Counting statistics in multiple path geometries and quantum stirring

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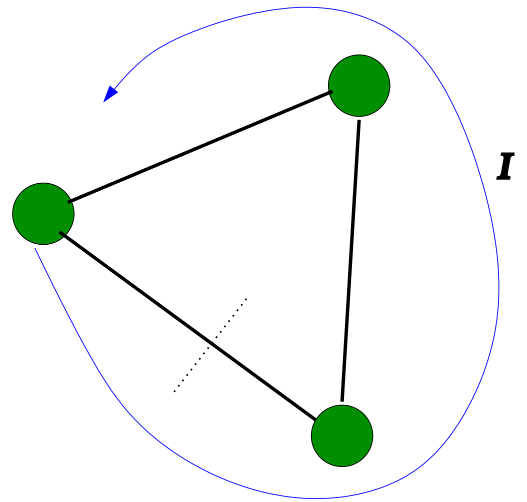


Refs:

Counting statistics for a coherent transition, MC and DC (PRA 2008)

Counting statistics in multiple path geometries, MC and DC (JPA 2008)

Quantum stirring of electrons in a ring, IS and DC (PRBs 2008)

http://www.bgu.ac.il/∼dcohen

$DIP, $BSF

Outline

The counting operator:

$$\mathcal{Q} \;=\; \int_0^t \mathcal{I}(t')dt'$$

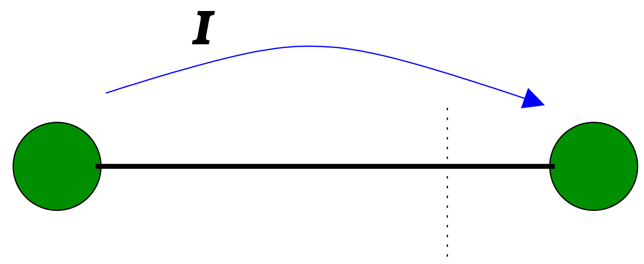
– Single path coherent transition

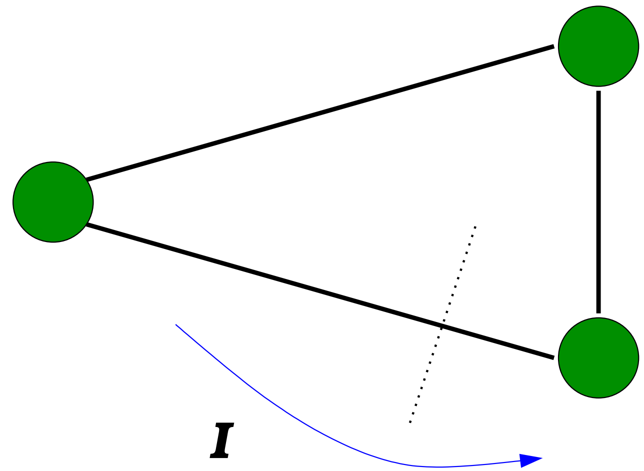
– Double path coherent transition

– Quantum stirring (full cycle)

– Condensed particles / interactions

$$\begin{aligned} \langle \mathcal{Q} \rangle &= ??? \\ \mathrm{Var}(Q) &= ??? \\ \mathrm{P}(Q) &= ??? \;\; [\mathrm{FCS}] \end{aligned}$$





Stirring = inducing DC current by AC driving.

$\mathcal{Q}$ is not an observable

The counting statistics can be determined using a continuous measurement scheme: The current induces a translation of a Von-Neumann pointer. At the final time, the position of the pointer is measured.

$$\mathrm{P}(Q) \; = \; \frac{1}{2\pi} \int \left\langle \left[\mathcal{T} \mathrm{e}^{-i(r/2)\mathcal{Q}}\right]^{\dagger} \left[\mathcal{T} \mathrm{e}^{+i(r/2)\mathcal{Q}}\right] \right\rangle \mathrm{e}^{-iQr} dr$$

$$\overline{Q} \quad = \quad \langle \mathcal{Q} \rangle$$

$$\overline{Q^2} \quad = \quad \langle \mathcal{Q}^2 \rangle$$

H. Everett, Rev. Mod. Phys. 29, 454 (1957).

L.S. Levitov and G.B. Lesovik, JETP Letters (1992/3).

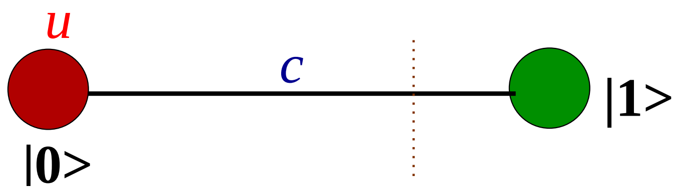
Y.V. Nazarov and M. Kindermann, EPJ B (2003).

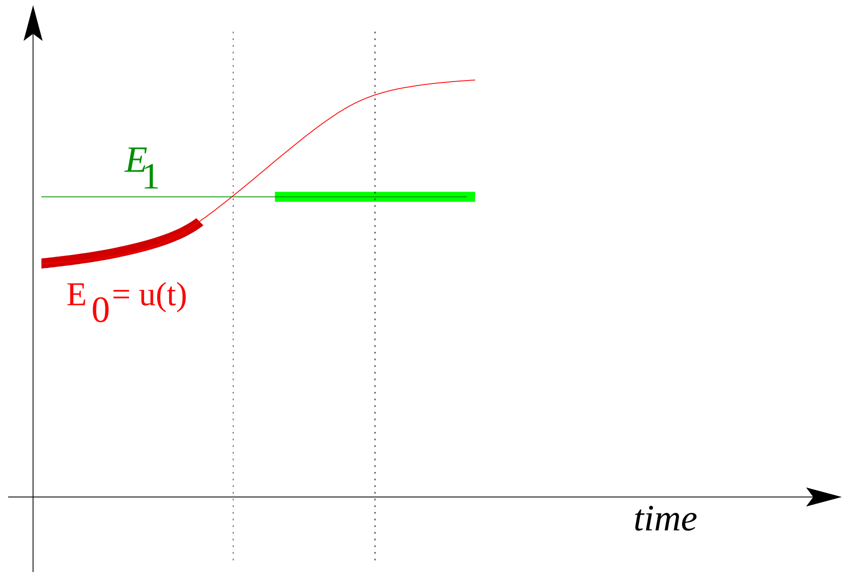
MC and DC, PRA (2008)

where $\quad \mathcal{Q} \; = \; \int_0^t \mathcal{I}(t')dt'$

Single path coherent transition

$$\langle \mathcal{N} \rangle = p = \text{occupation}$$
$$\langle \mathcal{Q} \rangle = p = \text{counting}$$
$$\mathrm{Var}(\mathcal{Q}) = \underset{\sim\sim\sim\sim\sim\sim}{(1-p)p}$$





classical:
$$N = 1_{\,[p]}, 0_{\,[1-p]}$$
$$Q = 1_{\,[p]}, 0_{\,[1-p]}$$

Quantum:
$$N = 1_{\,[p]}, 0_{\,[1-p]}$$
$$Q = \pm\sqrt{p}_{\,[(1\pm\sqrt{p})/2]}$$

$$p = 1 - P_{\mathrm{LZ}}$$
$$P_{\mathrm{LZ}} = \mathrm{e}^{-2\pi \frac{c^2}{\dot{u}}}$$

Restricted Quantum to Classical Correspondence (QCC)

$\mathcal{N}$ = occupation operator (eigenvalues $= 0, 1$)

$\mathcal{I}$ = current operator

Heisenberg equation of motion: $\frac{d}{dt}\mathcal{N}(t) = \mathcal{I}(t)$

Counting vs change in Occupation: $\mathcal{N}(t) - \mathcal{N}(0) = \mathcal{Q}$

