One body decoherence: fluctuations, recurrences and statistics

Doron Cohen, Ben-Gurion University

Maya Chuchem (BGU/Phys) [1,3,4]

Amichay Vardi (BGU/Chem) [1,2,3,5]

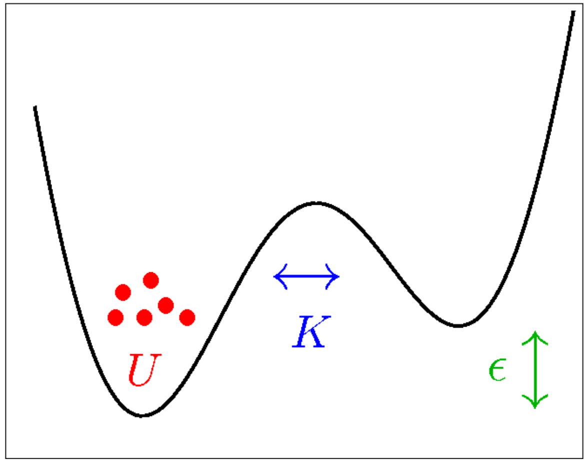
Erez Boukobza (BGU/Chem) [1,2,5]

Christine Khripkov (BGU/Chem) [6]

Tsampikos Kottos (Wesleyan) [3,4]

Katrina Smith-Mannschott (Wesleyan) [3,4]

Moritz Hiller (Freiburg) [3,4]



\$BSF, \$DIP, \$FOR760

[1] Dynamics & fluctuations (PRL 2009)

[2] Dynamics & fluctuations (PRA 2009)

[3] Dynamics & fluctuations (PRA 2010)

[4] Sweep operation, Landau-Zener (PRL 2009)

[5] Periodic driving, Chaos (PRL 2010)

[6] Erratic vs Noisy driving, Zeno (2011)

The Bose-Hubbard Hamiltonian (BHH) for a dimer

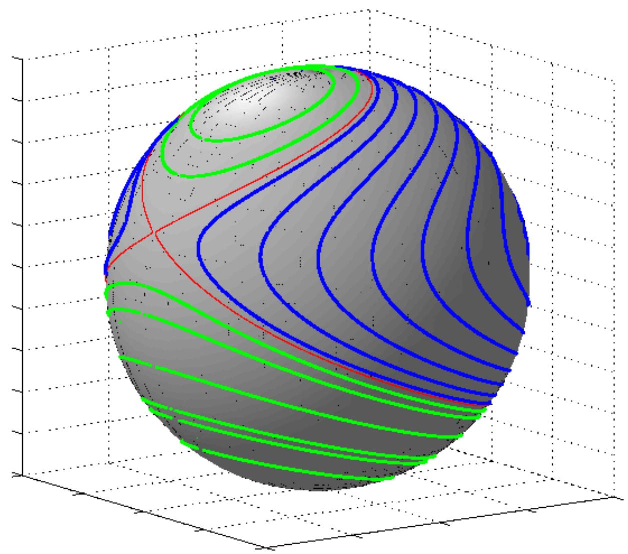
$$\mathcal{H} = \sum_{i=1,2} \left[\mathcal{E}_i \hat{n}_i + \frac{U}{2}\hat{n}_i(\hat{n}_i - 1)\right] - \frac{K}{2}(\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2)$$

K = hopping
U = interaction
$\mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1$ = bias

N particles in a double well is like spin $j = N/2$ system

$$u \equiv \frac{NU}{K}, \qquad \varepsilon \equiv \frac{\mathcal{E}}{K}$$

$$\mathcal{H} = -\mathcal{E}\hat{J}_z + U\hat{J}_z^2 - K\hat{J}_x$$



Similar to the Josephson Hamiltonian

$$\mathcal{H}(\hat{n}, \varphi) = U(\hat{n} - \epsilon)^2 - \frac{1}{2}KN\cos(\varphi)$$

$\hat{n} = \hat{J}_z =$ occupation difference

$\varphi =$ conjugate phase

Assuming $u > 1$ and $|\varepsilon| < \varepsilon_c$

Sea, Islands, Separatrix

Rabi regime: $u < 1$ (no islands)
Josephson regime: $1 < u < N^2$ (sea, islands, separatrix)
Fock regime: $u > N^2$ (empty sea)

$$\varepsilon_c = \left(u^{2/3} - 1\right)^{3/2}$$

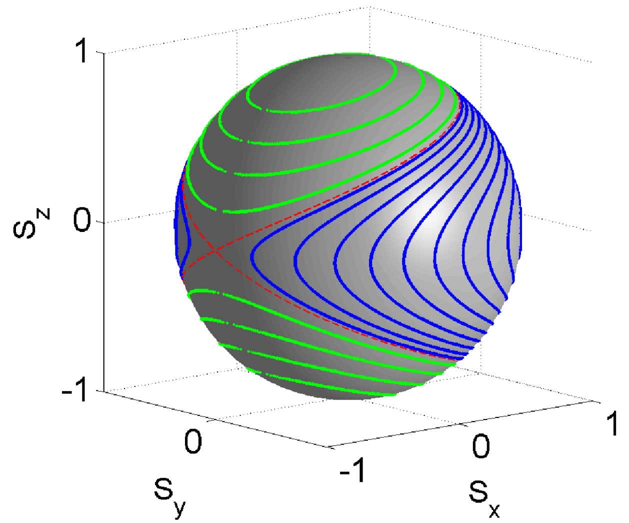
WKB quantization (Josephson regime)

$$h \;=\; \text{Planck cell area in steradians} \;=\; \frac{4\pi}{N+1}$$

$$A(E_\nu) \;=\; \left(\frac{1}{2}+\nu\right) h \qquad \nu = 0, 1, 2, 3, \ldots$$

$$\omega(E) \;\equiv\; \frac{dE}{d\nu} \;=\; \left[\frac{1}{h}A'(E)\right]^{-1}$$

Eigenstates $|E_\nu\rangle$ are like strips along contour lines of $\mathcal{H}$.

