BEC dynamics in a few site systems

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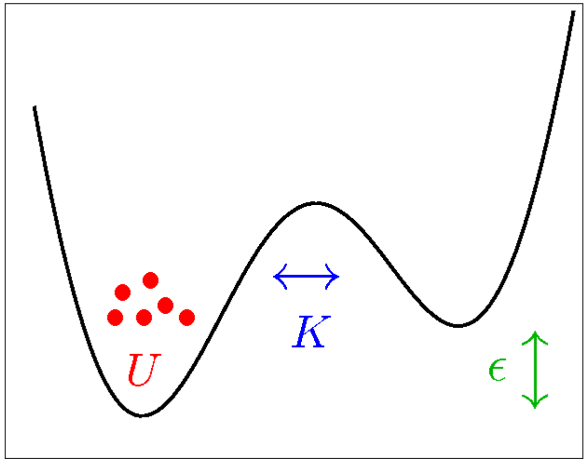


$BSF, $DIP, $FOR760

[1] Occupation dynamics & fluctuations (PRL 2009)

[2] Occupation dynamics & fluctuations (PRA 2009)

[3] Occupation dynamics & fluctuations (arXiv 2010)

[4] Landau-Zener dynamics (PRL 2009)

[5] Quantum stirring (EPL 2008)

[6] Quantum stirring (PRA 2008)

The Bose-Hubbard Hamiltonian (BHH) for a dimer

$$\mathcal{H} \;=\; \sum_{i=1,2}\left[\mathcal{E}_i \hat{n}_i + \frac{U}{2}\hat{n}_i(\hat{n}_i-1)\right] - \frac{K}{2}(\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2)$$

K = hopping
U = interaction
$\mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1$ = bias

N particles in a double well is like spin $j = N/2$ system

$$\mathcal{H} \;=\; -\mathcal{E}\hat{J}_z + U\hat{J}_z^2 - K\hat{J}_x + \text{const}$$

$$u \equiv \frac{NU}{K}, \qquad \varepsilon \equiv \frac{\mathcal{E}}{K}$$

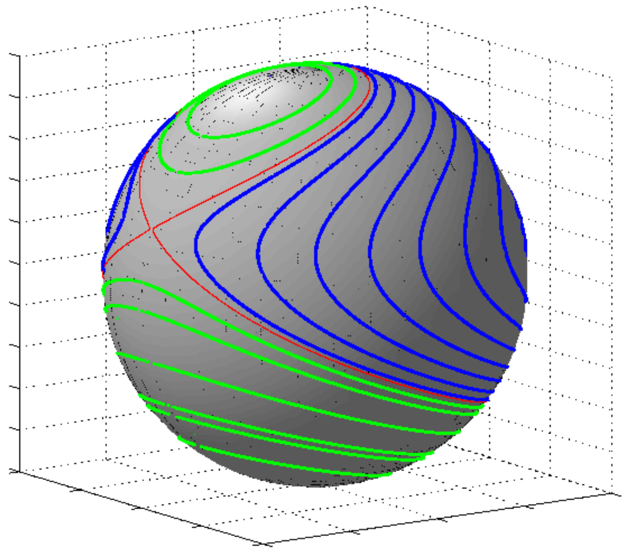
Classical phase space

$$\mathcal{H}(\theta,\varphi) \;=\; \frac{NK}{2}\left[\frac{1}{2}u(\cos\theta)^2 - \varepsilon\cos\theta - \sin\theta\cos\varphi\right]$$

$$\mathcal{H}(\hat{n},\varphi) \;=\; \text{(similar to Josephson/pendulum Hamiltonian)}$$

$$\hat{J}_z \;=\; (N/2)\cos(\theta) \qquad = \hat{n} = \text{occupation difference}$$

$$\hat{J}_x \;\approx\; (N/2)\sin(\theta)\cos(\varphi), \qquad \varphi = \text{relative phase}$$



Assuming $u>1$ and $|\varepsilon| < \varepsilon_c$
Sea, Islands, Separatrix

$$\varepsilon_c \;=\; \left(u^{2/3}-1\right)^{3/2}$$

$$A_c \;\approx\; 4\pi\left(1-u^{-2/3}\right)^{3/2}$$

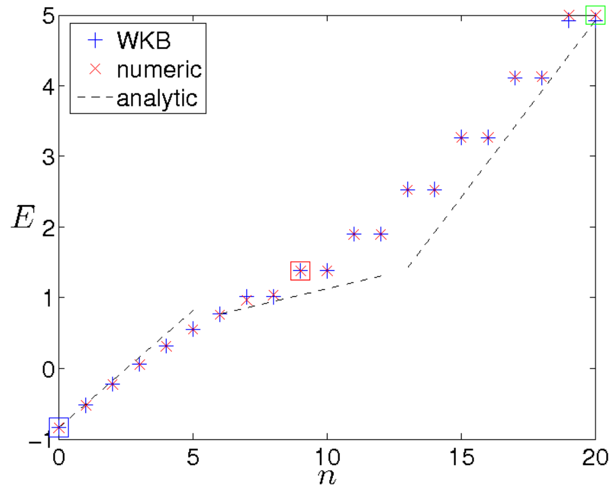
Rabi regime:	$u < 1$	(no islands)
Josephson regime:	$1 < u < N^2$	(sea, islands, separatrix)
Fock regime:	$u > N^2$	(empty sea)

WKB quantization (Josephson regime)

$$h \; = \; \text{Planck cell area in steradians} \; = \; \frac{4\pi}{N+1}$$

$$A(E_\nu) \; = \; \left(\frac{1}{2}+\nu\right) h \qquad \nu = 0, 1, 2, 3, \ldots$$

$$\omega(E) \; \equiv \; \frac{dE}{d\nu} \; = \; \left[\frac{1}{h}A'(E)\right]^{-1}$$

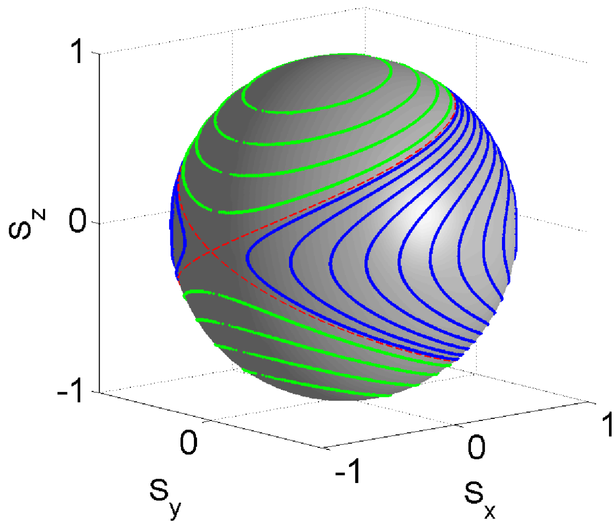


$$\omega_K \; \approx \; K \; = \; \text{Rabi Frequency}$$

$$\omega_J \; \approx \; \sqrt{NUK} \; = \; \sqrt{u}\,\omega_K$$

$$\omega_+ \; \approx \; NU \; = \; u\,\omega_K$$

$$\omega_x \; \approx \; \left[\log\left(\frac{N^2}{u}\right)\right]^{-1} \omega_J$$