Counting statistics = Occupation statistics:

$$\langle \mathcal{Q}^k \rangle \; = \; \left\langle (\mathcal{N}(t) - \mathcal{N}(0))^k \right\rangle \; = \; \langle \mathcal{N}^k \rangle_t \; = \; p \qquad \text{for } k = 1, 2 \text{ only}$$

$$\left\langle 0 \middle| \, (\mathcal{N}(t) - \mathcal{N}(0)) \; (\mathcal{N}(t) - \mathcal{N}(0)) \; (\mathcal{N}(t) - \mathcal{N}(0)) \, \middle| 0 \right\rangle \; \neq \; \left\langle 0 \middle| \mathcal{N}(t)^3 \middle| 0 \right\rangle$$

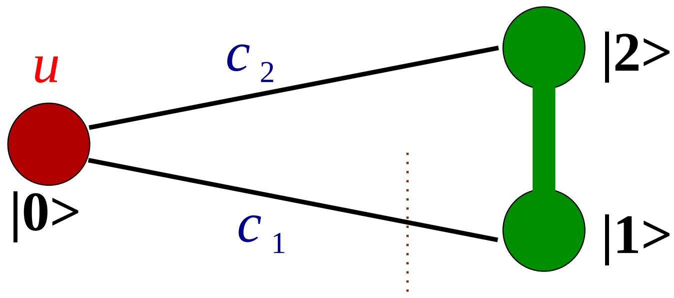
Restricted QCC is robust

Detailed QCC is fragile

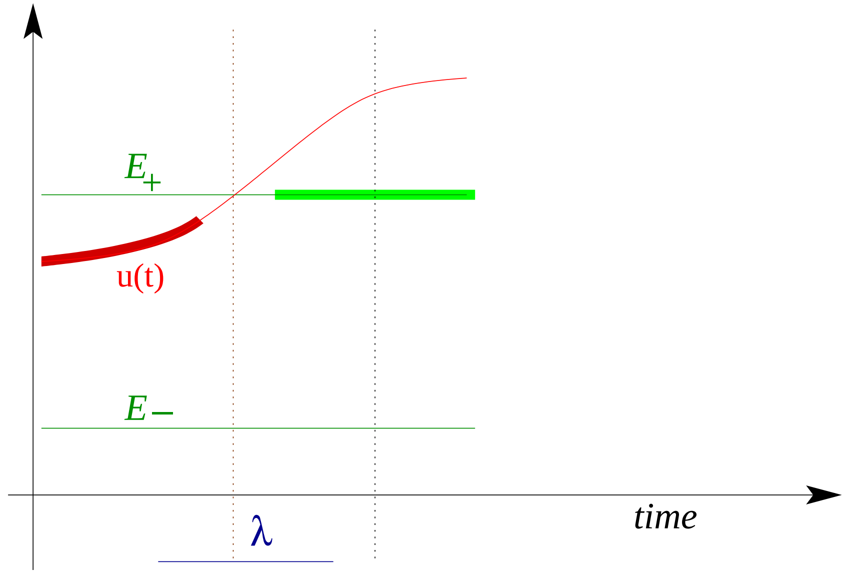
A. Stotland and D. Cohen, J. Phys. A 39, 10703 (2006).

Double path coherent transition

$$\langle \mathcal{N} \rangle = p$$
$$\langle \mathcal{Q} \rangle = \lambda p$$
$$\mathrm{Var}(\mathcal{Q}) = \lambda^2 (1-p)p$$



$$\lambda = \frac{c_1}{c_1 + c_2} = \text{splitting ratio}$$



Coherent splitting is not like incoherent partitioning:

$$\mathrm{Var}(\mathcal{Q}) \neq (1-\lambda p)\lambda p$$

$$\lambda \neq \frac{|c_1|^2}{|c_1|^2 + |c_2|^2}$$

$$p = 1 - P_{\mathrm{LZ}}$$
$$P_{\mathrm{LZ}} = \mathrm{e}^{-\pi \frac{(c_1+c_2)^2}{\dot{u}}}$$

Splitting vs Partitioning

$$|\Psi\rangle \quad = \quad \Big(|Q{=}0\rangle + |Q{=}1\rangle\Big) \ \otimes \ |q{=}0\rangle \qquad\qquad q = \text{pointer}$$

The Schrodinger-cat paradigm:

The state of the particle becomes mixed;

One measures $Q = 0, 1$ with 50%-50% probabilities.

$$|\Psi\rangle \quad = \quad |Q{=}0\rangle \otimes |q{=}0\rangle \ + \ |Q{=}1\rangle \otimes |q{=}1\rangle$$

The Born-Oppenheimer paradigm:

The state of the particle remains pure;

One measures $Q = \frac{1}{2}$ with 100% probability.

$$|\Psi\rangle \quad = \quad \Big(|Q{=}0\rangle + |Q{=}1\rangle\Big) \ \otimes \ |q{=}\tfrac{1}{2}\rangle$$

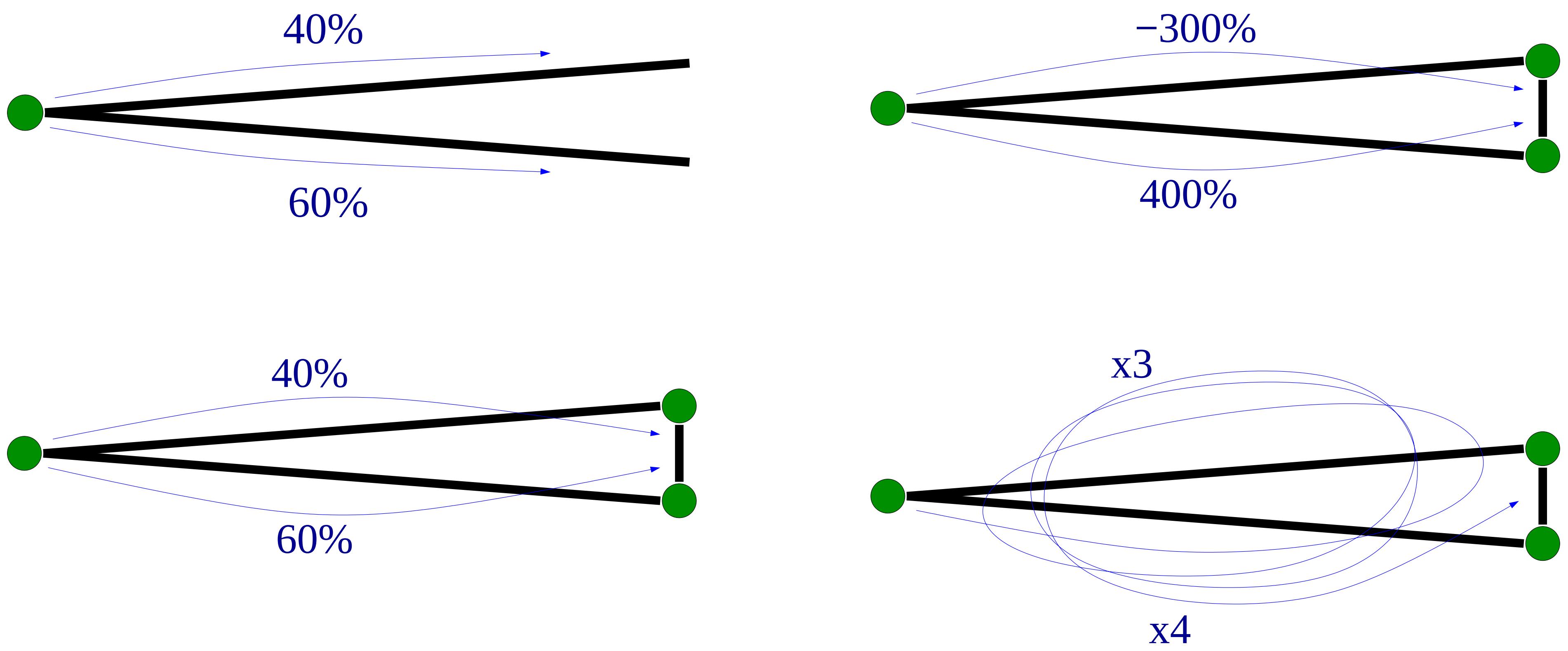
Splitting and stirring

The scattering point of view:

The particle has two paths to its destination.

