The conductance of small mesoscopic disordered rings

Doron Cohen, Ben-Gurion University

Collaborations:

Tsampikos Kottos (Wesleyan)
Holger Schanz (Gottingen)
Swarnali Bandopadhyay (BGU)
Yoav Etzioni (BGU)
Alex Stotland (BGU)
Rangga Budoyo (Wesleyan)
Tal Peer (BGU)

Discussions:

Michael Wilkinson (UK)
Bernhard Mehlig (Goteborg)
Yuval Gefen (Weizmann)
Shmuel Fishman (Technion)

$ISF, $GIF, $DIP, $BSF

The model

Non interacting "spinless" electrons in a ring.

$$\mathcal{H}(r, p; \Phi(t))$$

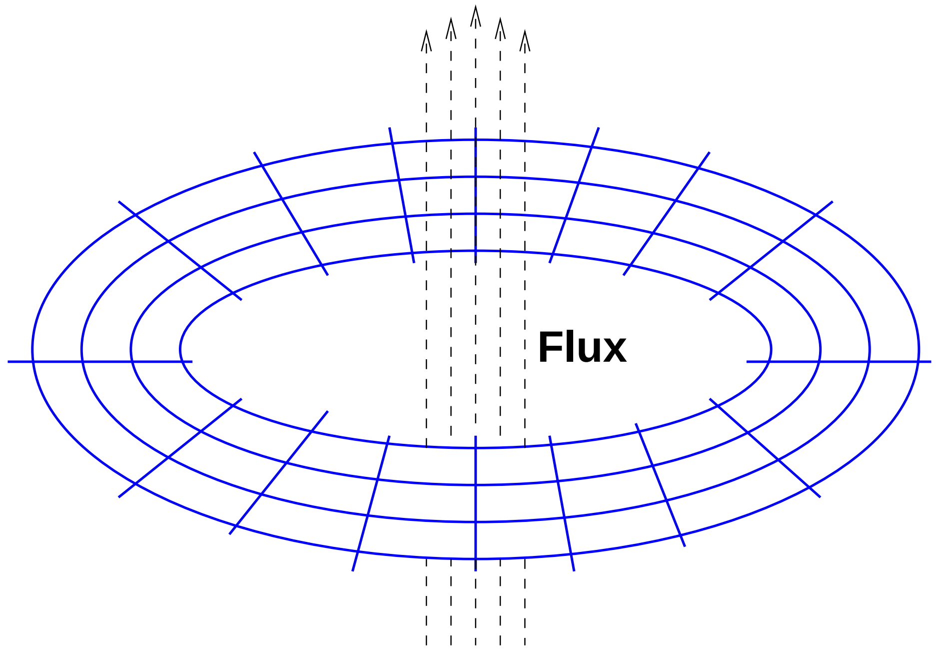
$$-\dot{\Phi} \quad = \quad \text{electro motive force (RMS)}$$

$$\boldsymbol{G}\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$

$$\boldsymbol{G} = \pi \left(\frac{e}{L}\right)^2 \mathrm{DOS}^2 \;\langle\langle|v_{mn}|^2\rangle\rangle$$