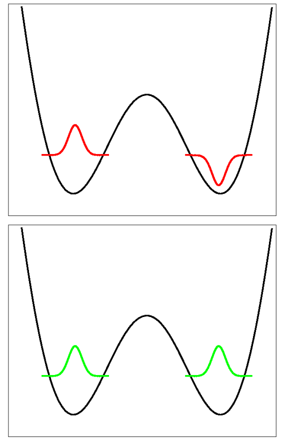


The preparations

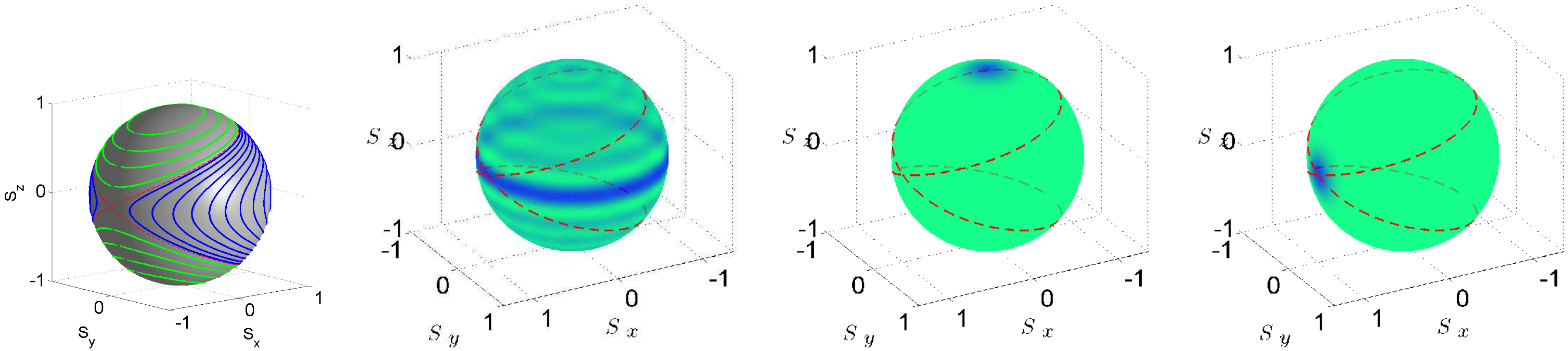
Eigenstates $|E_\nu\rangle$ are like strips along contour lines of $\mathcal{H}$.

Coherent state $|\theta\varphi\rangle$ is like a minimal Gaussian wavepacket.

Fock state $|n\rangle$ is like equi-latitude annulus.

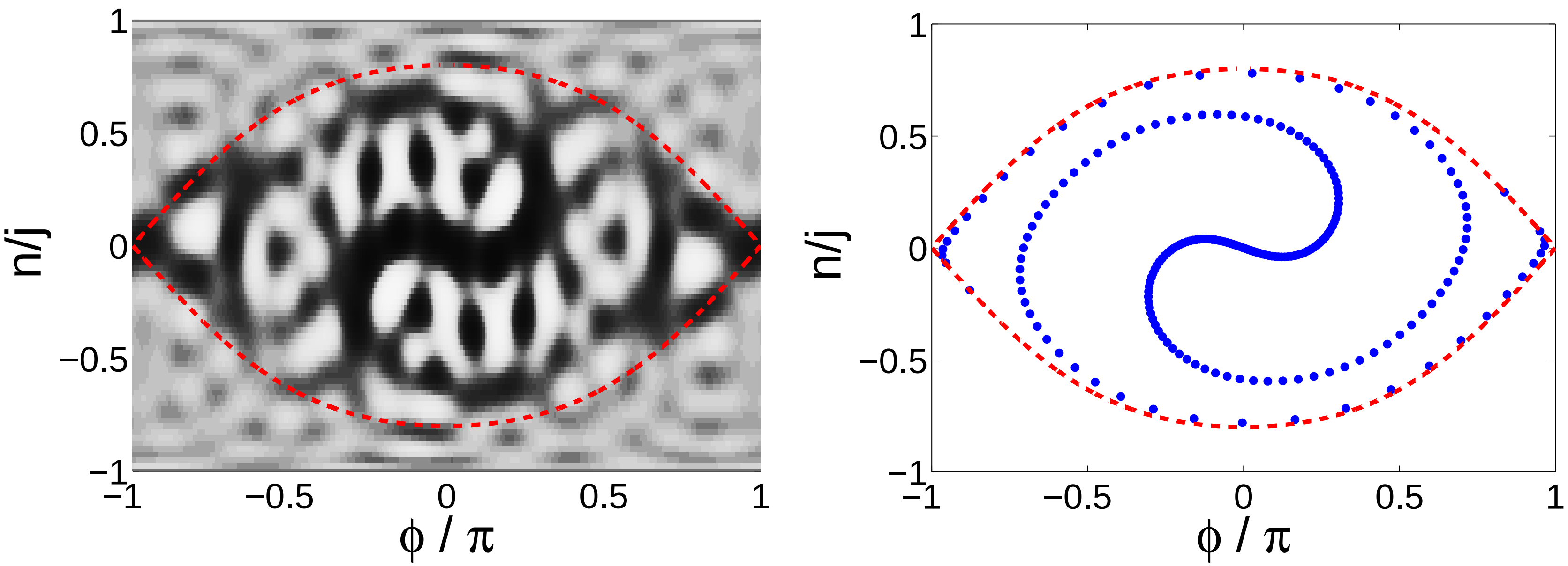


Fock $n=0$ preparation - exactly half of the particles in each site
Fock coherent $\theta=0$ preparation - all particles occupy the left site
Coherent $\varphi=0$ preparation - all particles occupy the symmetric orbital
Coherent $\varphi=\pi$ preparation - all particles occupy the antisymmetric orbital



Wavepacket dynamics

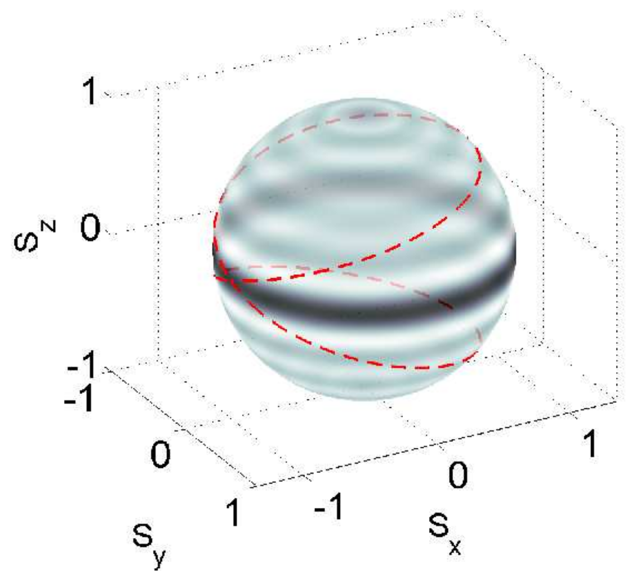
MeanField theory (GPE) = classical evolution of a point in phase space
SemiClassical theory = classical evolution of a distribution in phase space
Quantum theory = recurrences, fluctuations (WKB is very good)