The stirring point of view:

A circulating current is induced due to the driving.



The splitting ratio approach* to quantum stirring**

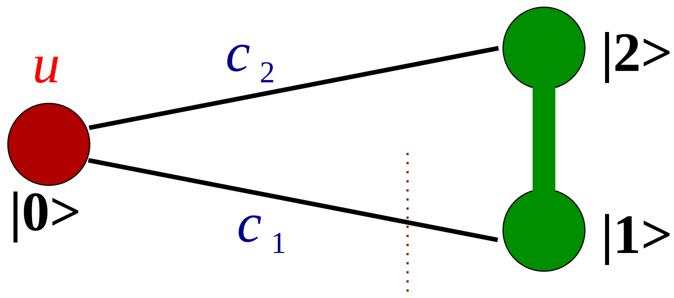
$$\langle \mathcal{N} \rangle \approx \left| \sqrt{P_{\mathrm{LZ}}^{\circlearrowleft}} - e^{i\varphi} \sqrt{P_{\mathrm{LZ}}^{\circlearrowright}} \right|^2$$

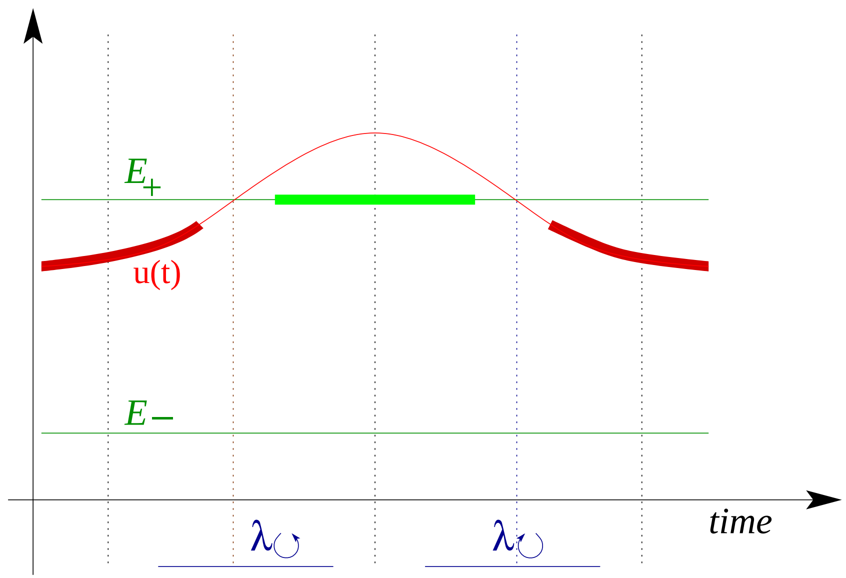
$$\langle \mathcal{Q} \rangle \approx \lambda_{\circlearrowleft} - \lambda_{\circlearrowright}$$

$$\mathrm{Var}(\mathcal{Q}) \approx \left| \tilde{\lambda}_{\circlearrowleft} \sqrt{P_{\mathrm{LZ}}^{\circlearrowleft}} + e^{i\varphi} \lambda_{\circlearrowright} \sqrt{P_{\mathrm{LZ}}^{\circlearrowright}} \right|^2$$

(*) In the classical context a similar approach has been independently proposed under the name *current decomposition formula*. S.Rahav, J.Horowitz, and C.Jarzynski1 (PRL 2008).

(**) The splitting ratio approach allows to bypass the Kubo formula approach to quantum stirring, D.Cohen (PRB 2003), which is based on the adiabatic transport formalism of Thouless (1983), Avron (1988), Berry and Robbins (1993).





$$\tilde{\lambda}_{\circlearrowleft} = \lambda_{\circlearrowleft} - 2\lambda_{\circlearrowright}$$

Derivation, using the adiabatic approximation

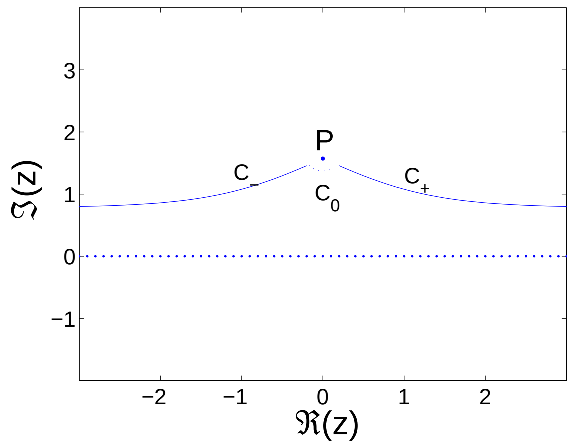
$$U(t) \approx \sum_n \Big|n(t)\Big\rangle \exp\left[-i\int_{t_0}^{t} E_n(t')dt'\right]\Big\langle n(t_0)\Big|$$

$$\mathcal{I}(t)_{nm} = \langle n|U(t)^\dagger \mathcal{I} U(t)|m\rangle \approx \langle n(t)|\mathcal{I}|m(t)\rangle \exp\left[i\int_{t_0}^{t} E_{nm}(t')dt'\right]$$

$$\mathcal{Q} \equiv \begin{pmatrix} +Q_{\parallel} & iQ_{\perp} \\ -iQ_{\perp}{}^{*} & -Q_{\parallel} \end{pmatrix}$$

$$\mathrm{Var}(\mathcal{Q}) = |Q_{\perp}|^2 \approx \left| \int_{-\infty}^{\infty} c\, \mathrm{e}^{i\Phi(t)} dt \right|^2$$

$$\Phi(t) \equiv \int_0^t \sqrt{u(t')^2 + (2c)^2}\; dt' \qquad \text{[for a single LZ transition]}$$

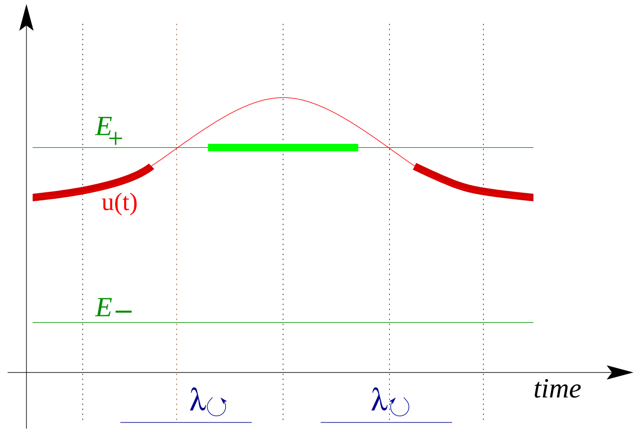


$$\mathrm{Var}(\mathcal{Q}) \approx \left|\int_{-\infty}^{\infty} c\,\mathrm{e}^{i\Phi(t)}dt\right|^2 = \left|\frac{2c^2}{\dot{u}}\int_{-\infty}^{\infty}\cosh(z)\,\mathrm{e}^{i\Phi(z)}dz\right|^2 \sim \left(\frac{2c^2}{\dot{u}}\right)^{2/3} \exp\left[-\pi\frac{c^2}{\dot{u}}\right]$$

$$P_{\mathrm{LZ}} \approx \left|\int_{-\infty}^{\infty}\frac{c\dot{u}}{u^2+(2c)^2}\,\mathrm{e}^{i\Phi(t)}dt\right|^2 = \left|\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{\cosh(z)}\,\mathrm{e}^{i\Phi(z)}dz\right|^2 \sim \left(\frac{\pi}{3}\right)^2 \exp\left[-\pi\frac{c^2}{\dot{u}}\right]$$

Derivation, using the adiabatic approximation (cont.)