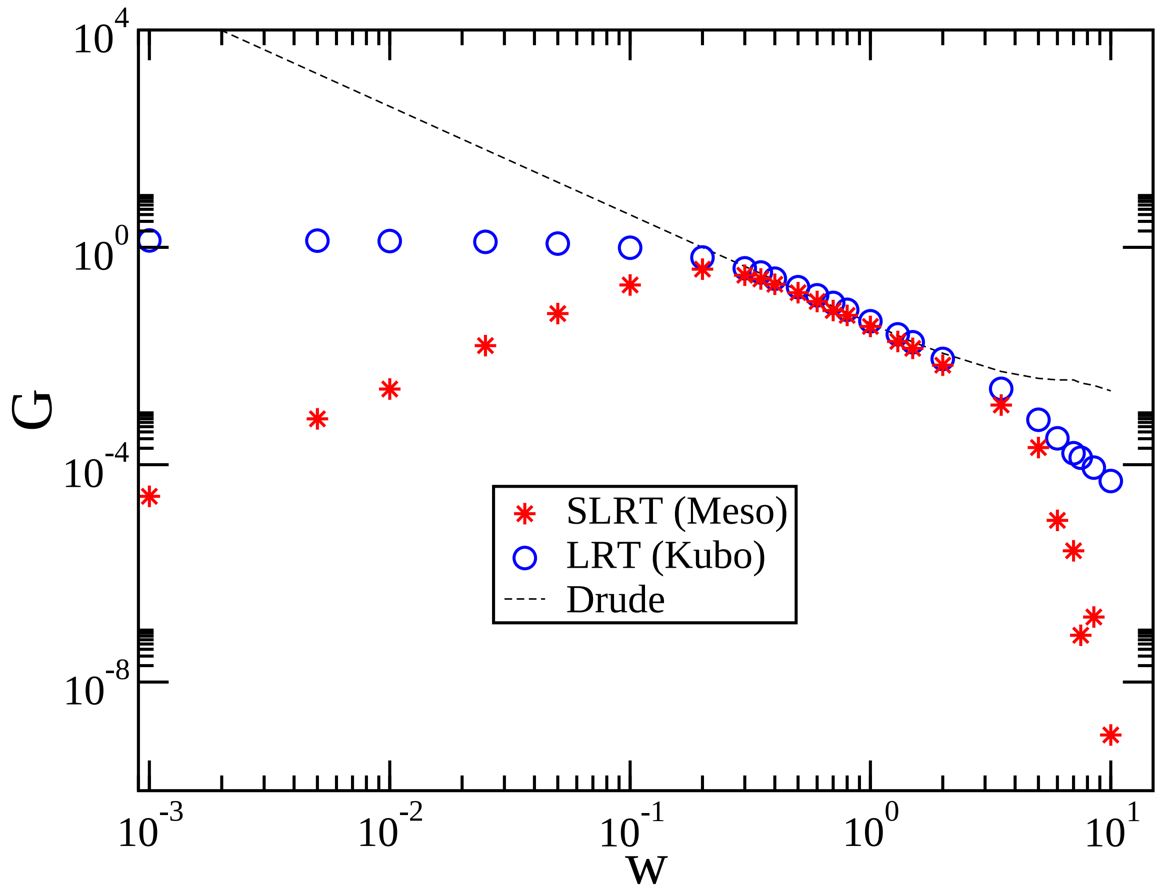
$$\langle\langle|v_{mn}|^2\rangle\rangle_{\text{harmonic}} \ll \langle\langle|v_{mn}|^2\rangle\rangle \ll \langle\langle|v_{mn}|^2\rangle\rangle_{\text{algebraic}}$$

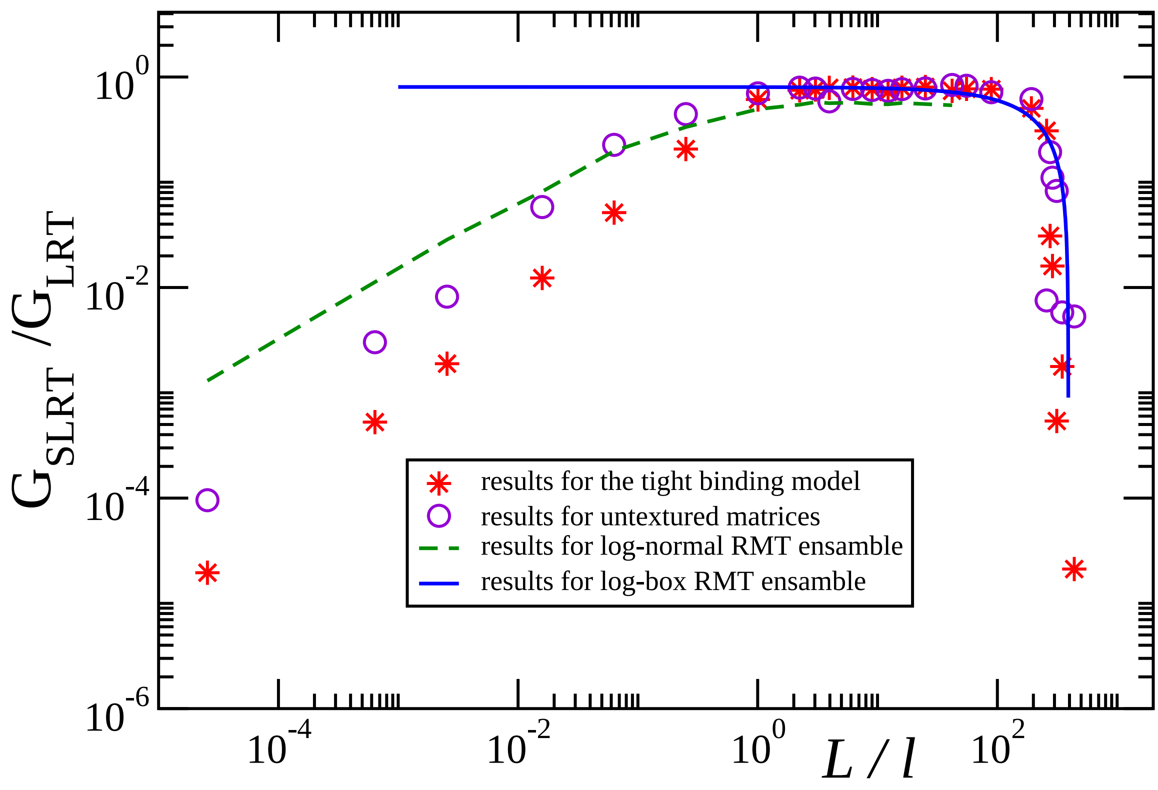
$\mathcal{M}$ mode ring of length L with disorder W