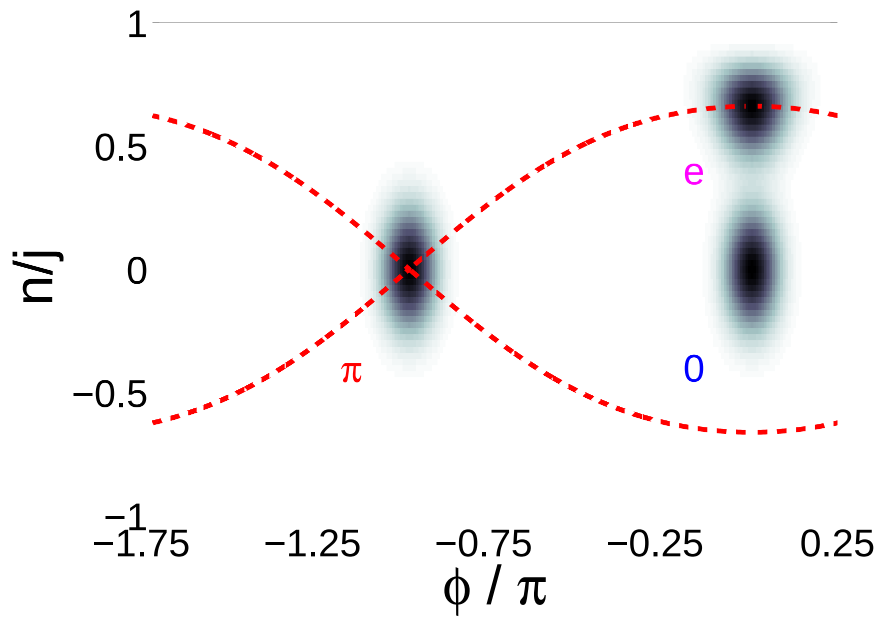


Any operator $\hat{A}$ can be presented by the phase-space function $A_{\mathrm{W}}(\Omega)$

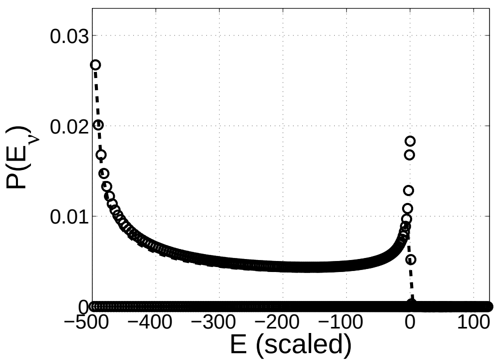
$$\langle \hat{A} \rangle \;=\; \mathrm{trace}[\hat{\rho}\;\hat{A}] \;=\; \int \frac{d\Omega}{h} \rho_{\mathrm{W}}(\Omega) A_{\mathrm{W}}(\Omega)$$

$P(E)$ = the LDOS of the preparations



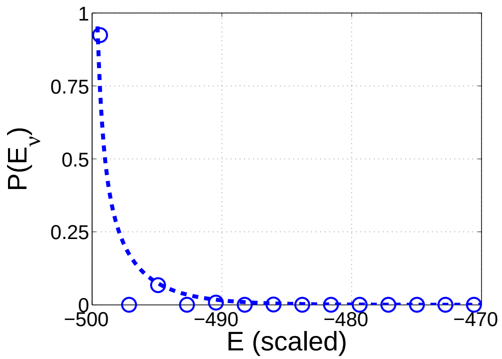


TwinFock preparation



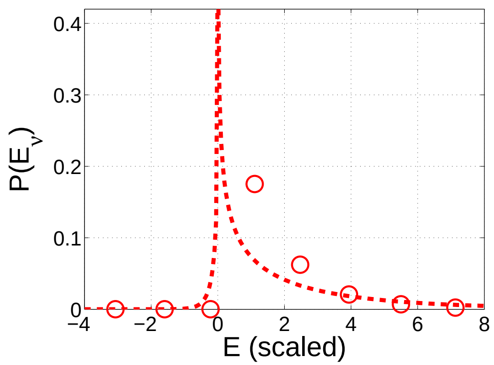
$$\sim \left[1-\left(\frac{2E}{NK}\right)^2\right]^{-1/2}$$

Zero preparation



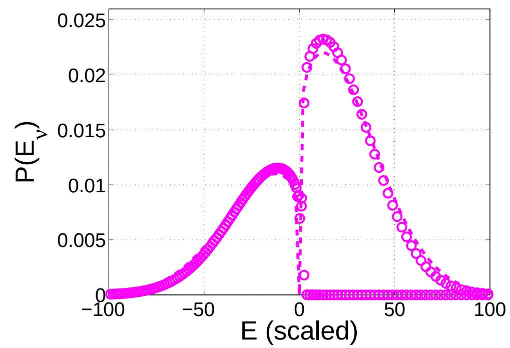
$$\sim \mathrm{BesselI}\left[\frac{E-E_-}{NU}\right]$$

Pi preparation



$$\sim \mathrm{BesselK}\left[\frac{E-E_x}{NU}\right]$$

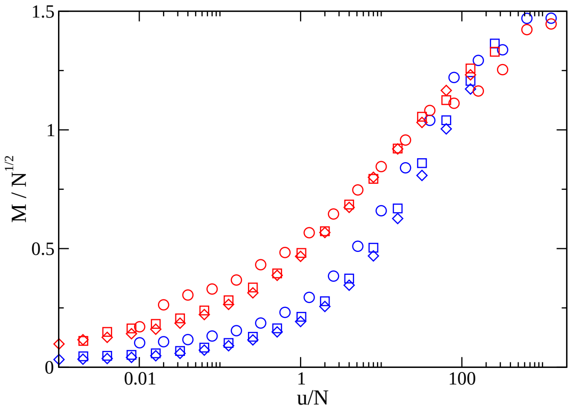
Edge preparation

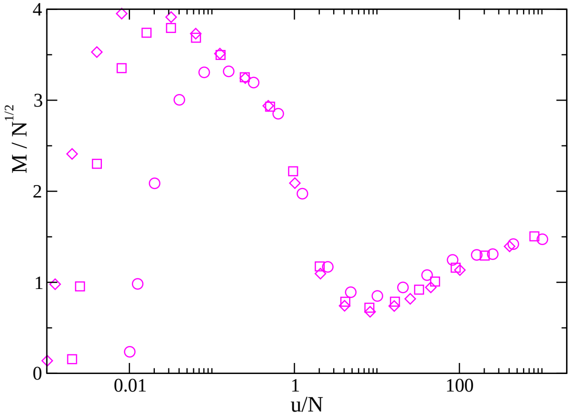


$$\sim \exp\left[-\frac{1}{N}\left(\frac{E-E_x}{\omega_J}\right)^2\right]$$

M = the participation number

$$M \;=\; \left[\sum_{\nu} \mathrm{P}(E_\nu)^2\right]^{-1} \;=\; \text{number of participating levels in the LDOS}$$





$$\frac{M}{N^{1/2}} \quad \text{vs} \quad \frac{u}{N}$$

for: Zero, Pi, Edge

In the semiclassical analysis there is scaling with respect to $(u/N)^{1/2}$ which is [the width of the wavepacket] / [the width of the separatix]

$$M \;\approx\; \left[\log\left(\frac{N}{u}\right)\right]\sqrt{u} \qquad \text{[Pi]} \qquad \leadsto \text{(quasi periodic large fluctuations)}$$

$$M \;\approx\; \left[\log\left(\frac{N}{u}\right)\right]\sqrt{N} \qquad \text{[Edge]} \qquad \leadsto \text{(easy to get the classical limit)}$$