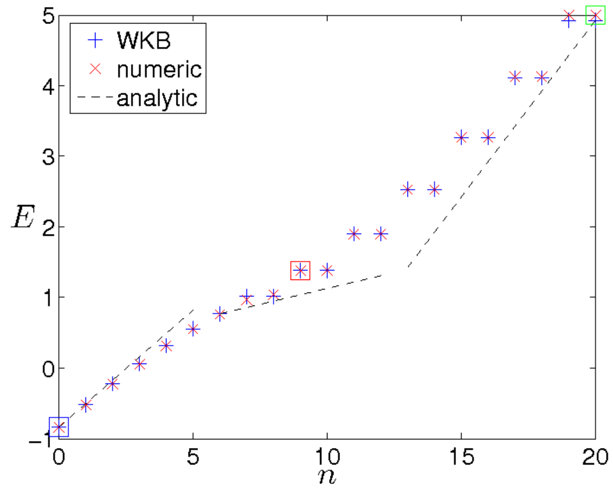


$$\omega_K \;\approx\; K \;=\; \text{Rabi Frequency}$$

$$\omega_J \;\approx\; \sqrt{NUK} \;=\; \text{Josephson Frequency}$$

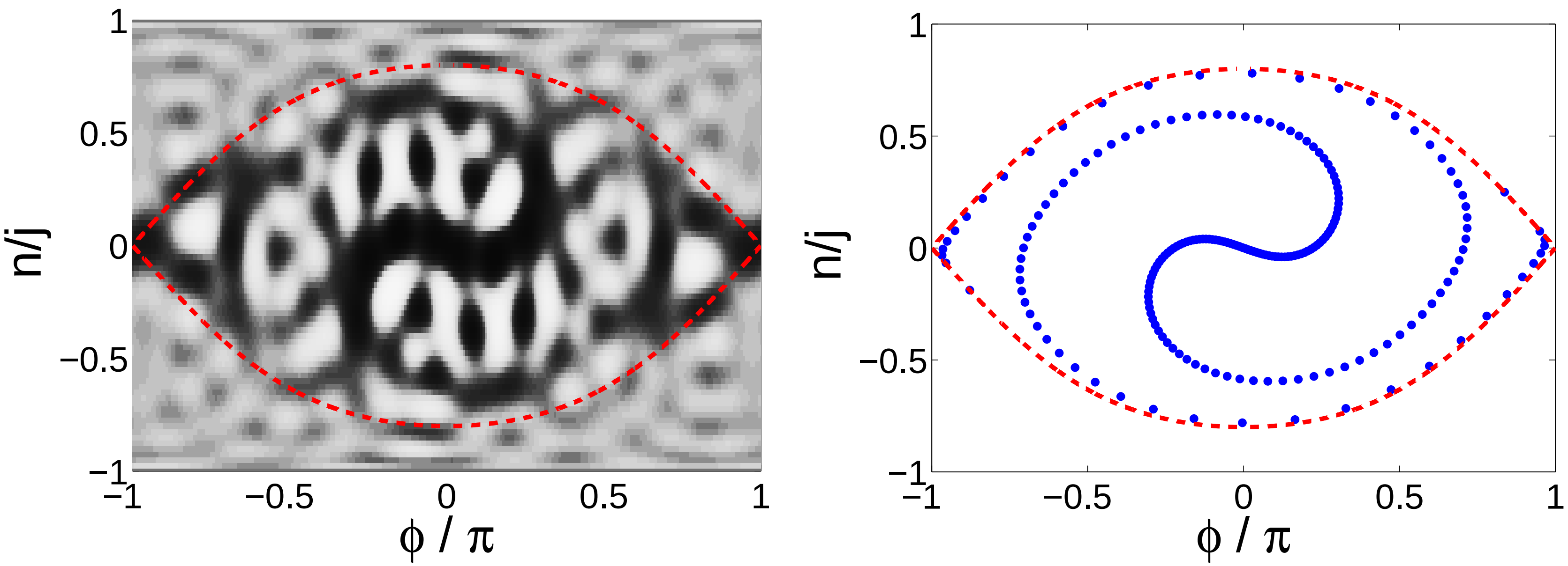
$$\omega_+ \;\approx\; NU \;=\; \text{Island Frequency}$$

$$\omega_{\mathrm{x}} \;\approx\; \left[\log\left(\frac{N^2}{u}\right)\right]^{-1} \omega_J$$



Wavepacket dynamics

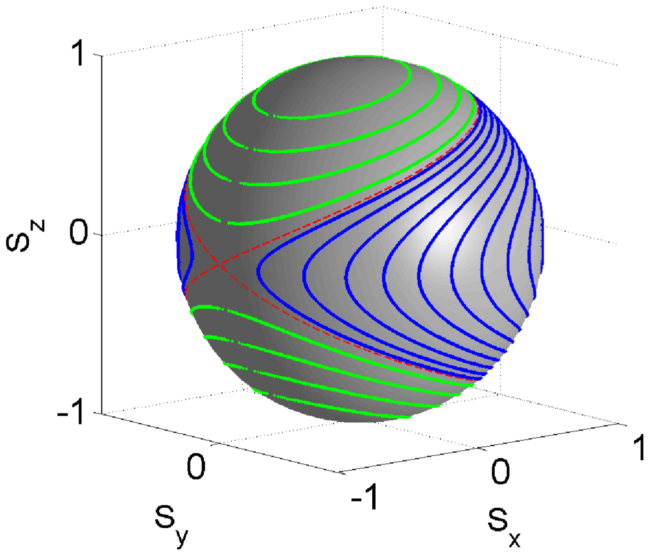
MeanField theory (GPE) = classical evolution of a point in phase space
SemiClassical theory = classical evolution of a distribution in phase space
Quantum theory = recurrences, fluctuations (WKB is very good)

