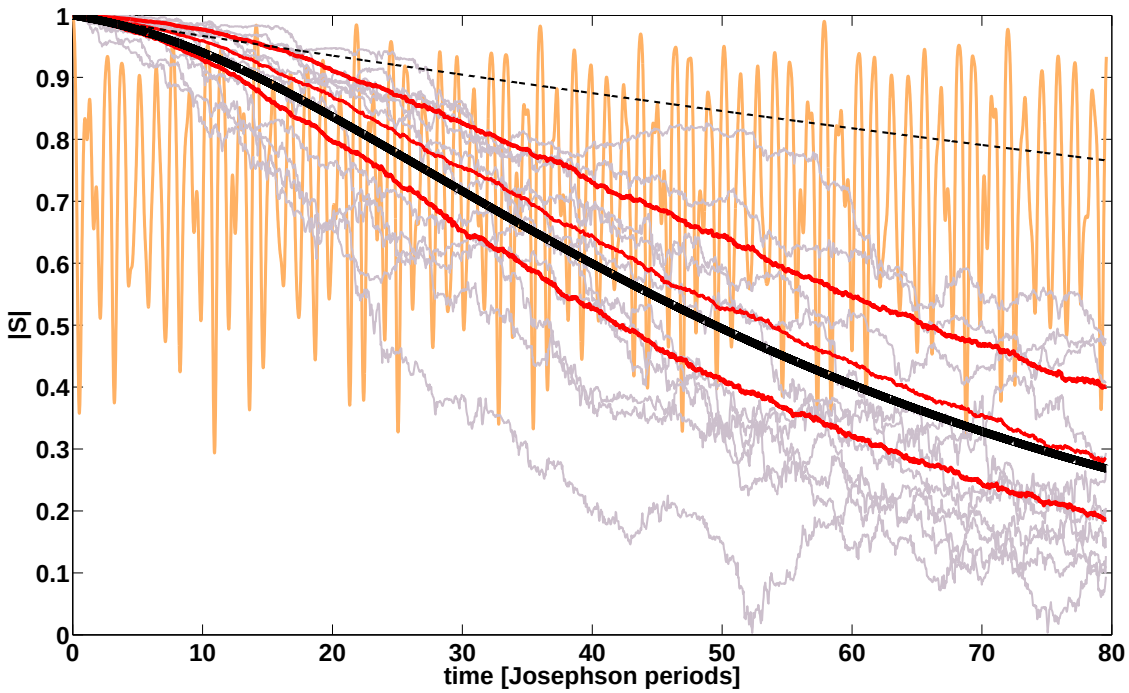


Erratic vs Noisy driving

$$\mathcal{H} = U\hat{J}_z^2 - (K + f(t))\hat{J}_x$$

$$\overline{f(t)f(t')} = 2D\delta(t-t')$$

Initial state: $S = (-1, 0, 0)$



Master Equation:

$$\frac{d\rho}{dt} = -i[\mathcal{H}_0, \rho] - D[J_x, [J_x, \rho]]$$

Quantum Zeno Effect:

[Khodorkovsky Kurizki Vardi 2008]

$$|S|_{\text{noise}} \approx \exp\left\{-\frac{1}{N}\frac{w_J^2}{D}t\right\}$$

Statistics for erratic driving:

$$|S|_{f(t)} \approx \exp\left\{-\frac{2}{N}\sinh^2(\Lambda)\right\}$$

$$|S|_{\text{median}} \approx \exp\left\{-\frac{2}{N}\sinh^2(\mu(t))\right\}$$

$$|S|_{\text{average}} \approx \exp\left\{-\frac{1}{N}\left[\mathrm{e}^{2\sigma(t)^2}\cosh(2\mu(t)) - 1\right]\right\}$$

Details of analysis (I)

Traditional approach to analyze the Quantum Zeno effect is based on an FGR picture.
Instead we use a semi-classical picture.

$$J_{\|} \;\mapsto\; [j(j+1)]^{1/2}\cos(r) \qquad \text{[Wigner-Weyl]}, \qquad r = \text{the radial coordinate} \quad r \equiv \theta \;\text{ if }\; J_{\|} \equiv J_z$$