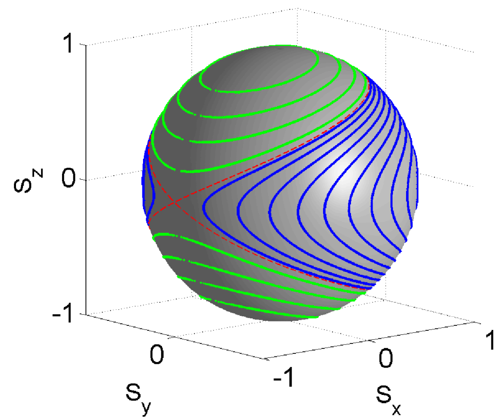
Recurrences and fluctuations

$$\vec{S} = \langle \vec{J} \rangle / (N/2) \quad = \quad (S_x, S_y, S_z) \quad = \quad \text{Bloch vector}$$

$$\text{OccupationDifference} \quad = \quad (N/2)\ \langle S_z \rangle$$

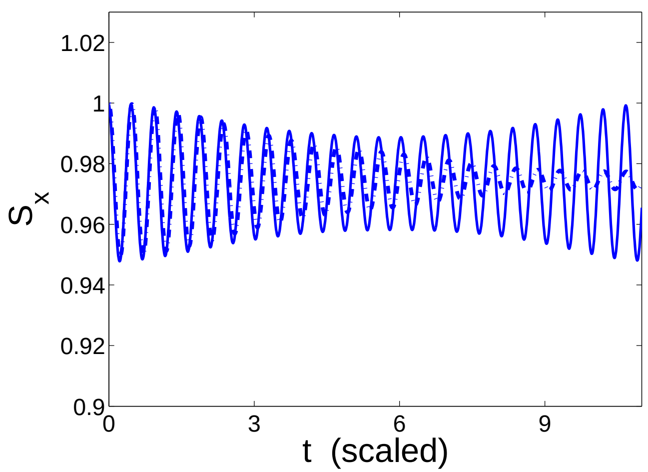
$$\text{OneBodyPurity} \quad = \quad (1/2)\left[1 + \langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2\right]$$

$$\text{FringeVisibility} \quad = \quad \left[\langle S_x \rangle^2 + \langle S_y \rangle^2\right]^{1/2}$$

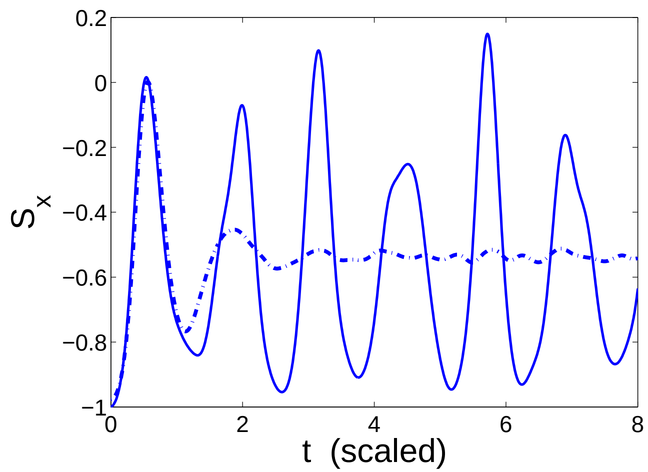


Spectral analysis of the fluctuations: dependence on u and on N, various preparations.