A sequence of two Landau Zener crossings:



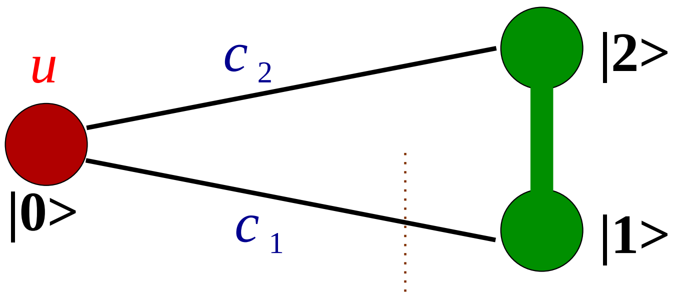
$$\langle \mathcal{Q} \rangle \quad \approx \quad \lambda_{\circlearrowleft} - \lambda_{\circlearrowright}$$

[assume for simplicity that only the splitting ratio is different]

$$\mathrm{Var}(\mathcal{Q}) \quad = \quad \left| \int_{-\infty}^{\infty} \lambda c \, \mathrm{e}^{i\Phi(t)} dt \right|^2 \quad \approx \quad \left| \lambda_{\circlearrowleft} \, \mathrm{e}^{i\varphi_1} + \lambda_{\circlearrowright} \, \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}$$

$$P_{\mathrm{LZ+LZ}} \quad = \quad \left| \int_{-\infty}^{\infty} \frac{c\dot{u}}{u^2 + (2c)^2} \, \mathrm{e}^{i\Phi(t)} dt \right|^2 \quad \approx \quad \left| \mathrm{e}^{i\varphi_1} - \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}$$

Derivation, using the splitting ratio approach



$$\mathcal{H} = \begin{pmatrix} u(t) & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u(t) & \frac{(c_1+c_2)}{\sqrt{2}} \\ \frac{(c_1+c_2)}{\sqrt{2}} & 1 \end{pmatrix}, \qquad \mathcal{I} = \frac{c_1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & 0 \end{pmatrix}, \qquad \mathcal{I} = \lambda \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

$$U_{\mathrm{LZ}} \approx \begin{pmatrix} \sqrt{P_{\mathrm{LZ}}} & -\sqrt{1-P_{\mathrm{LZ}}} \\ \sqrt{1-P_{\mathrm{LZ}}} & \sqrt{P_{\mathrm{LZ}}} \end{pmatrix}$$

$$U(\mathrm{cycle}) = \left[T\, U_{\mathrm{LZ}}^{\circlearrowleft}\, T\right] \mathrm{e}^{-i\varphi} \left[U_{\mathrm{LZ}}^{\circlearrowleft}\right]$$

$$\mathcal{Q} = \int \mathcal{I}(t)dt \approx \lambda_{\circlearrowleft} \mathcal{Q}_{\mathrm{LZ}}^{\circlearrowleft} - [T\mathrm{e}^{-i\varphi} U_{\mathrm{LZ}}^{\circlearrowleft}]^\dagger \lambda_{\circlearrowleft} \mathcal{Q}_{\mathrm{LZ}}^{\circlearrowleft} [T\mathrm{e}^{-i\varphi} U_{\mathrm{LZ}}^{\circlearrowleft}]$$

Long time Counting Statistics

Naive expectation:

Probabilistic point of view implies

$$\delta Q \quad \propto \quad \sqrt{t}$$

Quantum result:

The eigenvalues $Q_{\pm}$ of the $\mathcal{Q}$ operator grow linearly with the number of cycles

$$\delta Q \quad \propto \quad t$$

If we have good control over the preparation we can select it to be a Floque state of the quantum evolution operator. For such preparation the linear growth of δQ is avoided, and it oscillates around a residual value.

References and further work

Refs:

Counting statistics for a coherent transition, MC and DC (PRA 2008)

Counting statistics in multiple path geometries, MC and DC (JPA 2008, FQMT proc. 2009)

Quantum stirring of electrons in a ring, IS and DC (PRBs 2008)

Further work:

BEC in 2-sites - Bloch-Josephson oscillations, E.Boukobza, MC, DC and A.Vardi (PRL 2009)

BEC in 2-sites - Landau-Zener transitions, K.Smith-Mannschott, MC, M.Hiller, T.Kottos and DC (PRL 2009)

BEC in 3-sites - Quantum stirring, M.Hiller, T.Kottos and DC (EPL 2008 & PRA 2008)

URL:

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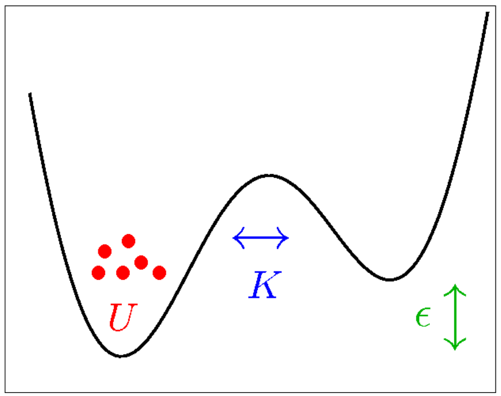
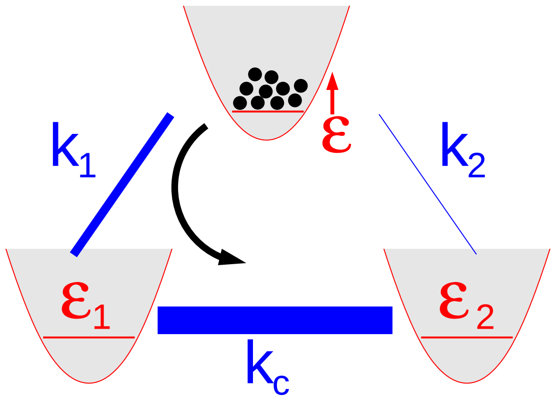




The Bose-Hubbard Hamiltonian (BHH) for a dimer

$$\mathcal{H} \;=\; \sum_{i=1,2}\left[\mathcal{E}_i\hat{n}_i + \frac{U}{2}\hat{n}_i(\hat{n}_i-1)\right] - \frac{K}{2}(\hat{a}_2^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_2)$$

K = hopping
U = interaction
$\mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1$ = bias