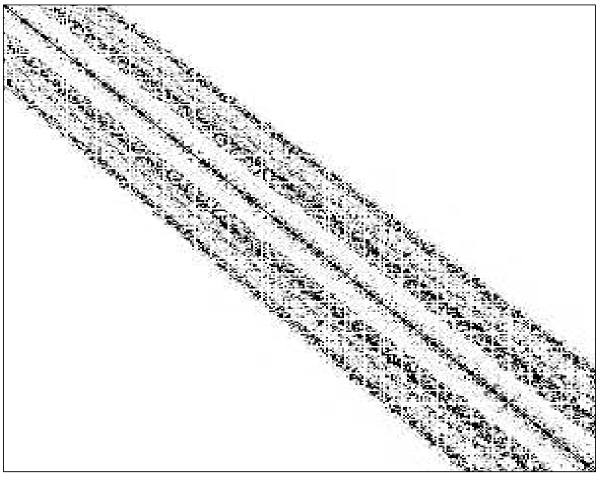
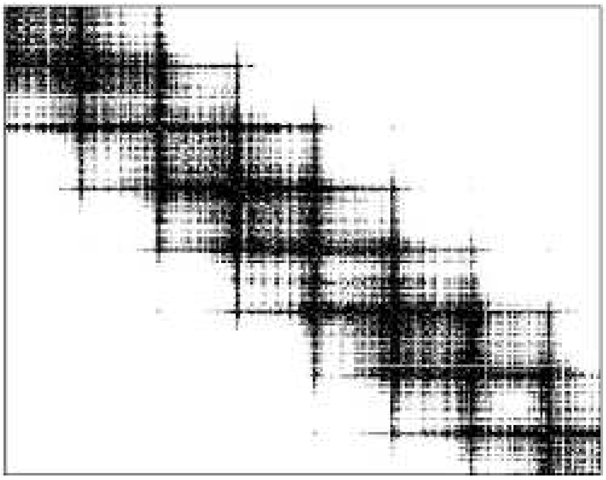
Numerical Results

Regimes: ballistic; diffusive; localizaion





Linear response theory (LRT)

$$\mathcal{H} = \{E_n\} - \frac{e}{L}\Phi(t)\{v_{nm}\}$$

$$\boldsymbol{G} \;=\; \pi\left(\frac{e}{L}\right)^2 \sum_{n,m} |v_{mn}|^2 \;\delta_T(E_n - E_F)\;\delta_\Gamma(E_m - E_n)$$

$$\boldsymbol{G} \;=\; \pi\left(\frac{e}{L}\right)^2 \mathrm{DOS}^2 \;\langle\langle |v_{mn}|^2 \rangle\rangle_{\mathrm{algebraic}}$$

applies if

EMF driven transitions $\ll$ relaxation

otherwise

connected sequences of transitions are essential.

leading to

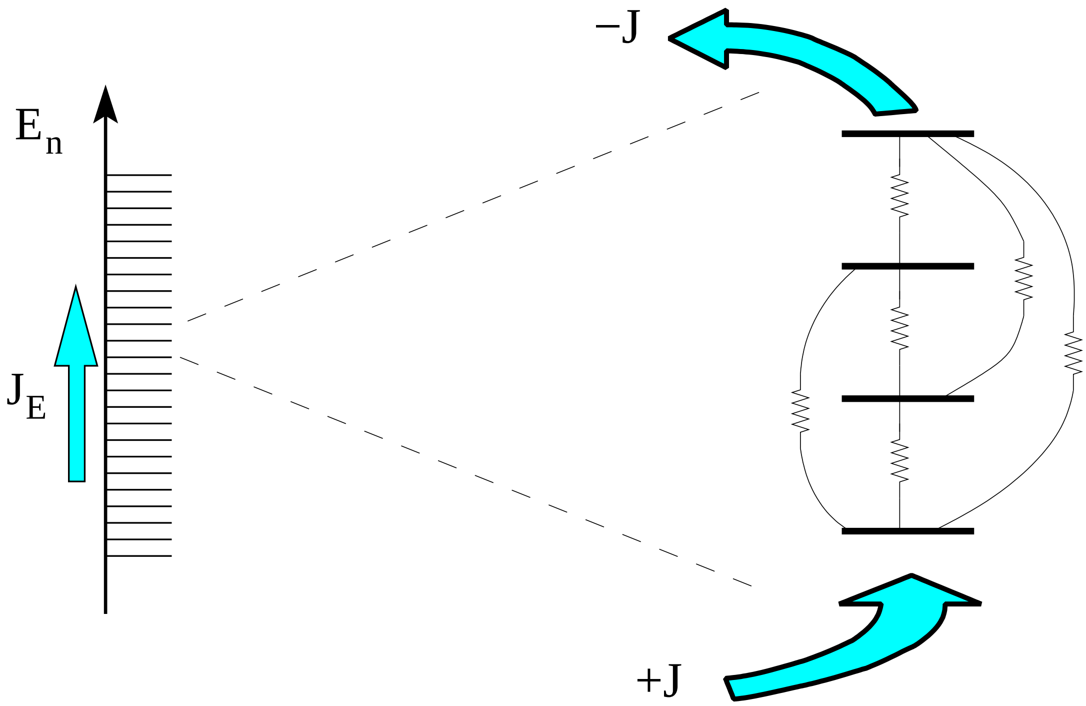
Semi Linear Response Theory (SLRT)