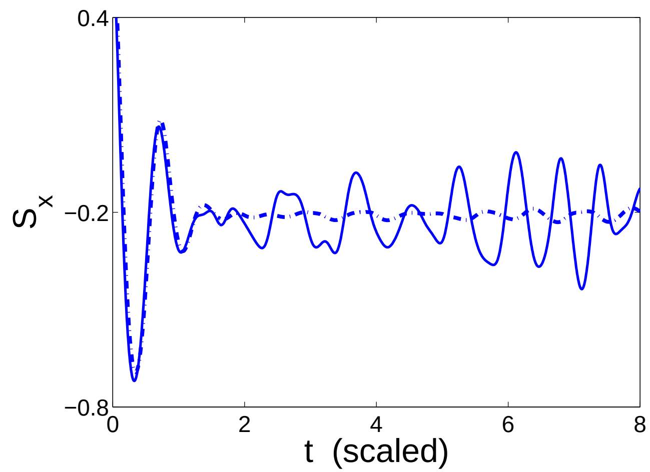
Zero prep

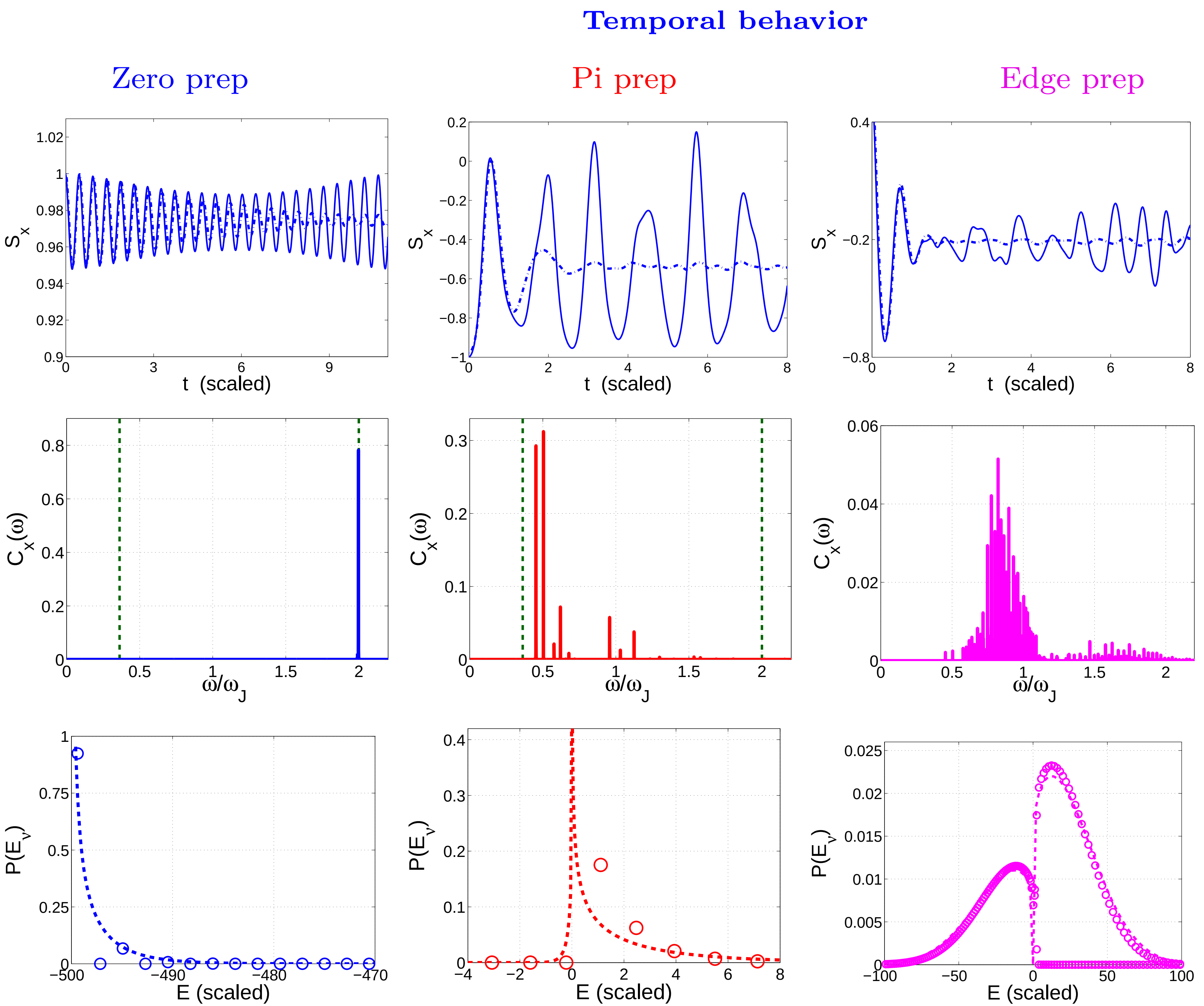


Pi prep



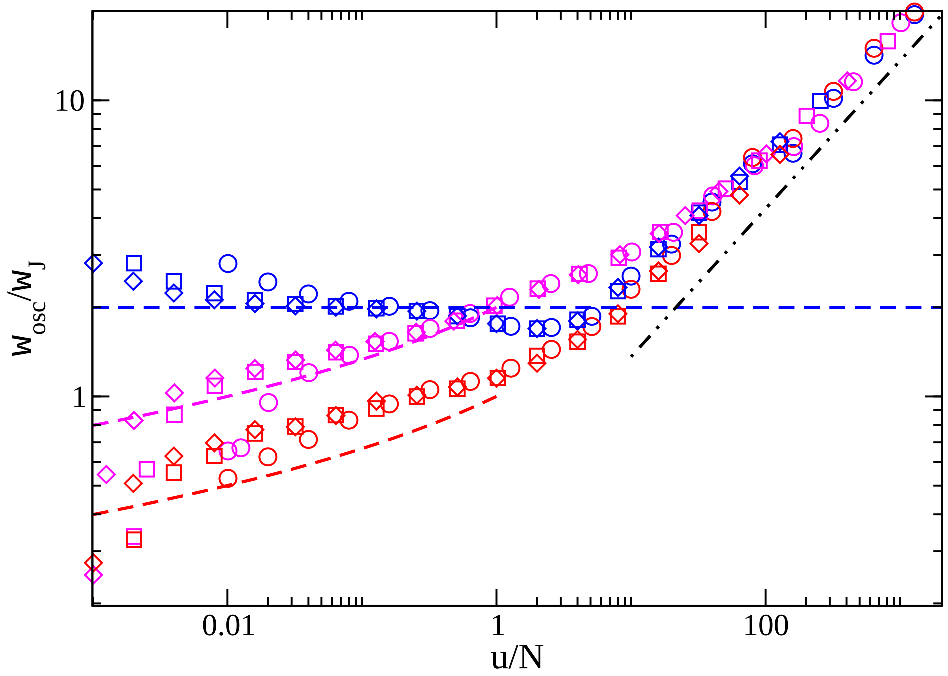
Edge prep





Temporal behavior
Zero prep
Pi prep
Edge prep
S_x
t (scaled)
C_x(ω)
ω/ω_J
P(E_ν)
E (scaled)

The spectral content of S_x



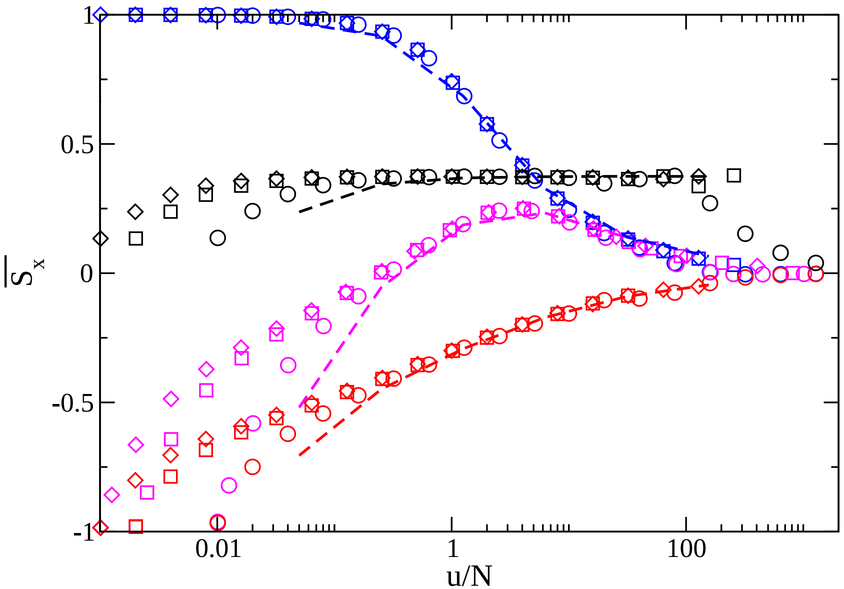
$$\omega_{\text{osc}} \approx 2\omega_J \qquad \text{[Zero]}$$

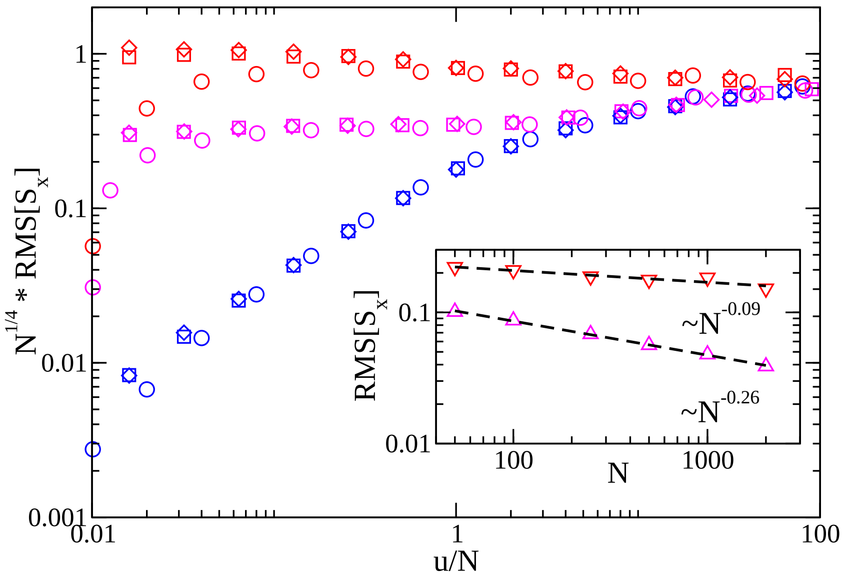
$$\omega_{\text{osc}} \approx 1\times \left[\log\left(\frac{N}{u}\right)\right]^{-1} 2\omega_J \qquad \text{[Pi]}$$

$$\omega_{\text{osc}} \approx 2\times \left[\log\left(\frac{N}{u}\right)\right]^{-1} 2\omega_J \qquad \text{[Edge]}$$

$$\omega_{\text{osc}} \approx \left(\frac{u}{N}\right)^{1/2} 2\omega_J \qquad [u \gg N]$$

Fluctuations of S_x





$N = 100$ ○, $N = 500$ □, $N = 1000$ ◇.