N particles in a double well is like spin $j = N/2$ system

$$\mathcal{H} \;=\; -\mathcal{E}\hat{J}_z + U\hat{J}_z^2 - K\hat{J}_x + \text{const}$$

$$u \equiv \frac{NU}{K}, \qquad \varepsilon \equiv \frac{\mathcal{E}}{K}$$

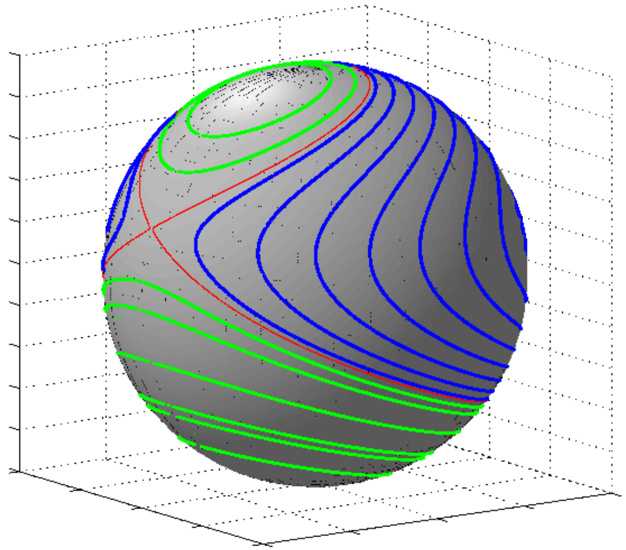
Classical phase space

$$\mathcal{H}(\theta,\varphi) \;=\; \frac{NK}{2}\left[\frac{1}{2}u(\cos\theta)^2 - \varepsilon\cos\theta - \sin\theta\cos\varphi\right]$$

$$\mathcal{H}(\hat{n},\varphi) \;=\; \text{(similar to Josephson/pendulum Hamiltonian)}$$

$$\hat{J}_z \;=\; (N/2)\cos(\theta) \;=\; \underline{\hat{n} = \text{occupation difference}}$$

$$\hat{J}_x \;\approx\; (N/2)\sin(\theta)\cos(\varphi), \qquad \underline{\varphi = \text{relative phase}}$$



Assuming $u>1$ and $|\varepsilon| < \varepsilon_c$

Sea, Islands, Separatrix

Rabi regime:	$u < 1$	(no islands)
Josephson regime:	$1 < u < N^2$	(sea, islands, separatrix)
Fock regime:	$u > N^2$	(empty sea)

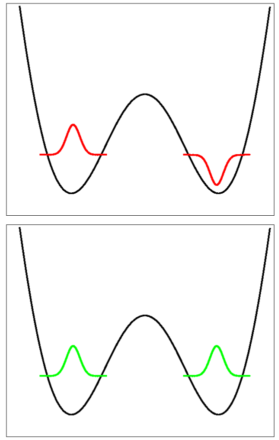
$$\varepsilon_c \;=\; \left(u^{2/3} - 1\right)^{3/2}$$

$$A_c \;\approx\; 4\pi\left(1 - u^{-2/3}\right)^{3/2}$$

Wavepacket dynamics

Coherent state $|\theta\varphi\rangle$ is like a minimal Gaussian wavepacket

Fock state $|n\rangle$ is like equi-latitude annulus

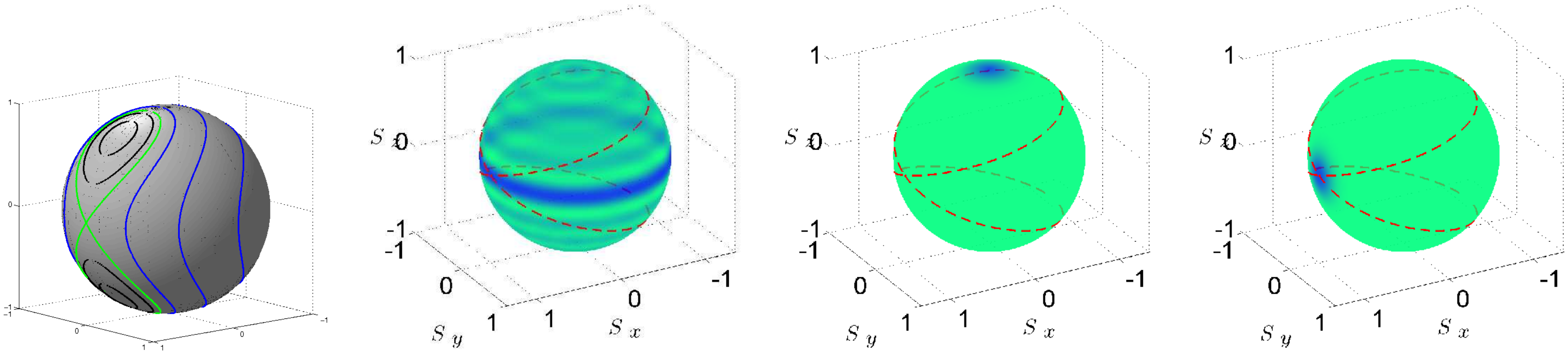


Fock $n{=}0$ preparation - exactly half of the particles in each site

Fock coherent $\theta{=}0$ preparation - all particles occupy the left site

Coherent $\varphi{=}0$ preparation - all particles occupy the symmetric orbital

Coherent $\varphi{=}\pi$ preparation - all particles occupy the antisymmetric orbital



MeanField theory (GPE) = classical evolution of a point in phase space

SemiClassical theory = classical evolution of a distribution in phase space

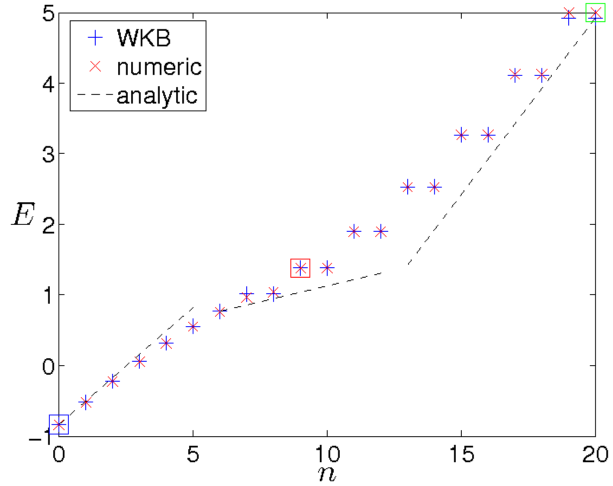
Quantum theory = recurrences, fluctuations (WKB is very good!)

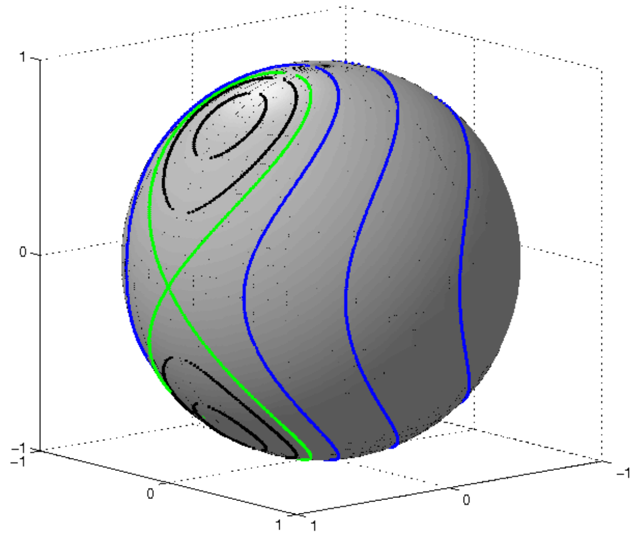
WKB quantization (Josephson regime)