Semi Linear Response Theory (SLRT)

$$\mathcal{H} = \{E_n\} - \frac{e}{L}\Phi(t)\{v_{nm}\}$$

$$\frac{dp_n}{dt} = -\sum_m w_{nm}(p_n - p_m)$$

$$w_{nm} \quad = \quad \text{const} \times \mathrm{g}_{nm} \times \text{EMF}^2$$

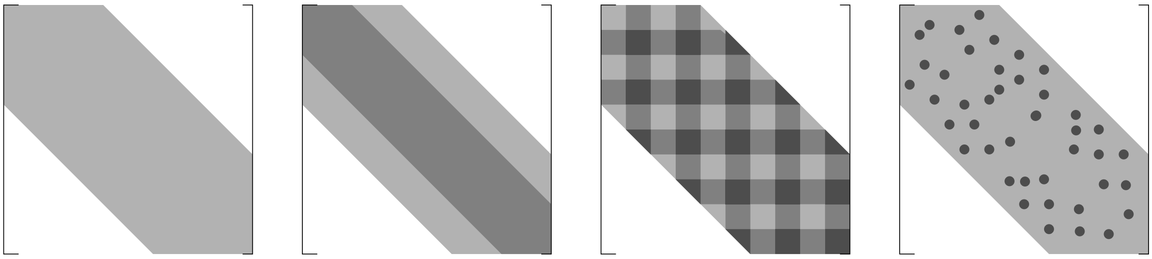


$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3}\ \frac{|v_{nm}|^2}{(E_n-E_m)^2}\ \delta_\Gamma(E_n-E_m)$$

$$\langle\langle|v_{mn}|^2\rangle\rangle \ \equiv \ \text{inverse resistivity of the network}$$

$$\boldsymbol{G} \ = \ \pi\left(\frac{e}{L}\right)^2 \text{DOS}^2\ \langle\langle|v_{mn}|^2\rangle\rangle$$

Bandprofile, sparsity and texture



$$\boldsymbol{G} \;=\; \pi \left(\frac{e}{L}\right)^2 \mathrm{DOS}^2 \; \langle\langle |v_{mn}|^2 \rangle\rangle$$

$$\langle\langle |v_{mn}|^2 \rangle\rangle \;\equiv\; \text{inverse resistivity of the network}$$

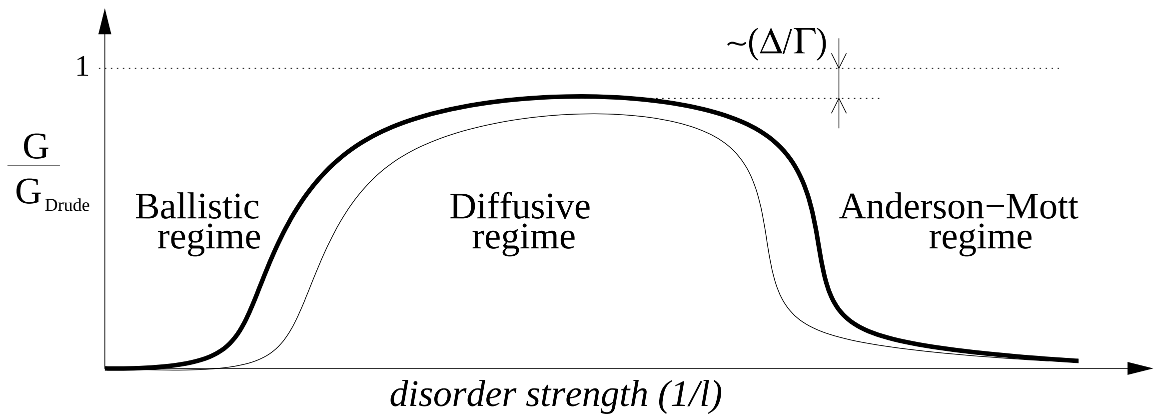
Bounds:

$$\langle\langle |v_{mn}|^2 \rangle\rangle_{\text{harmonic}} \ll \langle\langle |v_{mn}|^2 \rangle\rangle \ll \langle\langle |v_{mn}|^2 \rangle\rangle_{\text{algebraic}}$$