Naive expectation: phase spreading diminishes coherence.
In the Fock regime $\langle S_x \rangle_\infty \approx 0$ [Leggett's "phase diffusion"]
In the Josephson regime $\langle S_x \rangle_\infty$ is determined by u/N.

$$\overline{S_x} \approx 1/3 \quad \text{[TwinFock]}$$

$$\overline{S_x} \approx \exp[-(u/N)] \quad \text{[Zero]}$$

$$\overline{S_x} \approx -1 - 4/\log\left[\frac{1}{32}(u/N)\right] \quad \text{[Pi]}$$

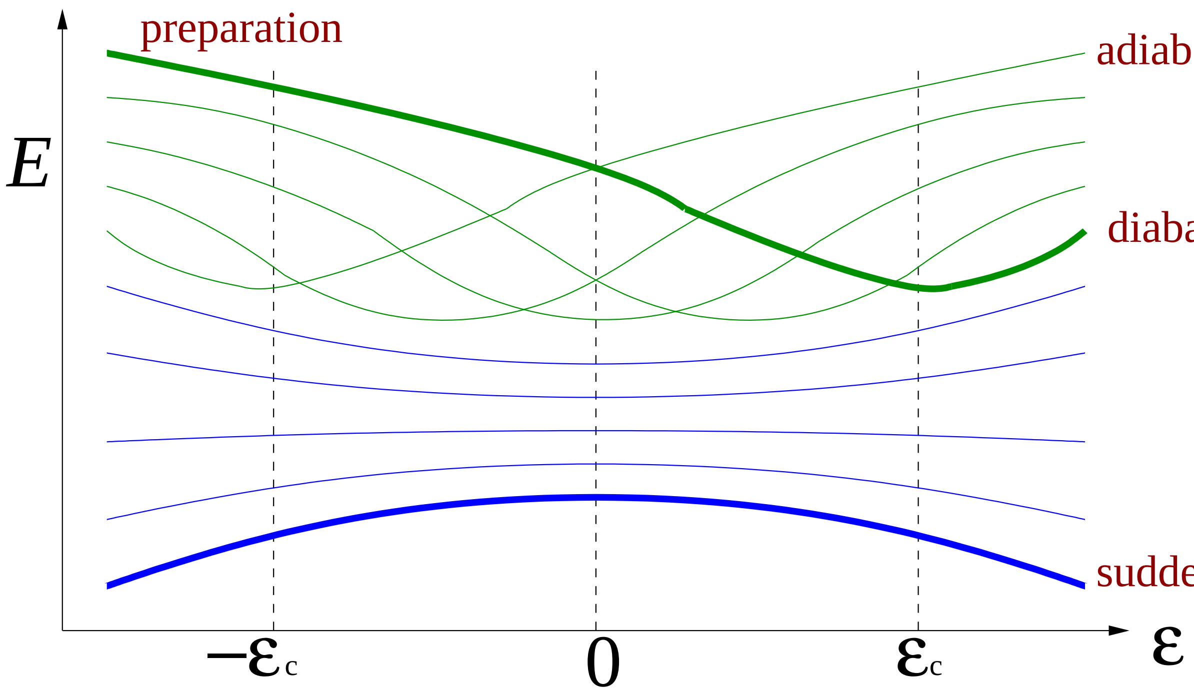
$$\text{RMS}\left[\langle A \rangle_t\right] = \left[\frac{1}{M}\int \tilde{C}_{\text{cl}}(\omega)d\omega\right]^{1/2}$$

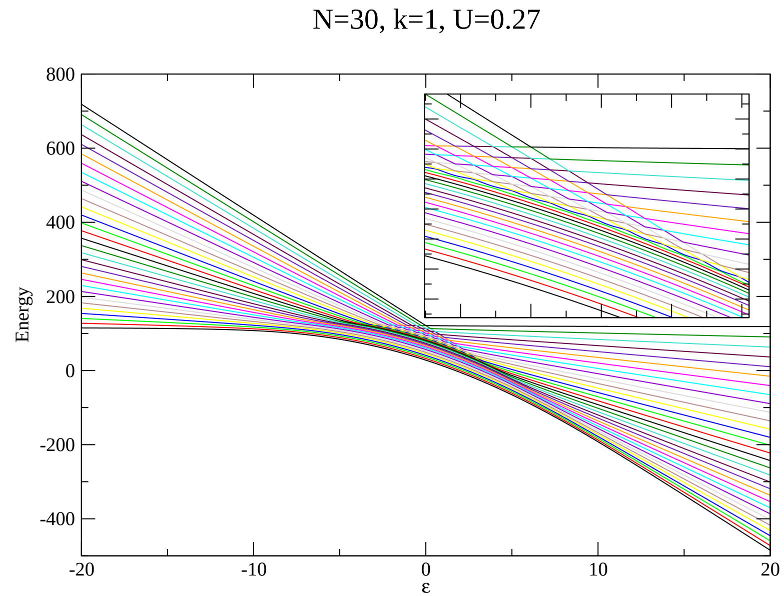
$$\text{RMS}\left[S_x(t)\right] \sim N^{-1/4} \quad \text{[Edge]}$$

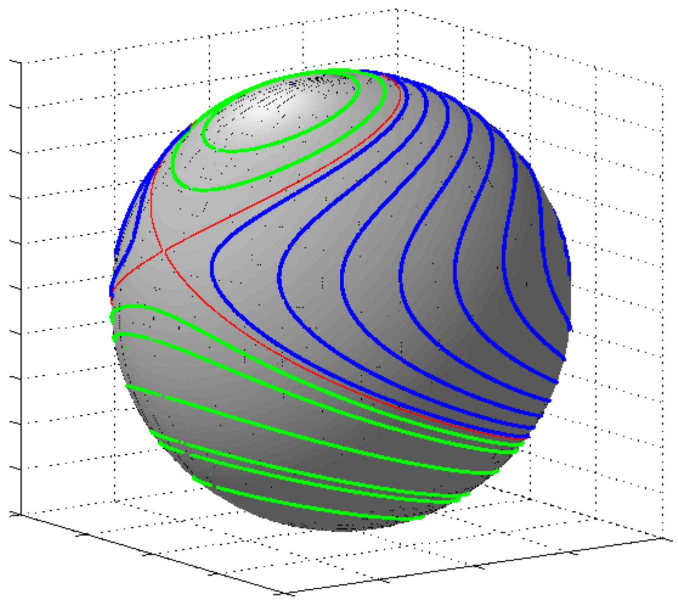
$$\text{RMS}\left[S_x(t)\right] \sim (\log(N))^{-1/2} \quad \text{[Pi]}$$

TwinFock: Self induced coherence leading to $\overline{S_x} \approx 1/3$.
Zero: Coherence maintained if $u/N < 1$ (phase locking).
Pi: Fluctuations are suppressed by u.
Edge: Fluctuations are suppressed by N (classical limit).