$$h = \text{Planck cell area in steradians} = \frac{4\pi}{N+1}$$

$$A(E_n) = \left(\frac{1}{2}+n\right)h$$

$$\omega(E) \equiv \frac{dE}{dn} = \left[\frac{1}{h}A'(E)\right]^{-1}$$





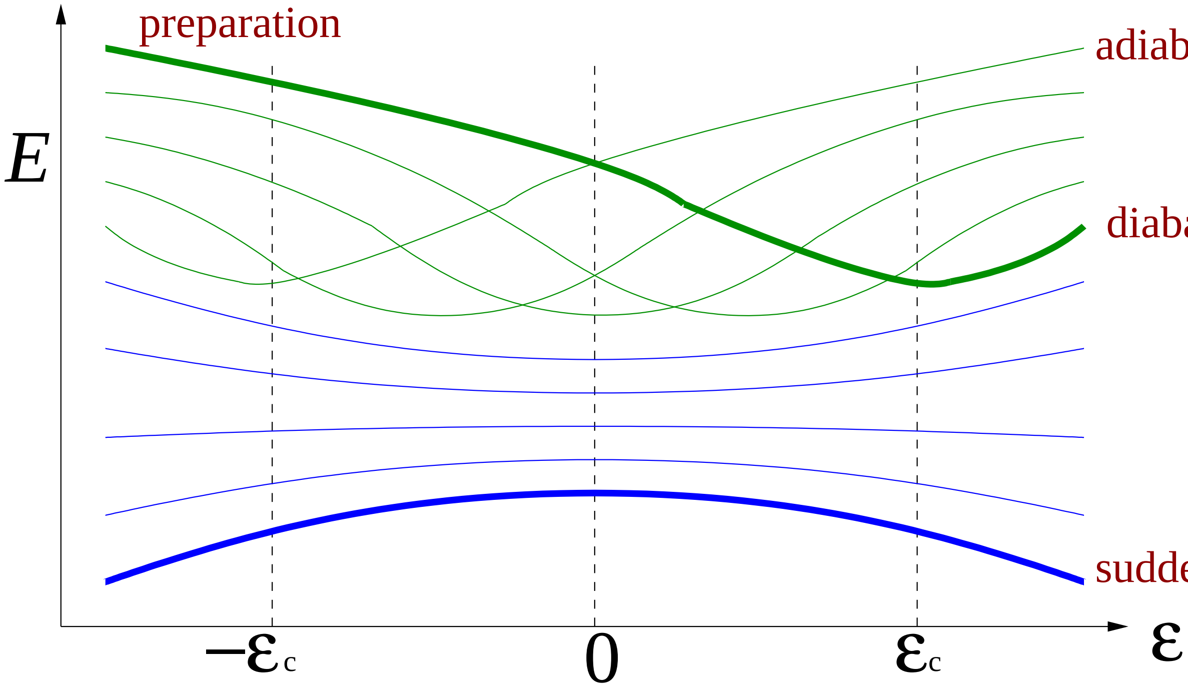
$$\omega_K \approx K = \text{Rabi Frequency}$$

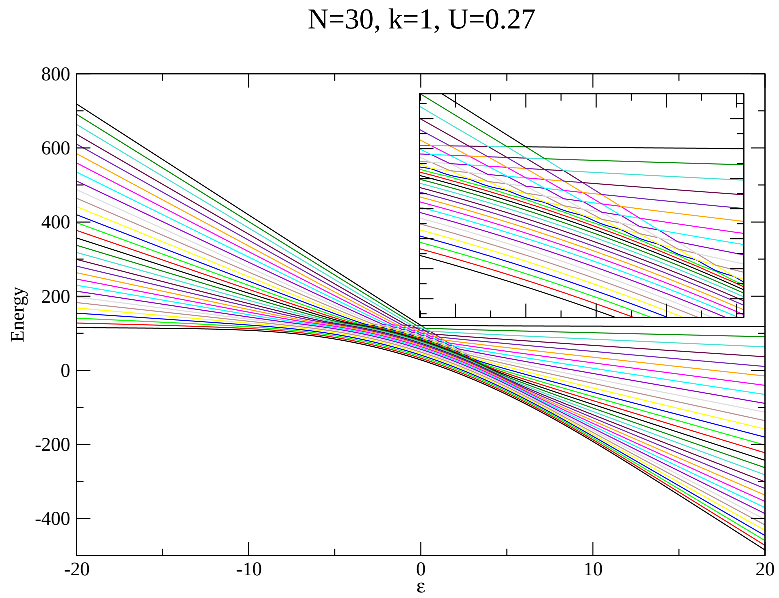
$$\omega_J \approx \sqrt{NUK} = \sqrt{u}\,\omega_K$$

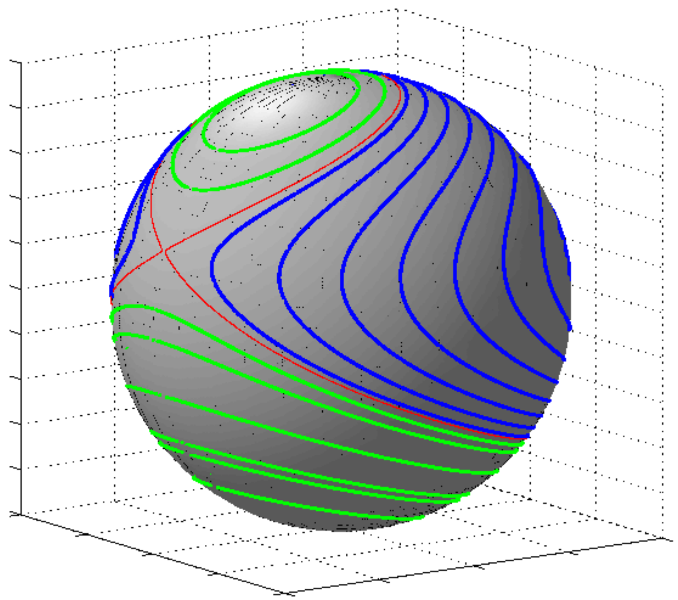
$$\omega_+ \approx NU = u\,\omega_K$$

$$\omega_x \approx \left[\frac{1}{2}\log\left(\frac{N^2}{u}\right)\right]^{-1}\omega_J$$