Analytical estimates:

- Mixed average scheme
- Variable range hopping scheme

Conductance versus disorder



Naive expectation (assuming $\Gamma > \Delta$):

$$G = \frac{e^2}{2\pi\hbar}\mathcal{M}\frac{\ell}{L} + \mathcal{O}\left(\frac{\Delta}{\Gamma}\right)$$

$$L = \text{perimeter of the ring}$$

$$\ell = \text{mean free path} \propto W^2$$

$$\ell_\infty = \text{localization length} \approx \mathcal{M}\ell$$

Ballistic regime: $L \ll \ell$

Diffusive regime: $\ell \ll L \ll \ell_\infty$

Anderson regime: $\ell_\infty \ll L$

Strategy of analysis

Given W...

Characterization of the eigenstates:

- participation ratio (PR)

Characterization of v_{nm} and RMT modeling

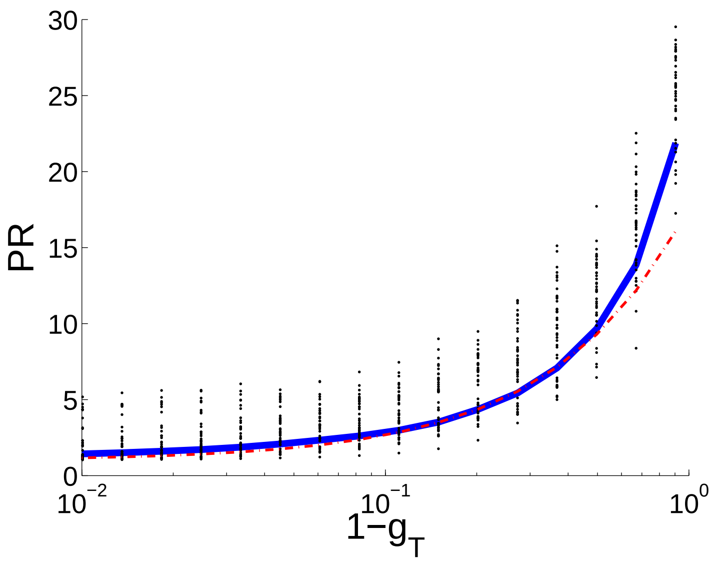
- bandwidth
- sparsity (p)
- texture

Approximation schemes for G

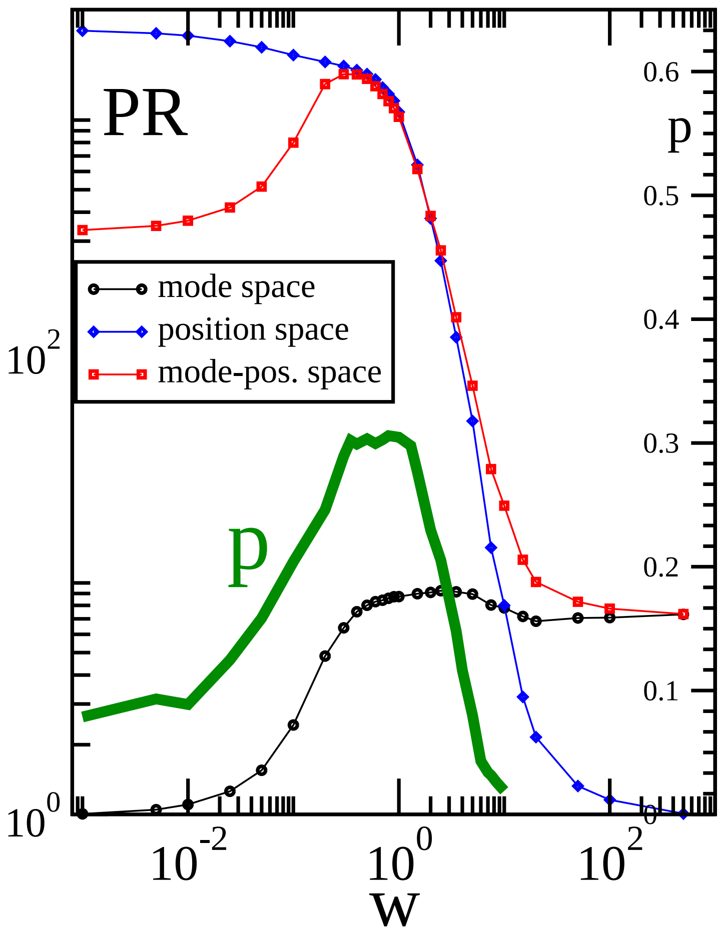
- Mixed average
- Variable range hopping estimate

Ergodicity of the eigenstates

- Weak disorder (ballistic rings): Wavefunctions are localized in mode space.
- Strong disorder (Anderson localization): Wavefunctions are localized in real space.