The many body Landau-Zener transition



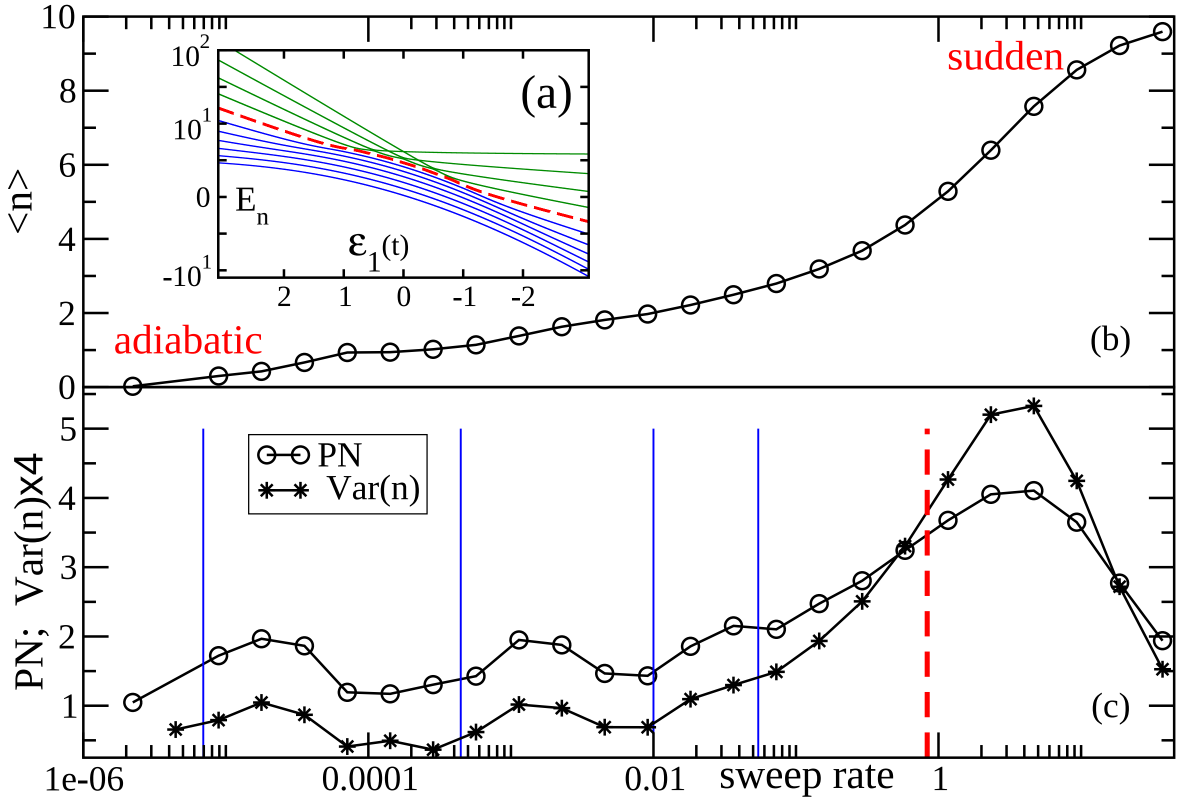


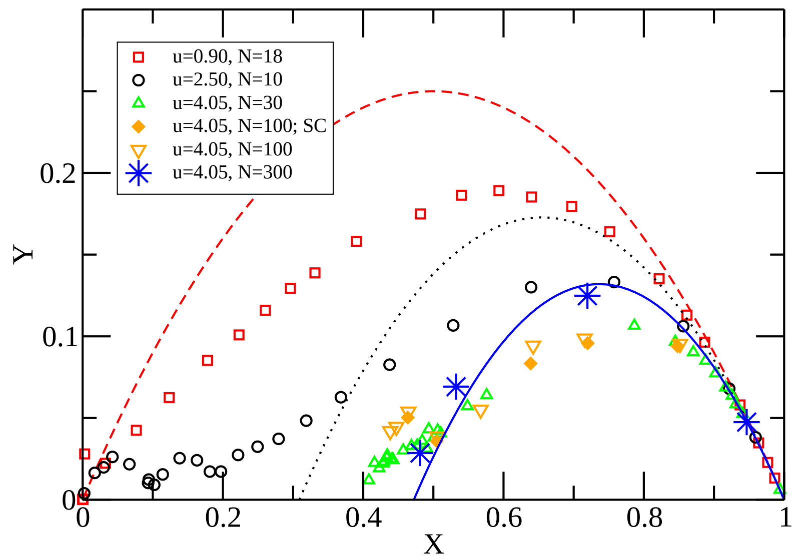


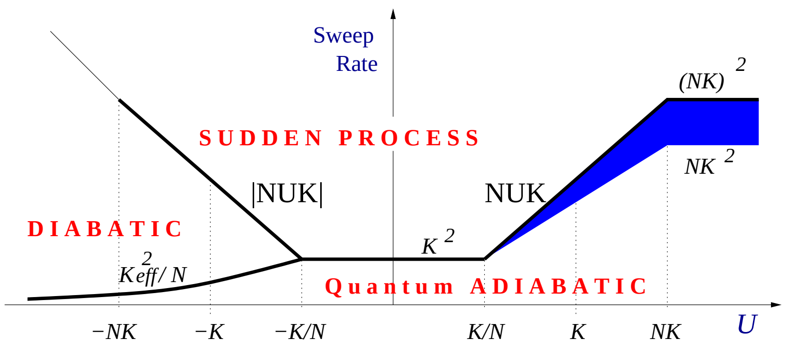
Dynamical scenarios:

adiabatic/diabatic/sudden

Occupation Statistics





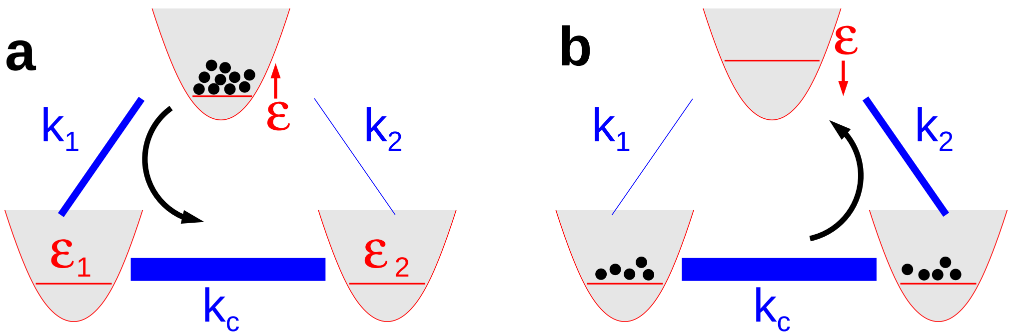


Adiabtic-diabatic (quantum) crossover

Diabatic-sudden (semiclassical) crossover

Sub-binomial scaling of Var(n) versus $\langle n \rangle$

Quantum Stirring in a 3 site system



Control parameters:

$$X_1 = \left(\frac{1}{k_2} - \frac{1}{k_1}\right)$$

$$X_2 = \mathcal{E}_0 \qquad (\mathcal{E}_1{=}\mathcal{E}_2{=}0)$$

$$\boldsymbol{X} = (X_1, X_2)$$

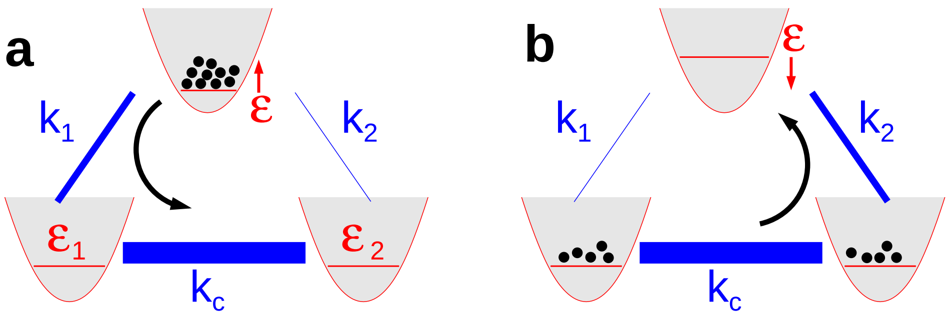
U = the inter-atomic interaction

$$\hat{\mathcal{H}} = \sum_{i=0}^{2} \mathcal{E}_i n_i + \frac{U}{2}\sum_{i=0}^{2} \hat{n}_i(\hat{n}_i - 1) - k_c(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) - k_1(\hat{b}_0^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{b}_0) - k_2(\hat{b}_0^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_0)$$

The induced current: $I = -G\dot{\mathcal{E}}$ $\qquad (G = G_2)$

The pumped particles: $Q = \oint I dt = \oint \boldsymbol{G} \cdot \boldsymbol{dX}$ $\qquad$ (per cycle)

Stirring of BEC

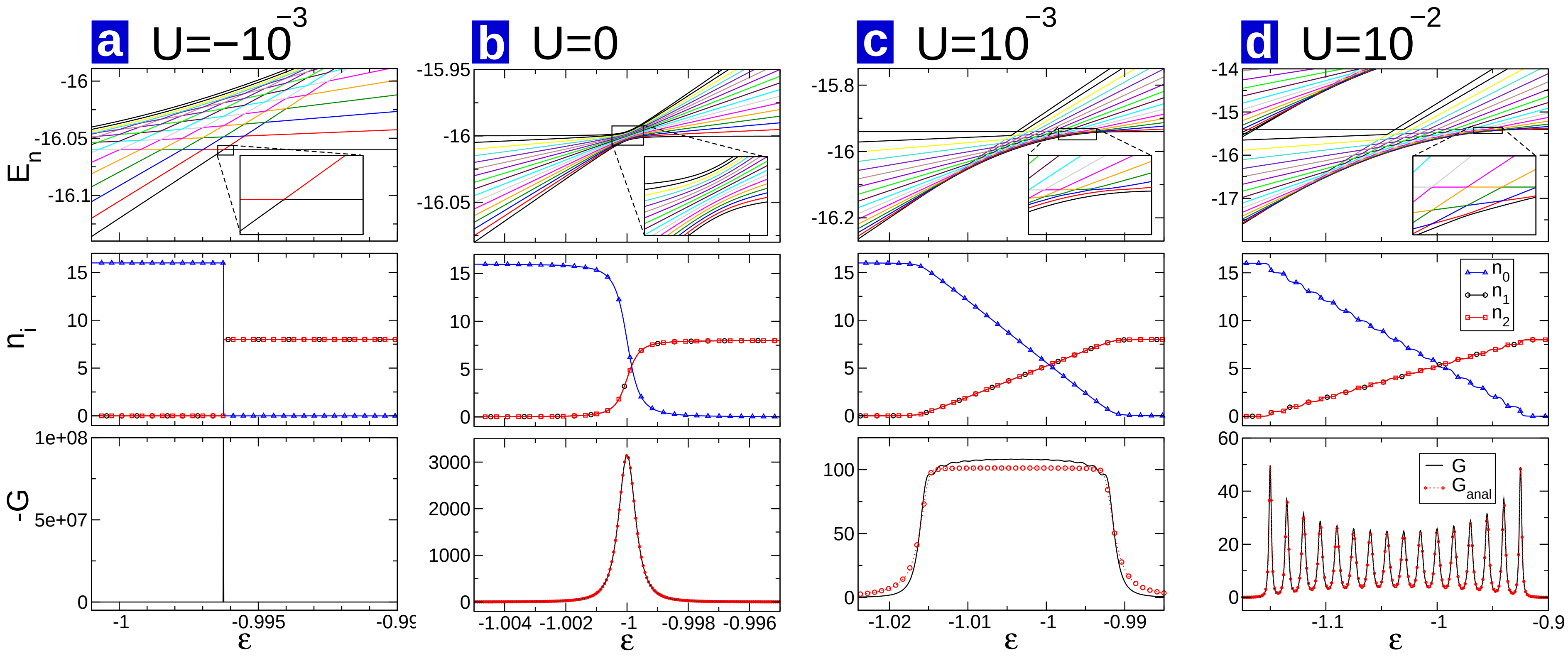


strong attractive interaction: classical ball dynamics

negligible interaction ($|U| \ll \kappa/N$): mega-crossing

weak repulsive interaction: gradual crossing

strong repulsive interaction ($U \gg N\kappa$): sequential crossing



Results for the geometric conductance

$$G(R) = -N\frac{(k_1^2-k_2^2)/2}{[(\varepsilon-\varepsilon_-)^2+2(k_1+k_2)^2]^{3/2}}$$ mega crossing

$$G(J) \approx -\left[\frac{k_1-k_2}{k_1+k_2}\right]\frac{1}{3U}$$ gradual crossing

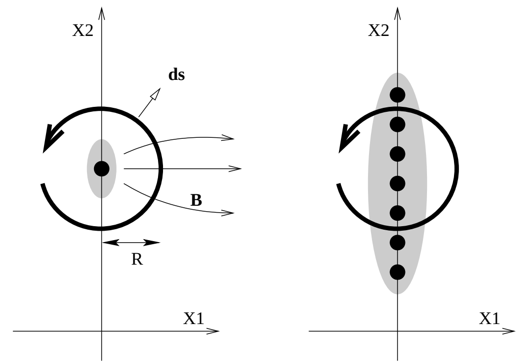
$$G(F) = -\left(\frac{k_1-k_2}{k_1+k_2}\right)\sum_{n=1}^{N}\frac{(\delta\varepsilon_n)^2}{[(\varepsilon-\varepsilon_n)^2+(2\delta\varepsilon_n)^2]^{3/2}},$$ sequential crossing

where:

R = Rabi regime ($U \ll \kappa/N$)

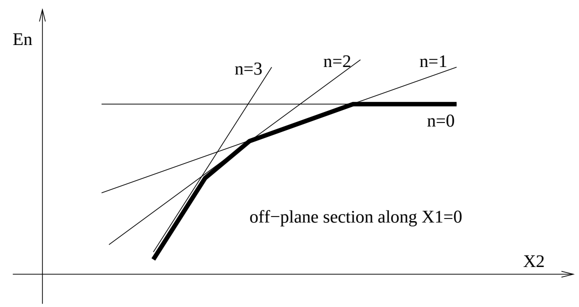
J = Josephson regime ($\kappa/N \ll U \ll N\kappa$)

F = Fock regime ($U \gg N\kappa$)



Observation:

It is possible to pump $Q \gg N$ per cycle.



Summary

- Semiclassical and WKB analysis of the dynamics
- Occupation statistics in a time dependent scenario
- The adiabatic / diabatic / sudden crossovers
- Quantum stirring: mega / gradual / sequential crossings