For squeezing along orthogonal directions with $e^{\pm\Lambda}$

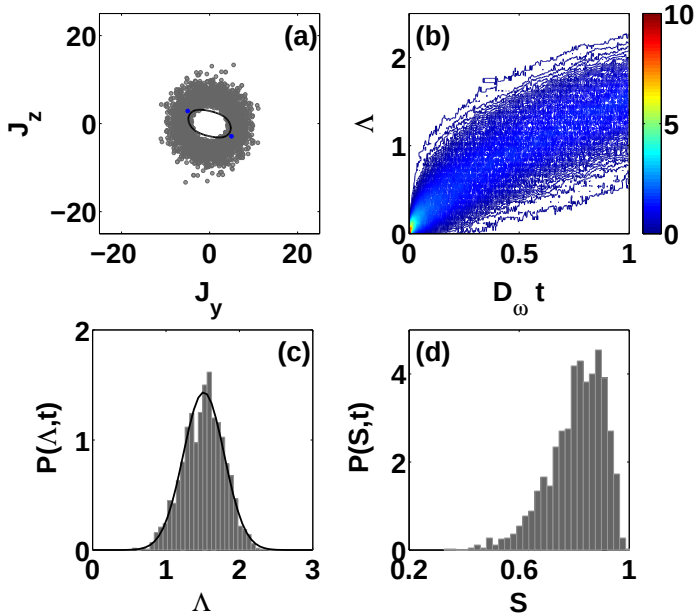
$$|S| \;=\; \left[1+\frac{2}{N}\right]^{1/2}\Big\langle \cos(r)\Big\rangle \;=\; \exp\left\{-\frac{1}{2}\left(\langle r^2\rangle - \frac{2}{N}\right)\right\} \;=\; \exp\left\{-\frac{2}{N}\sinh^2(\Lambda)\right\}$$

(a) For a pure squeezing scenario

$$|S| \;=\; \exp\left\{-\frac{2}{N}\cot^2(2\Theta)\,\sinh^2(w_J t)\right\}$$

$$w_J = \sqrt{(NU-K)K}, \quad \Theta = \tan^{-1}(w_J/K)$$

(b) For an erratic squeezing scenario:
one has to figure out the Λ statistics.
$\rightsquigarrow$ log-wide distribution.



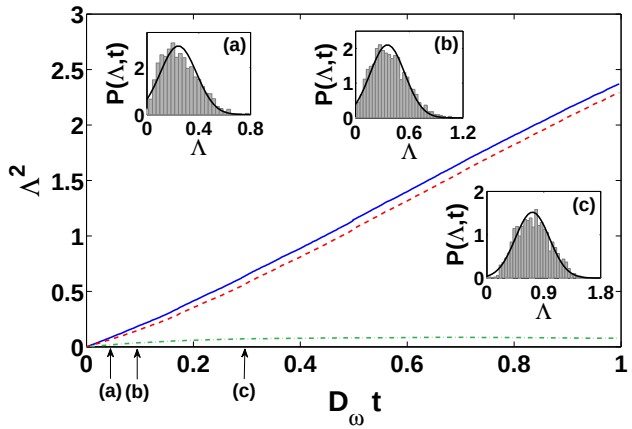
Details of analysis (II)

$$|S| \;=\; \left[1+\frac{2}{N}\right]^{1/2}\left\langle \cos(r)\right\rangle \;=\; \exp\left\{-\frac{1}{2}\left(\langle r^2\rangle - \frac{2}{N}\right)\right\} \;=\; \exp\left\{-\frac{2}{N}\sinh^2(\Lambda)\right\}$$

(c) For a noisy squeezing scenario there are two options for analysis.

Via Λ statistics:

$$|S|_{\text{average}} \quad\approx\quad \exp\left\{-\frac{1}{N}\left[e^{2\sigma(t)^2}\cosh(2\mu(t))-1\right]\right\}$$



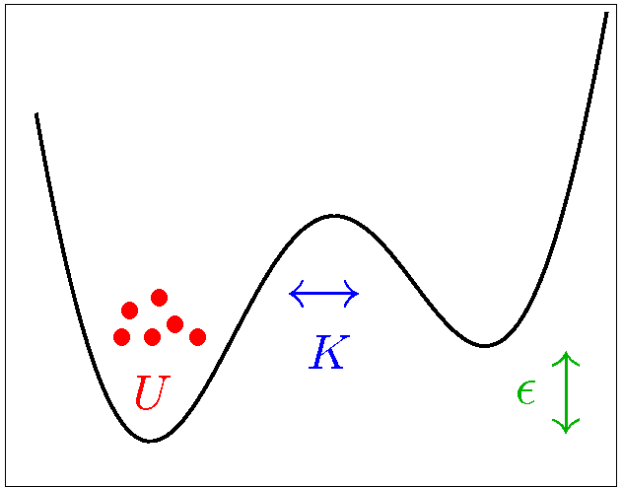
Via convolution of small steps:

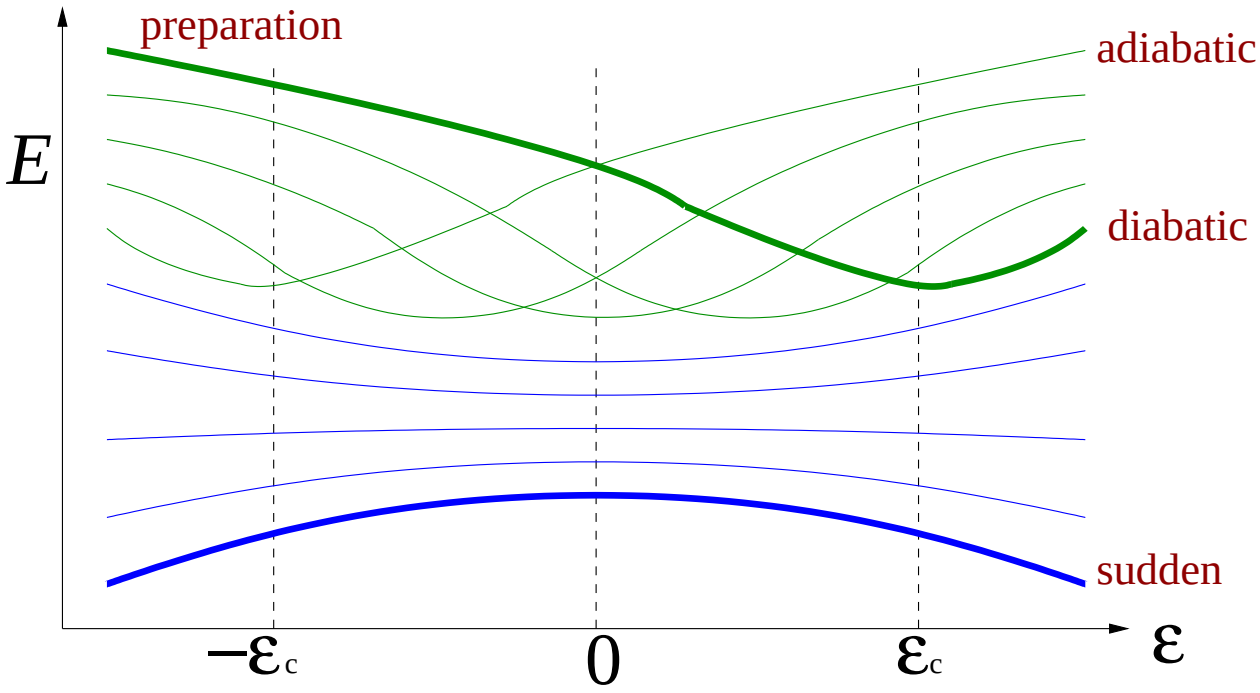
$$D_w \;=\; [\cot^2(2\Theta)]\;\frac{w_J^2}{8D}$$ log-radial diffusion coefficient in multiplicative process

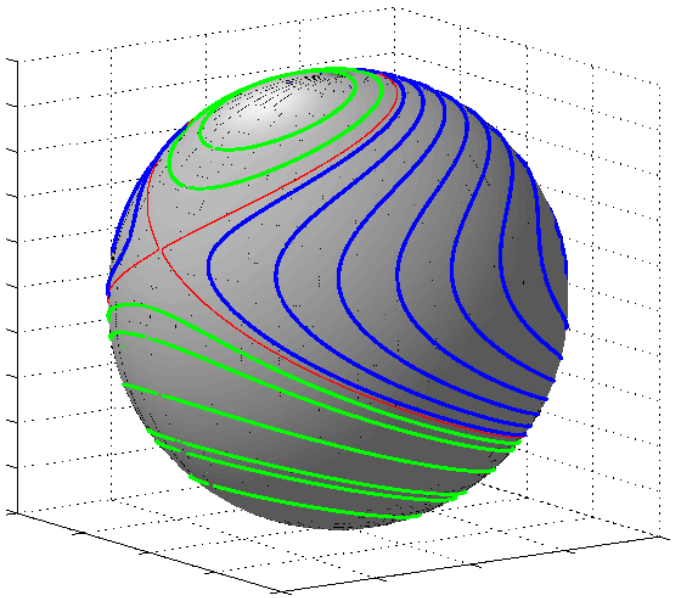
$$t_D \;=\; \frac{1}{2D}$$ time to randomize direction due to transverse diffusion

$$|S|_{\text{average}} \;=\; \exp\left\{-\frac{1}{N}\left[\exp\left(8D_w t\right)-1\right]\right\}$$ should be contrasted with $\exp\left\{-\frac{1}{N}8D_w t\right\}$