The PR of eigenstates of a ring with a single scatterer. The horizontal axis is the reflection of the scatterer.



The PR of eigenstates of a ring with disorder. The horizontal axis is W.

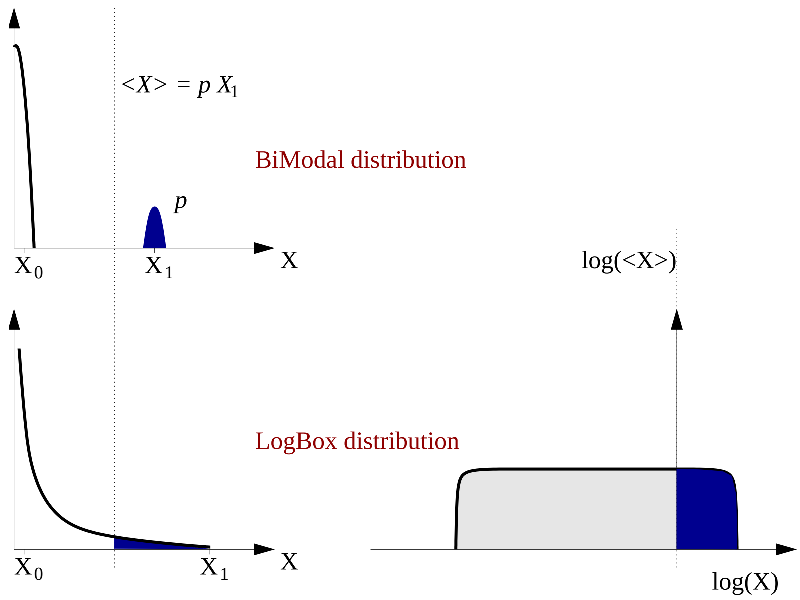
The sparsity (p) of the perturbation matrix is related to the ergodicity of the eigenstates.

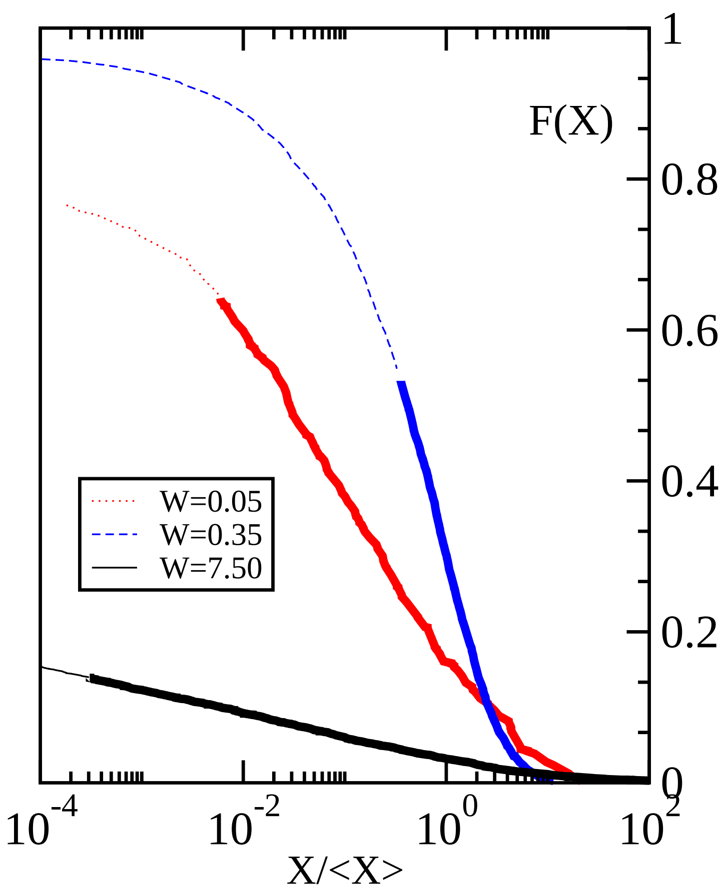
$\{|v_{nm}|^2\}$ **as a random matrix** $\{X\}$

The fraction of "large" elements:

$p \equiv \mathrm{F}(\langle X \rangle)$

Sparsity: $p \ll 1$.