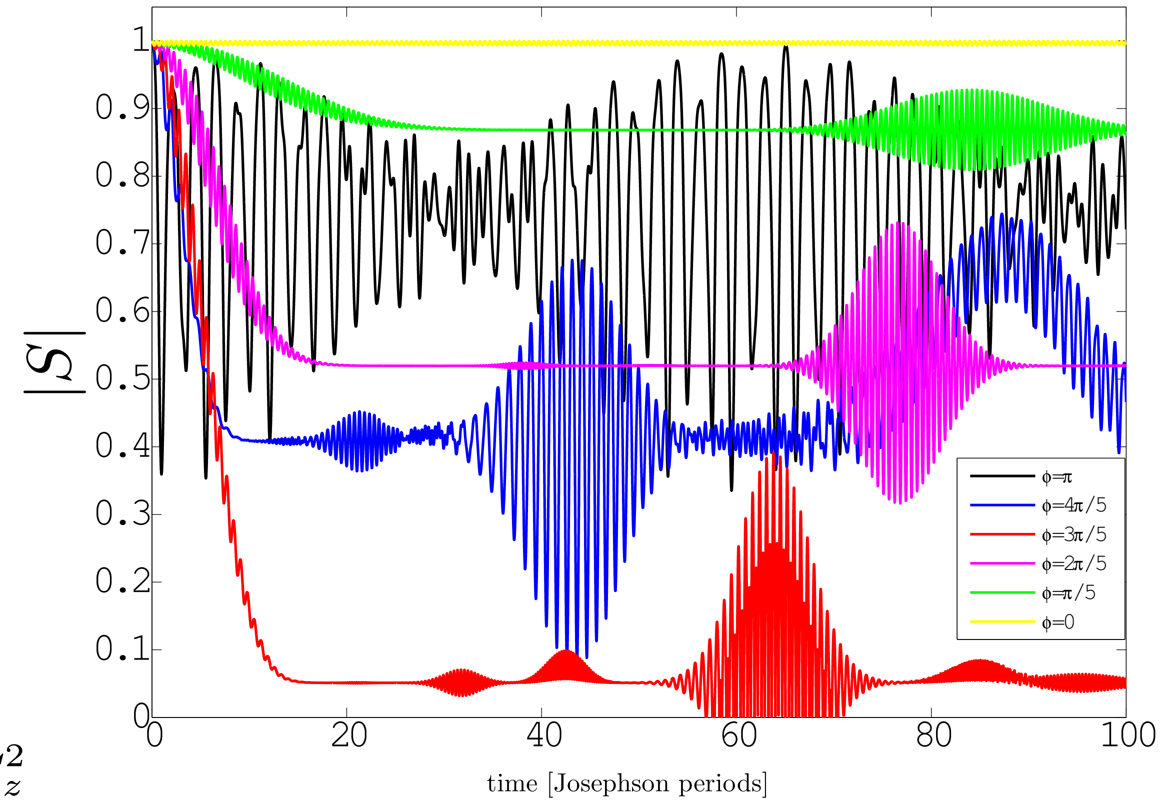


Any operator $\hat{A}$ can be presented by the phase-space function $A_{\rm W}(\Omega)$

$$\langle \hat{A} \rangle \;=\; \mathrm{trace}[\hat{\rho}\;\hat{A}] \;=\; \int \frac{d\Omega}{h}\rho_{\rm W}(\Omega)A_{\rm W}(\Omega)$$

Recurrences and fluctuations





$$\vec{\boldsymbol{S}} = \langle \vec{\boldsymbol{J}} \rangle / (N/2) = (S_x, S_y, S_z)$$

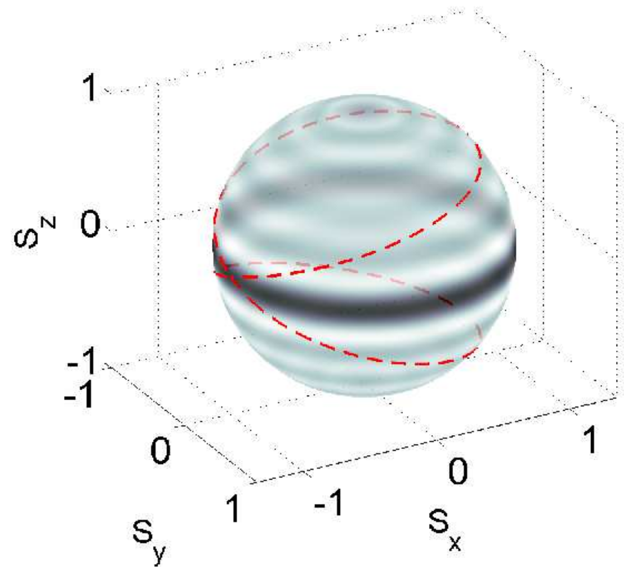
$$\text{OccupationDifference} = (N/2)\ S_z$$

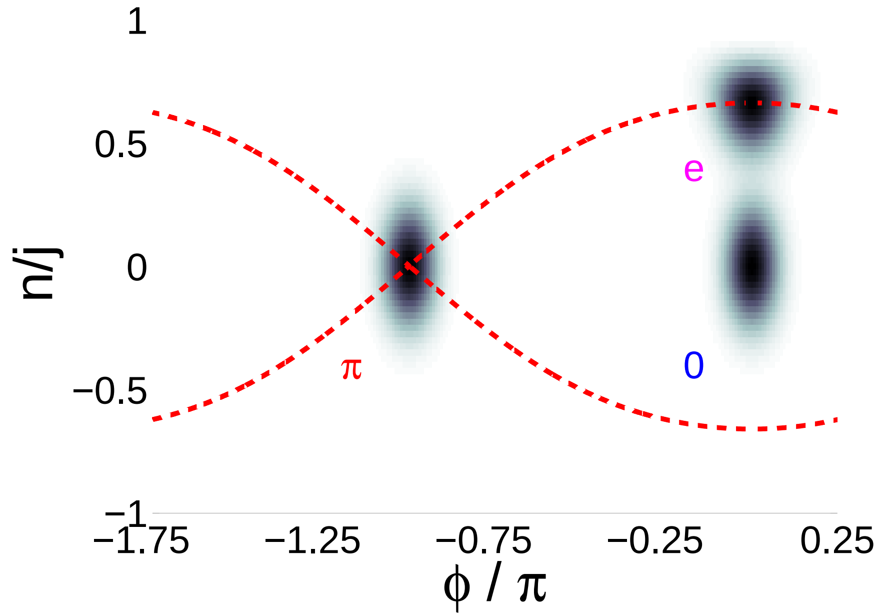
$$\text{OneBodyCoherence} = S_x^2 + S_y^2 + S_z^2$$