The many body Landau-Zener transition



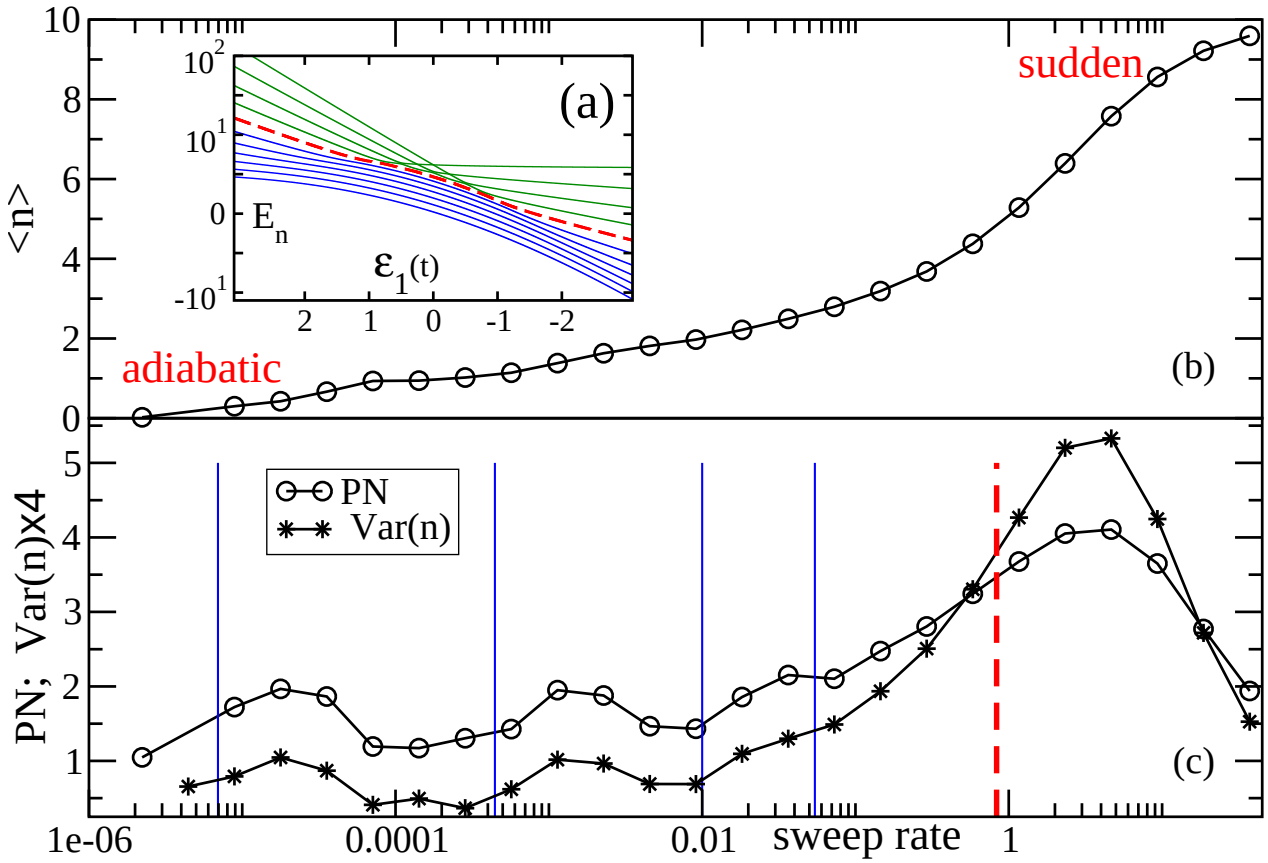




Dynamical scenarios:

adiabatic/diabatic/sudden

Occupation Statistics



Adiabtic-diabatic (quantum) crossover

Diabatic-sudden (semiclassical) crossover

Summary

- Semiclassical analysis (WKB and Wigner-Weyl are beyond MFT)
- The dependence of the participation number M on u and on N.
- Fluctuations and recurrences, study of $\omega_{\rm osc}$ and $\overline{S(t)}$ and $\mathrm{Var}[S(t)]$
- Noise driven dimer: Improved Quantum Zeno effect analysis for $\overline{S(t)}$
- Erratic driving: analysis of the statistics of $|S(t)|$ - challenging the system-bath paradigm
- Occupation statistics in a time dependent Landau-Zener scenario: identification of the adiabatic / diabatic / sudden crossovers.