Histograms of X:

Ballistic:

$X \sim \mathrm{LogNormal}$

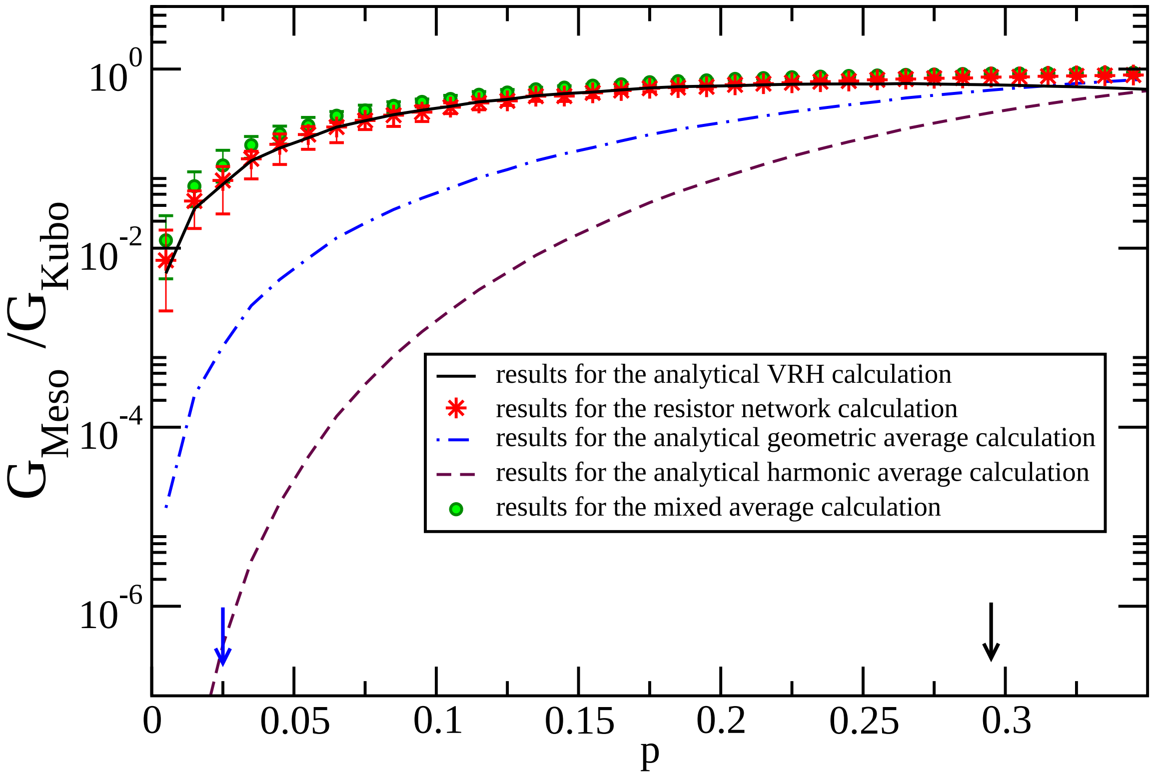
Localization:

$X \sim \mathrm{LogBox}$

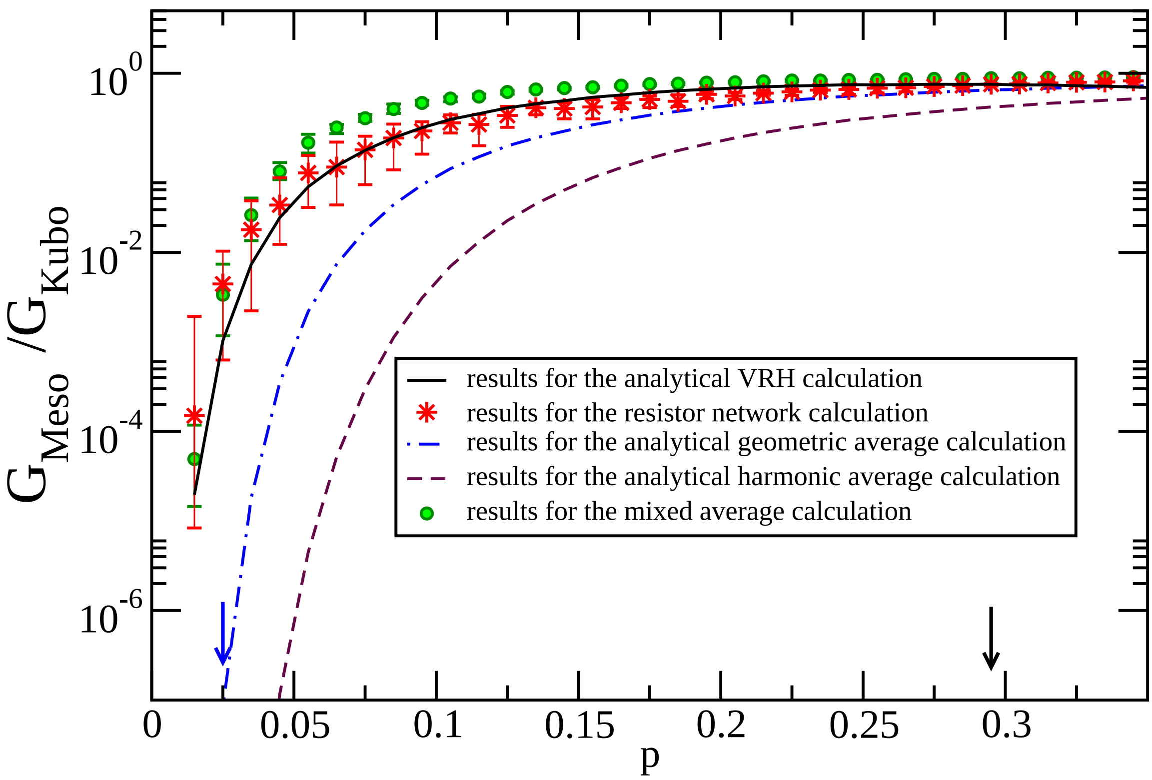
RMT based prediction for $G_{\mathbf{SLRT}}/G_{\mathbf{LRT}}$