$$\text{FringeVisibility} = \left[S_x^2 + S_y^2\right]^{1/2}$$

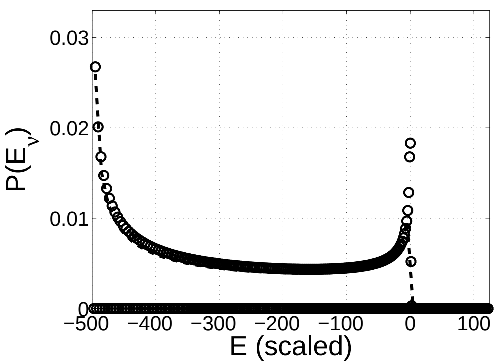
Spectral analysis of the fluctuations: dependence on u and on N, various preparations.

The preparations, and their LDOS $P(E)$



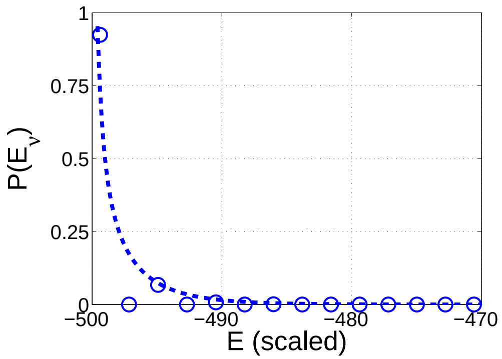


TwinFock preparation



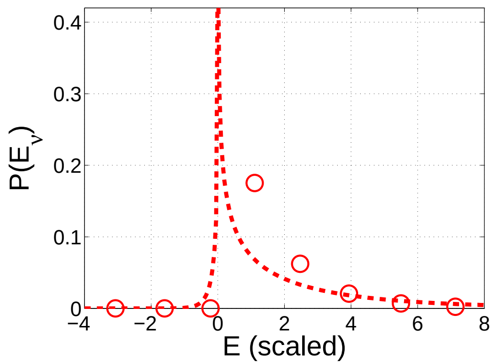
$$\sim \left[1-\left(\frac{2E}{NK}\right)^2\right]^{-1/2}$$

Zero preparation



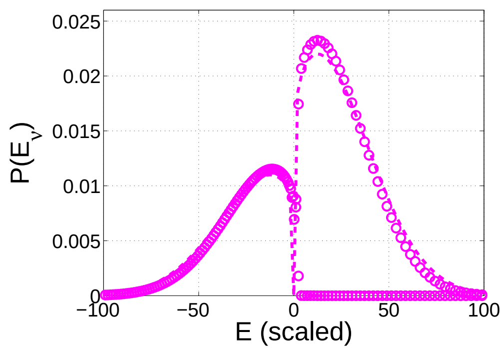
$$\sim \text{BesselI}\left[\frac{E-E_-}{NU}\right]$$

Pi preparation



$$\sim \text{BesselK}\left[\frac{E-E_x}{NU}\right]$$

Edge preparation



$$\sim \exp\left[-\frac{1}{\mathcal{N}}\left(\frac{E-E_x}{\omega_J}\right)^2\right]$$

The participation number M

$$M \equiv \left[\sum_{\nu} \mathrm{P}(E_\nu)^2\right]^{-1} = \text{number of participating levels in the LDOS}$$

In the semiclassical analysis there is scaling with respect to $(u/N)^{1/2}$ which is [the width of the wavepacket] / [the width of the separatix]

$$M \approx \text{ClassicalPrefactor} \times N \qquad \text{[TwinFock preparation]}$$

$$M \approx \sqrt{N} \qquad \text{[CoherentState] (for } (u/N)^{1/2} > 1\text{)}$$

$$M \approx \left[\log\left(\frac{N}{u}\right)\right]\sqrt{N} \qquad \text{[Edge]} \rightsquigarrow \text{(easy to get the classical limit)}$$

$$M \approx \left[\log\left(\frac{N}{u}\right)\right]\sqrt{u} \qquad \text{[Pi]} \rightsquigarrow \text{(quasi periodic large fluctuations)}$$