The many body Landau-Zener transition



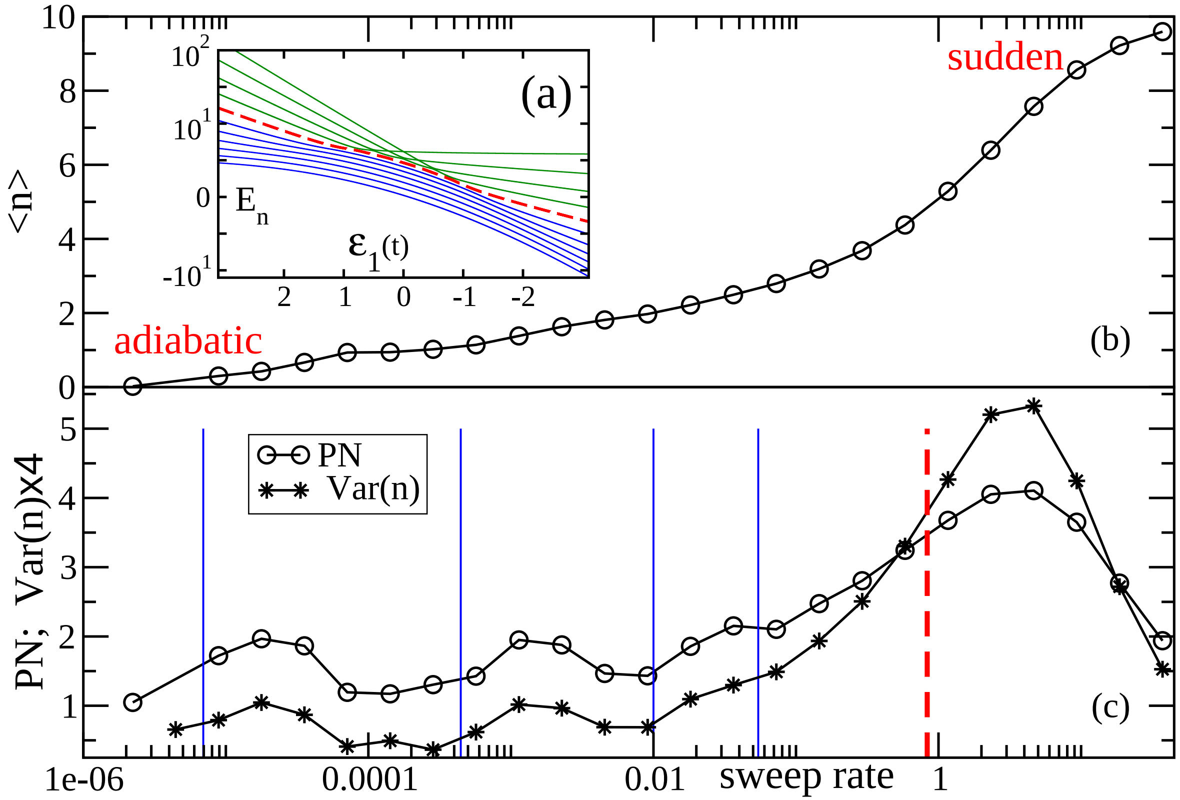


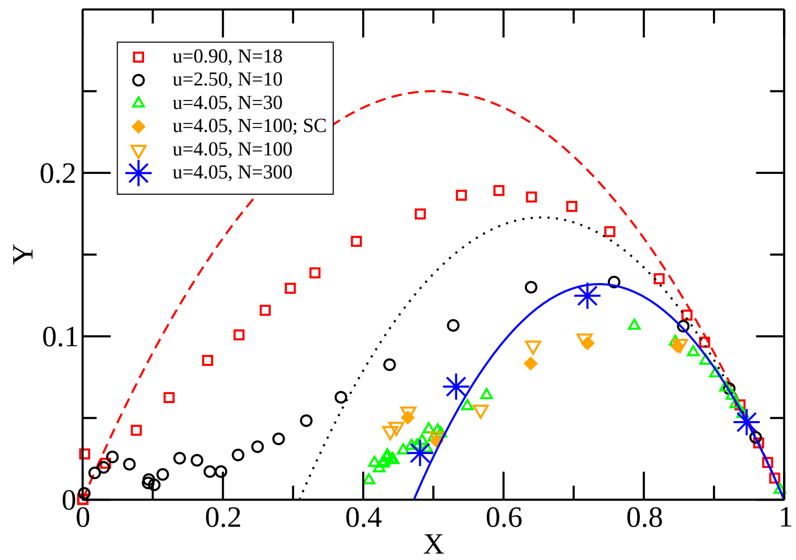


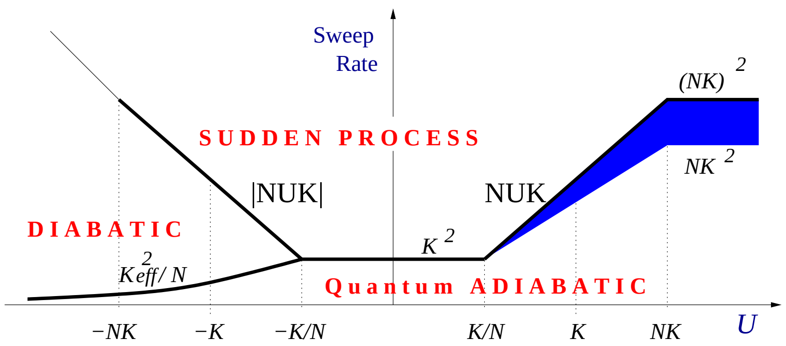
Dynamical scenarios:

adiabatic/diabatic/sudden

Occupation Statistics





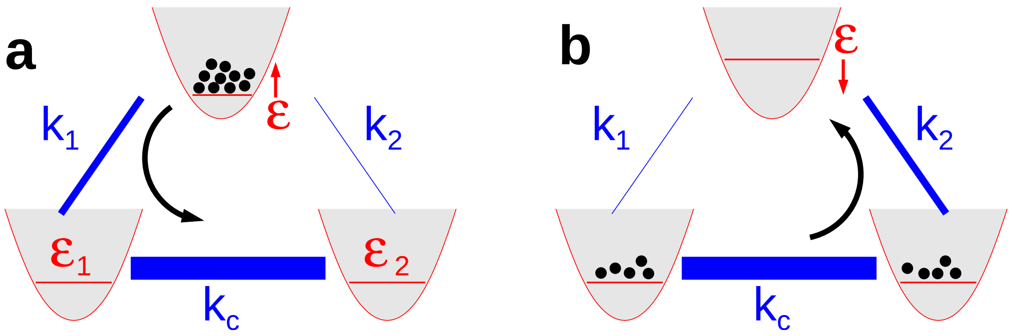


Adiabtic-diabatic (quantum) crossover

Diabatic-sudden (semiclassical) crossover

Sub-binomial scaling of Var(n) versus $\langle n \rangle$

Quantum Stirring in a 3 site system



Control parameters:

$$X_1 = \left(\frac{1}{k_2} - \frac{1}{k_1}\right)$$

$$X_2 = \mathcal{E}_0 \qquad (\mathcal{E}_1{=}\mathcal{E}_2{=}0)$$

$$\boldsymbol{X} = (X_1, X_2)$$

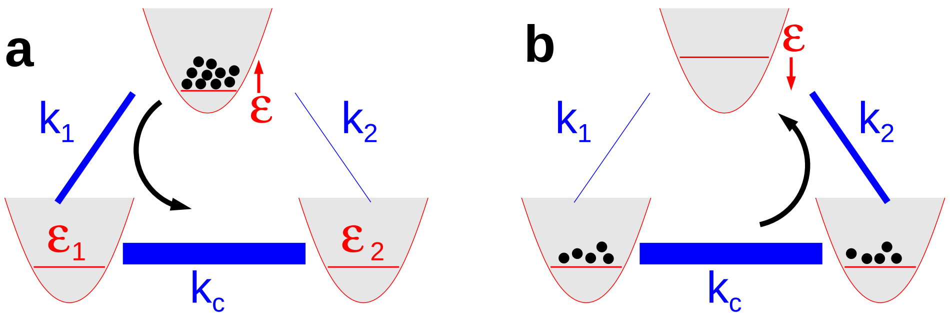
U = the inter-atomic interaction

$$\hat{\mathcal{H}} = \sum_{i=0}^{2} \mathcal{E}_i n_i + \frac{U}{2}\sum_{i=0}^{2} \hat{n}_i(\hat{n}_i - 1) - k_c(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) - k_1(\hat{b}_0^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{b}_0) - k_2(\hat{b}_0^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_0)$$

The induced current: $I = -G\dot{\mathcal{E}}$ $\quad (G = G_2)$

The pumped particles: $Q = \oint I dt = \oint \boldsymbol{G} \cdot \boldsymbol{dX}$ $\quad$ (per cycle)