RMT implied dependence on p

Log-normal distribution:



Log-box distribution:



The VRH estimate

$$\boldsymbol{G} \; = \; \pi\hbar \left(\frac{e}{L}\right)^2 \sum_{n,m} |v_{mn}|^2 \; \delta_T(E_n - E_F) \; \delta_\Gamma(E_m - E_n)$$

$$\boldsymbol{G} \; = \; \frac{1}{2} \left(\frac{e}{L}\right)^2 \varrho_{\mathrm{F}} \int \tilde{C}_{\mathrm{qm}}(\omega) \; \delta_\Gamma(\omega) \; d\omega$$

$$\tilde{C}_{\text{qm-LRT}}(\omega) \quad \equiv \quad 2\pi \varrho_{\mathrm{F}} \, \langle X \rangle$$

$$\tilde{C}_{\text{qm-SLRT}}(\omega) \quad \equiv \quad 2\pi \varrho_{\mathrm{F}} \, \overline{X}$$

where by definition: $\left(\frac{\omega}{\Delta}\right) \mathrm{Prob}\left(X > \overline{X}\right) \; \sim \; 1$

For strong disorder we get:

$$\overline{X} \quad \approx \quad v_{\mathrm{F}}^2 \, \exp\left(-\frac{\Delta_\ell}{\omega}\right)$$

$$\boldsymbol{G} \quad \propto \quad \int \exp\left(-\frac{\Delta_\ell}{|\omega|}\right) \, \exp\left(-\frac{|\omega|}{\omega_c}\right) \, d\omega$$

LRT, SLRT and beyond

$$-\dot{\Phi} \quad = \quad \text{electro motive force (RMS)}$$
$$G\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$

Semi linear response theory

[1] D. Cohen, T. Kottos and H. Schanz,
"Rate of energy absorption by a closed ballistic ring",
(JPA 2006)

[2] S. Bandopadhyay, Y. Etzioni and D. Cohen,
The conductance of a multi-mode ballistic ring,
(EPL 2006)

[3] M. Wilkinson, B. Mehlig, D. Cohen,
The absorption of metallic grains,
(EPL 2006)

[4] D. Cohen,
"From the Kubo formula to variable range hopping",
(PRB 2007)

[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen,
The conductance of disordered rings,
(JPA / FTC 2008)

Beyond (semi) linear response theory

[6] D. Cohen and T. Kottos,
"Non-perturbative response of Driven Chaotic Mesoscopic Systems",
(PRL 2000)

[7] A. Stotland and D. Cohen,
"Diffractive energy spreading and its semiclassical limit",
(JPA 2006)

[8] A. Silva and V.E. Kravtsov,
Beyond FGR,
(PRB 2007)

[9] D.M. Basko, M.A. Skvortsov and V.E. Kravtsov,
Dynamical localization,
(PRL 2003)

Conclusions

(*) Wigner ($\sim$ 1955):
The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.

Not always...

1. Ballistic ring $\Longrightarrow$ log-normal distribution.
2. Strong localization $\Longrightarrow$ log-box distribution.
3. Resistors network calculation to get G_{SLRT}.
4. Generalization of the VRH estimate
5. SLRT is essential whenever the distribution of matrix elements is wide ("sparsity") or if the matrix has "texture".
6. Other applications of SLRT...