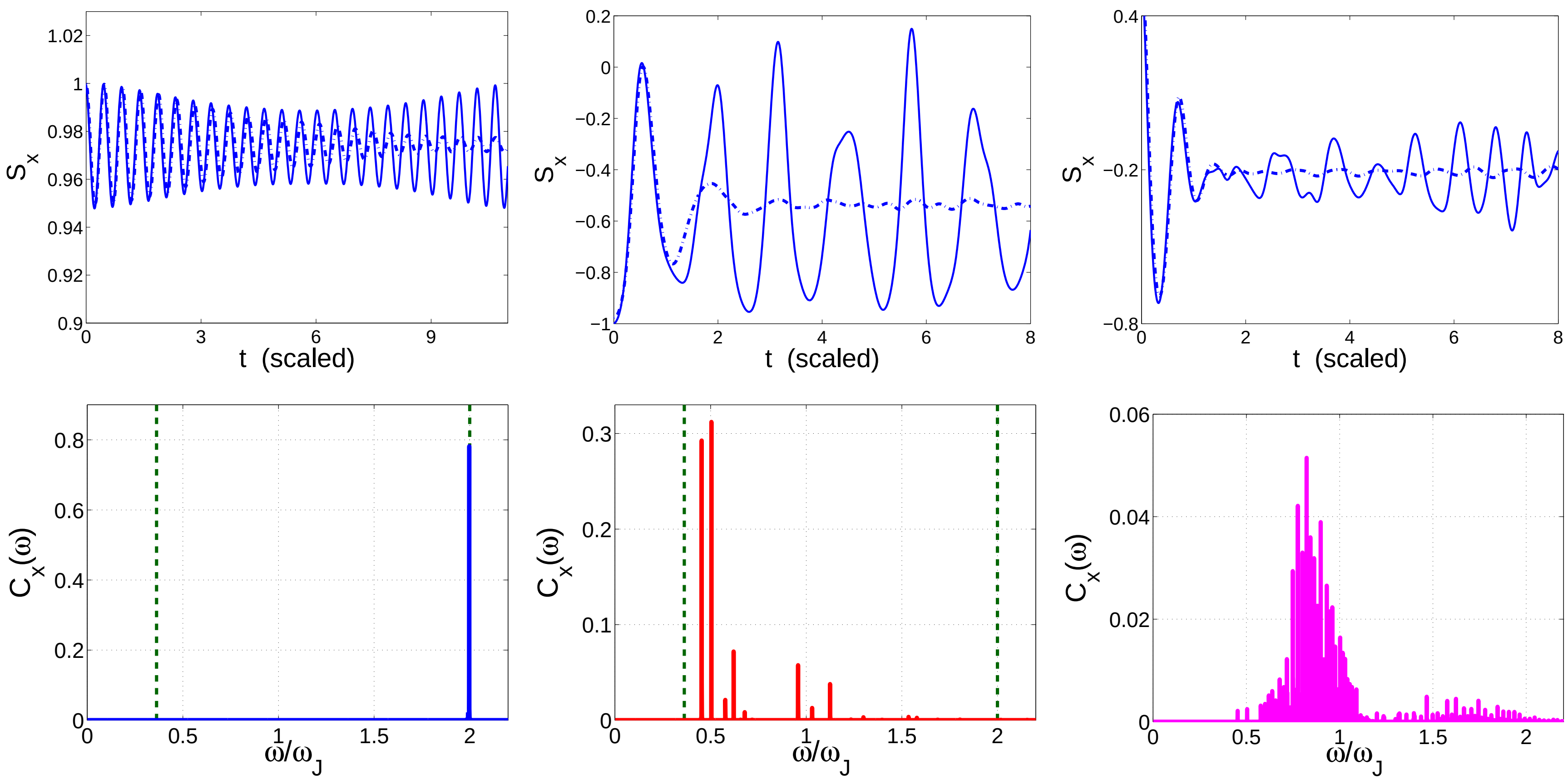
$$M \approx \sqrt{u} \qquad \text{[Zero]} \rightsquigarrow \text{(locking)}$$

Analysis

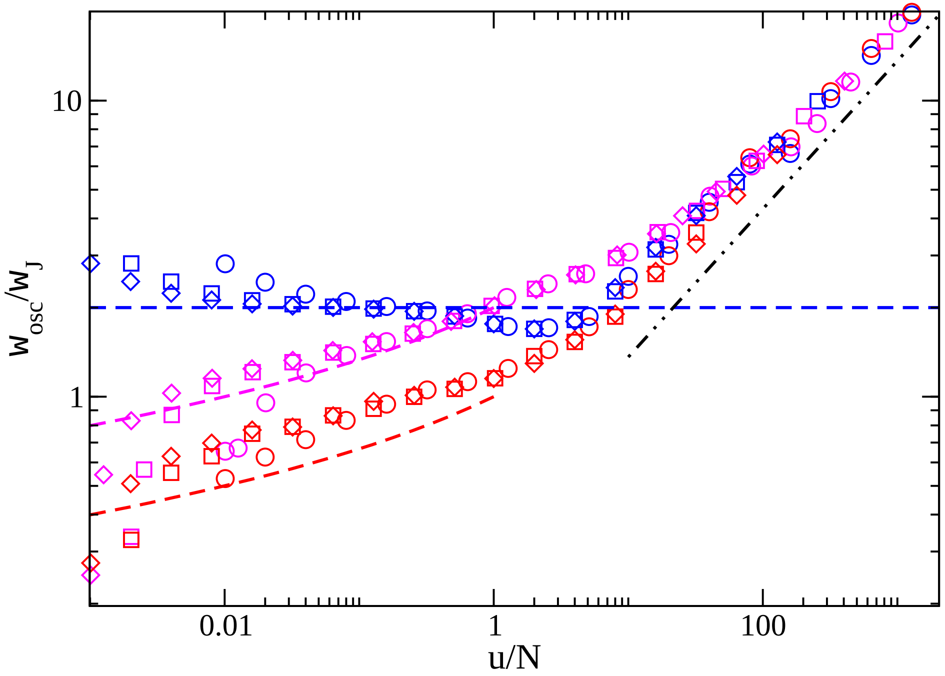
* Spectral content: characteristic frequency ω_{osc}