Stirring of BEC

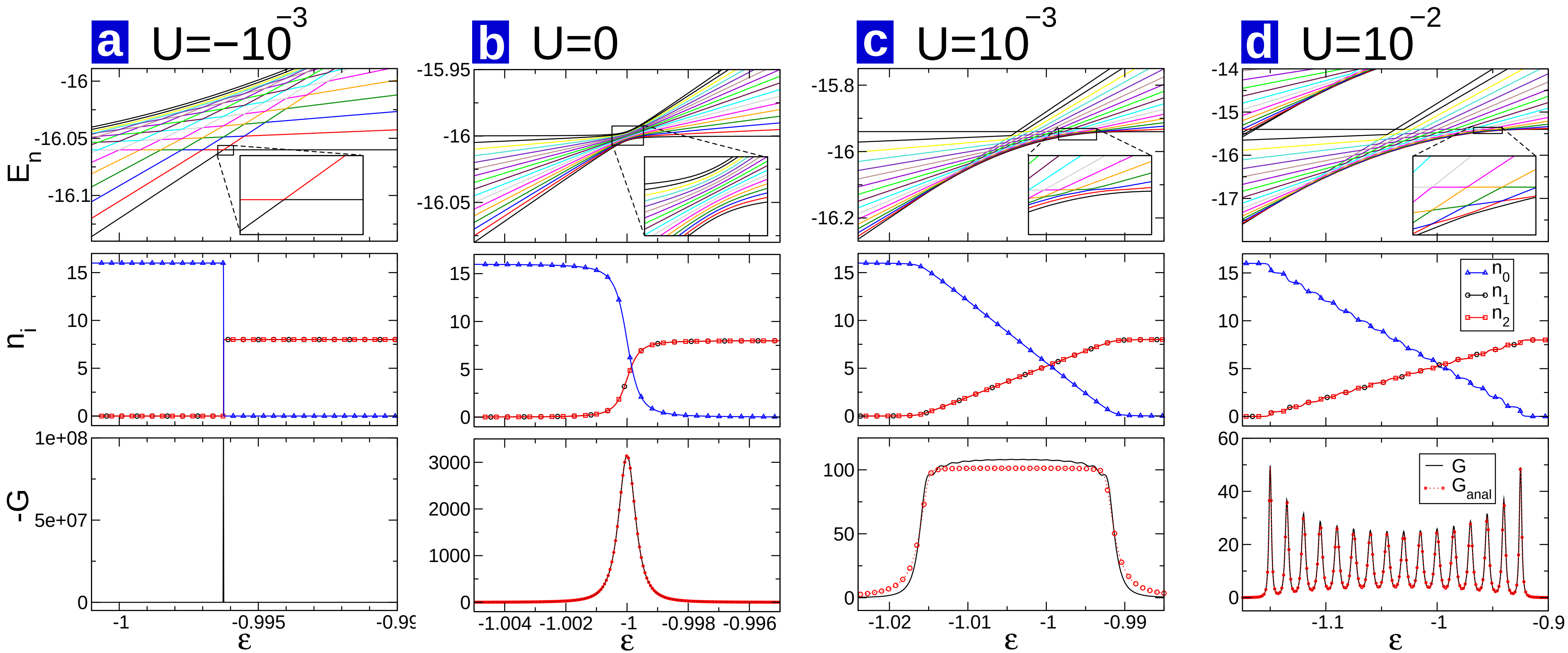


strong attractive interaction: classical ball dynamics

negligible interaction ($|U| \ll \kappa/N$): mega-crossing

weak repulsive interaction: gradual crossing

strong repulsive interaction ($U \gg N\kappa$): sequential crossing



Results for the geometric conductance

$$G(R) = -N \frac{(k_1^2 - k_2^2)/2}{[(\varepsilon - \varepsilon_-)^2 + 2(k_1+k_2)^2]^{3/2}}$$ mega crossing

$$G(J) \approx -\left[\frac{k_1 - k_2}{k_1 + k_2}\right] \frac{1}{3U}$$ gradual crossing

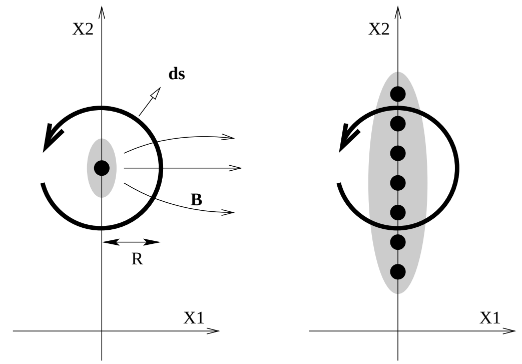
$$G(F) = -\left(\frac{k_1 - k_2}{k_1 + k_2}\right) \sum_{n=1}^{N} \frac{(\delta\varepsilon_n)^2}{[(\varepsilon - \varepsilon_n)^2 + (2\delta\varepsilon_n)^2]^{3/2}},$$ sequential crossing

where:

R = Rabi regime ($U \ll \kappa/N$)

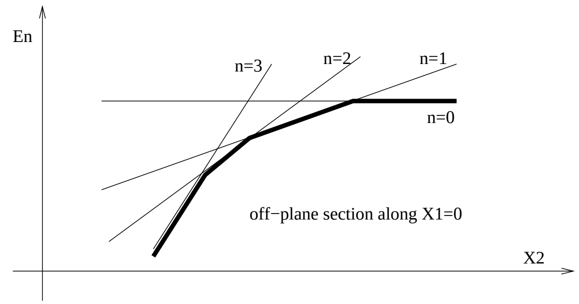
J = Josephson regime ($\kappa/N \ll U \ll N\kappa$)

F = Fock regime ($U \gg N\kappa$)



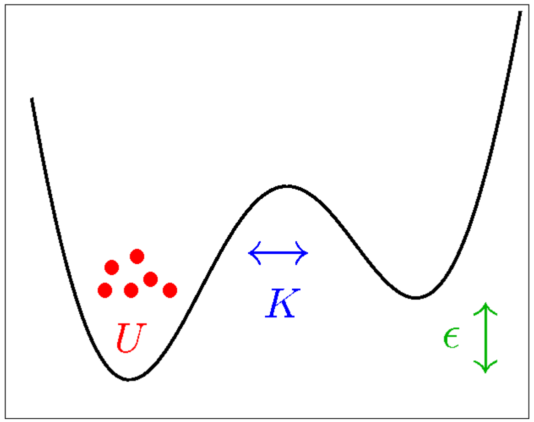
Observation:

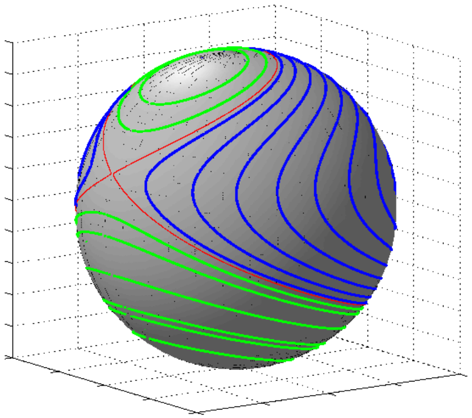
It is possible to pump $Q \gg N$ per cycle.

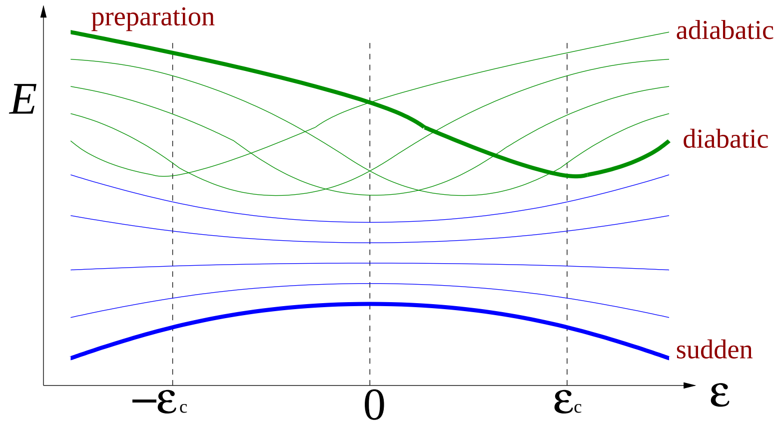


Summary

- Semiclassical and WKB analysis of the dynamics
- Occupation statistics in a time dependent scenario
- The adiabatic / diabatic / sudden crossovers
- Quantum stirring: mega / gradual / sequential crossings







Main messages

- FCS for a 2-site coherent transition - the simplest solvable model.
- Counting statistics for a 3-site system - multiple path geometry.
- Restricted QCC fails for multiple path transitions.
- Coherent splitting is not like incoherent partioning.
- Splitting ratio approach to quantum stirring (vs Kubo).
- Interference in the calculation of $\mathrm{Var}(\mathcal{Q})$.
- Exact vs adiabatic results for $\mathrm{Var}(\mathcal{Q})$.
- Long time counting statistics for multiple cycle stirring process.
- Stirring of BEC - the effect of interactions.