* Fluctuations: $\overline{S(t)}$ and $\mathrm{RMS}[S(t)]$

Zero prep | Pi prep | Edge prep



The spectral content of S_x



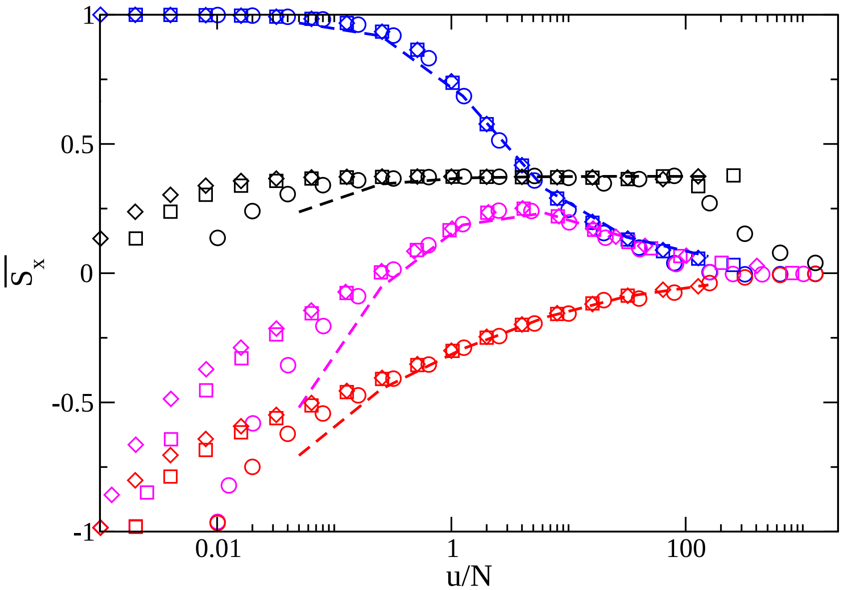
$$\omega_{\mathrm{osc}} \approx 2\omega_J \qquad \text{[Zero]}$$

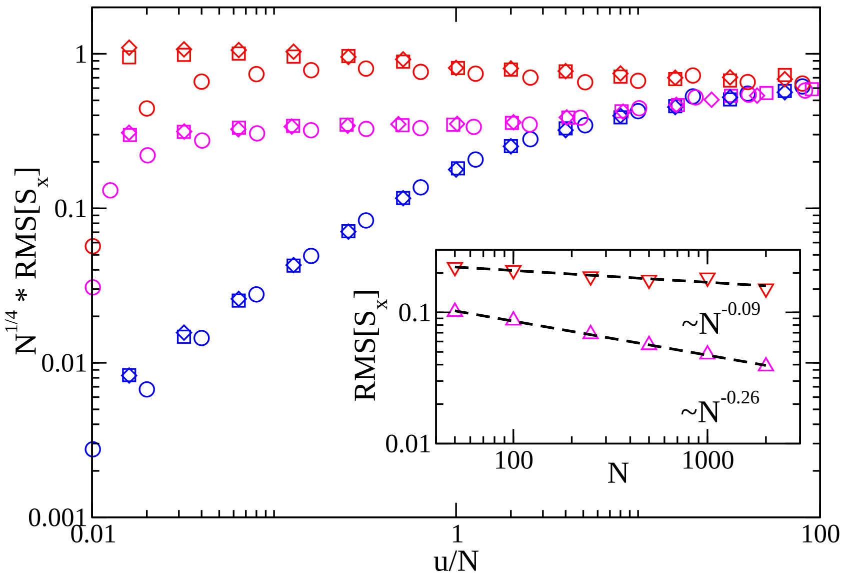
$$\omega_{\mathrm{osc}} \approx 1\times \left[\log\left(\frac{N}{u}\right)\right]^{-1} 2\omega_J \qquad \text{[Pi]}$$

$$\omega_{\mathrm{osc}} \approx 2\times \left[\log\left(\frac{N}{u}\right)\right]^{-1} 2\omega_J \qquad \text{[Edge]}$$

$$\omega_{\mathrm{osc}} \approx \left(\frac{u}{N}\right)^{1/2} 2\omega_J \qquad [u \gg N]$$

Fluctuations of S_x





$N = 100$ $\circ$, $N = 500$ $\square$, $N = 1000$ $\diamond$.

Naive expectation: phase spreading diminishes coherence.
In the Fock regime $\langle S_x \rangle_\infty \approx 0$ [Leggett's "phase diffusion"]
In the Josephson regime $\langle S_x \rangle_\infty$ is determined by u/N.

$$\overline{S_x} \approx 1/3 \qquad \text{[TwinFock]}$$

$$\overline{S_x} \approx \exp[-(u/N)] \qquad \text{[Zero]}$$

$$\overline{S_x} \approx -1 - 4/\log\left[\frac{1}{32}(u/N)\right] \qquad \text{[Pi]}$$

$$\text{RMS}\left[\langle A \rangle_t\right] = \left[\frac{1}{M}\int \tilde{C}_{\text{cl}}(\omega)d\omega\right]^{1/2}$$

$$\text{RMS}\left[S_x(t)\right] \sim N^{-1/4} \qquad \text{[Edge]}$$

$$\text{RMS}\left[S_x(t)\right] \sim (\log(N))^{-1/2} \qquad \text{[Pi]}$$

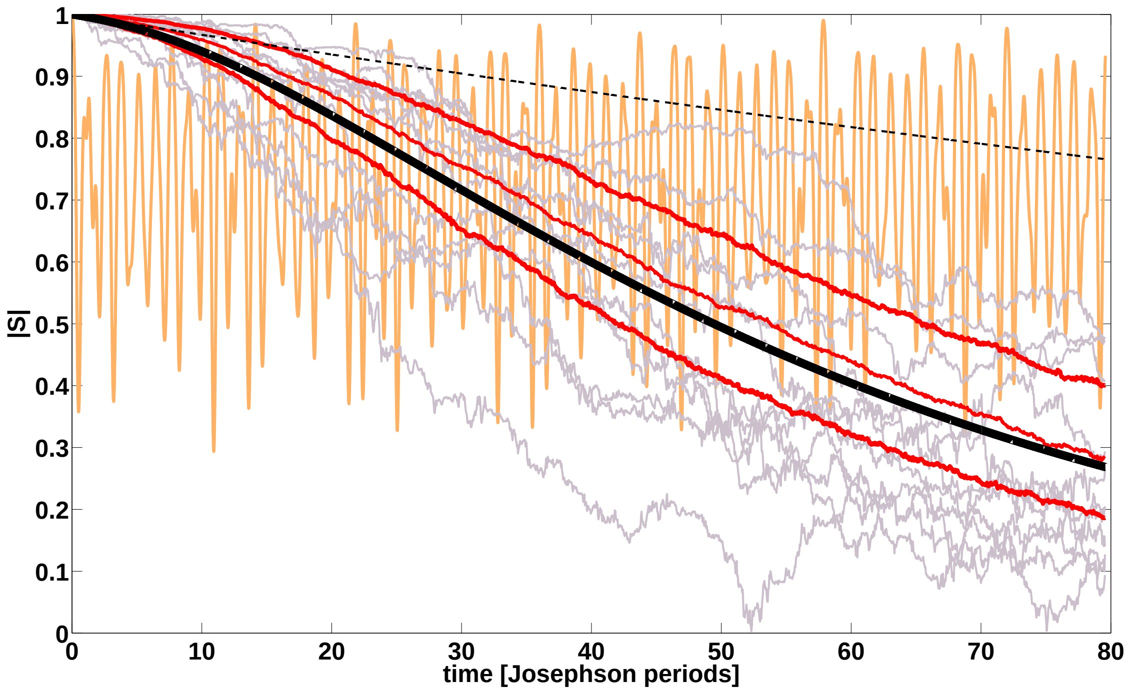
TwinFock: Self induced coherence leading to $\overline{S_x} \approx 1/3$.
Zero: Coherence maintained if $u/N < 1$ (phase locking).
Pi: Fluctuations are suppressed by u.
Edge: Fluctuations are suppressed by N (classical limit).

Erratic vs Noisy driving

$$\mathcal{H} = U\hat{J}_z^2 - (K + f(t))\hat{J}_x$$

$$\overline{f(t)f(t')} = 2D\delta(t-t')$$

Initial state: $S = (-1, 0, 0)$



Master Equation:

$$\frac{d\rho}{dt} = -i[\mathcal{H}_0, \rho] - D[J_x, [J_x, \rho]]$$

Quantum Zeno Effect:

[Khodorkovsky Kurizki Vardi 2008]

$$|S|_{\text{noise}} \approx \exp\left\{-\frac{1}{N}\frac{w_J^2}{D}t\right\}$$

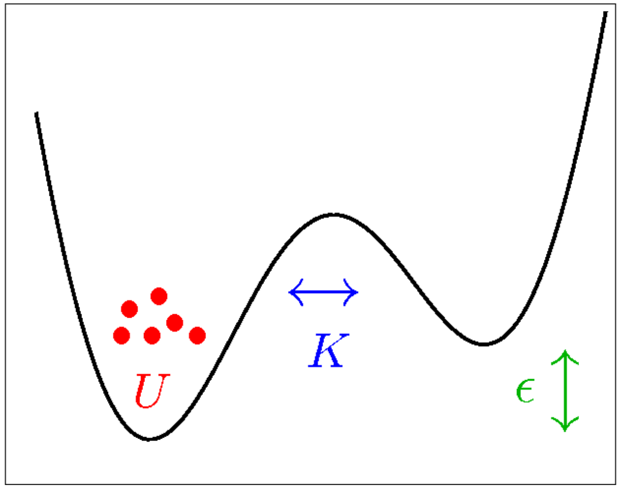
Statistics for erratic driving:

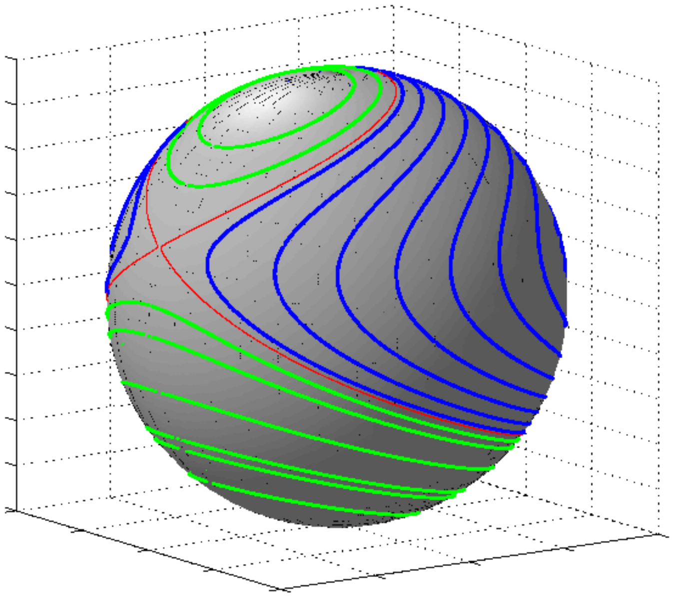
$$|S|_{f(t)} \approx \exp\left\{-\frac{2}{N}\sinh^2(\Lambda)\right\}$$

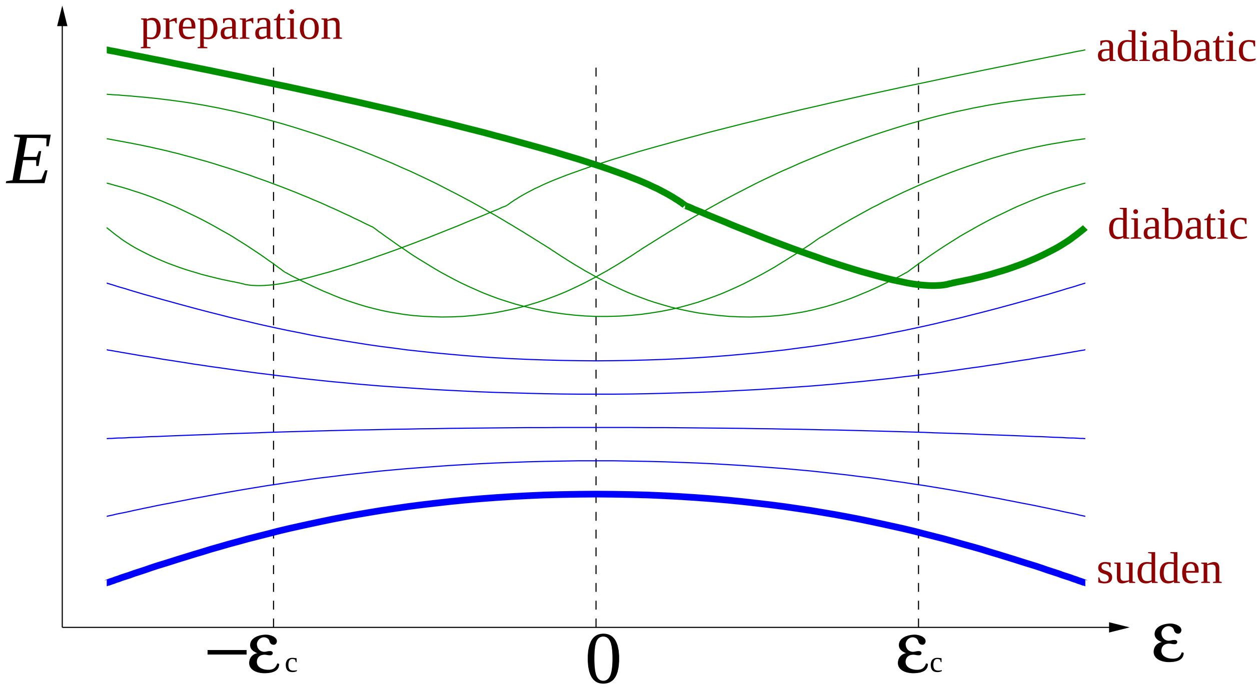
$$|S|_{\text{median}} \approx \exp\left\{-\frac{2}{N}\sinh^2(\mu(t))\right\}$$

$$|S|_{\text{average}} \approx \exp\left\{-\frac{1}{N}\left[e^{2\sigma(t)^2}\cosh(2\mu(t)) - 1\right]\right\}$$

The many body Landau-Zener transition

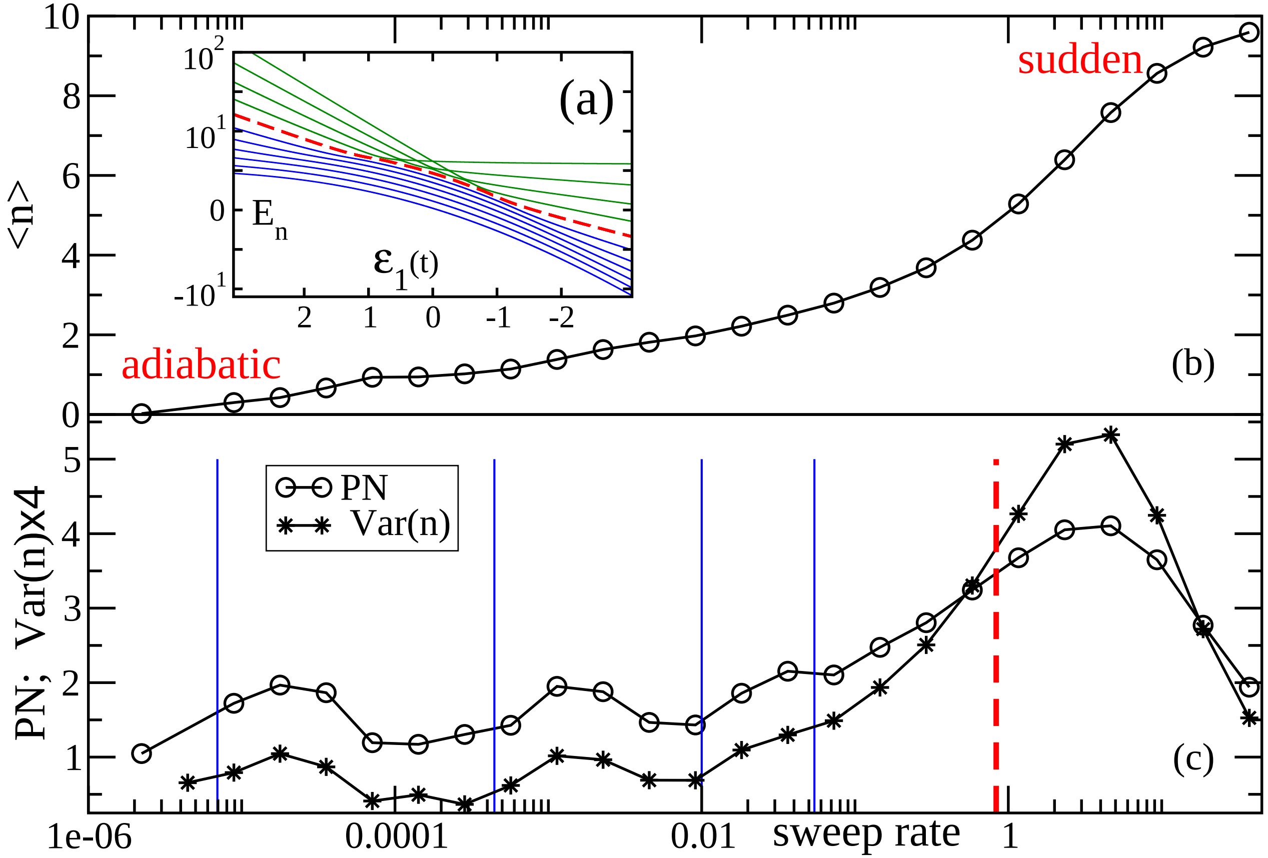






Dynamical scenarios:
adiabatic/diabatic/sudden

Occupation Statistics



Adiabtic-diabatic (quantum) crossover

Diabatic-sudden (semiclassical) crossover

Summary

- Semiclassical analysis (WKB and Wigner-Weyl are beyond MFT)
- The dependence of the participation number M on u and on N.
- Fluctuations and recurrences, study of ω_{osc} and $\overline{S(t)}$ and $\text{RMS}[S(t)]$
- Noise driven dimer: Improved Quantum Zeno effect analysis for $\overline{S(t)}$
- Erratic driving: analysis of the statistics of $|S(t)|$ - challenging the system-bath paradigm
- Occupation statistics in a time dependent Landau-Zener scenario: identification of the adiabatic / diabatic / sudden crossovers.