Many-body dynamical localization and thermalization

Doron Cohen, Ben-Gurion University

Based on:

Semiclassical theory of strong localization for quantum thermalization,
Christine Khripkov, Amichay Vardi, Doron Cohen, Phys. Rev. E 97, 022127 (2018).

Many-body dynamical localization and thermalization,
Christine Khripkov, Amichay Vardi, Doron Cohen, Phys. Rev. A 101, 043603 (2020).

Inspired by the studies of Shmuel Fishman and followers regarding “Dynamical localization”.

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Many-body dynamical localization and thermalization

Christine Khripkov,[1,2] Amichay Vardi,[1] and Doron Cohen[2]
[1]*Department of Chemistry, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel*
[2]*Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel*

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The study of *dynamical* localization in a low-dimensional chaotic system was pioneered in the publication "Chaos, Quantum Recurrences, and Anderson Localization" by Fishman, Grempel, and Prange [1], which had been motivated by the puzzling numerical observation of quantum-suppressed chaotic diffusion, by Casati, Chirikov, Izrailev, and Ford [2].

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Chaos, Quantum Recurrences, and Anderson Localization

Shmuel Fishman, D. R. Grempel, and R. E. Prange
Department of Physics and Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742
(Received 6 April 1982)

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The present line of study has been inspired by discussions with S. Fishman, who passed away recently. This research was supported by the Israel Science Foundation (Grant No. 283/18).

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Dynamical localization

The kicked rotor model can be regarded as a 2 dof system with time-independent Hamiltonian, whose energy surface is a cylinder of infinite length in the n (angular momentum) coordinate. The observation of Fishman et al was that the eigenstates are n-localized, as in the Anderson model.

Features:

- Classically we have a chaotic sea in the range $-\infty < n < \infty$.
- Classical: the spreading of a wavepacket feature diffusion with coefficient D.
- Quantum: the spreading stops after a breaktime t^* that reflects the localization length ξ.

Issues:

- Consider generic models that have a finite chaotic sea (how to define localization?).
- Consider generic models that have more than 2 dof.
- Consider specifically thermalization scenario where the reaction coordinate exhibits diffusion.
- Generalize relation between diffusion (D) and localization (ξ).
- Define localization as lack of ergodicity (memory of initial conditions).
- Define localization as opposed to ergodicity in the classical limit (mobility border?).
- Exclude the possibility of localization due to fragmentation of the energy surface.
- Exclude the possibility of localization due to dynamical stability (mixed phase space).
- Exclude the possibility of perturbative localization.

Manifestation of localization in thermalization?

$$\frac{\partial f(x)}{\partial t} = \frac{\partial}{\partial x}\left(g(x)D(x)\frac{\partial}{\partial x}\left(\frac{1}{g(x)}f(x)\right)\right)$$

$g(x)$ - local density of states

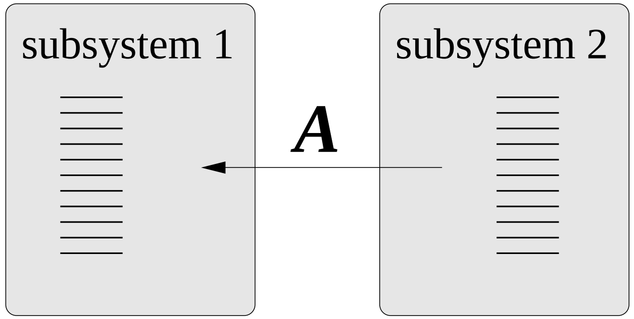
$$D(x) = \int_{-\infty}^{\infty} \omega^2 S_1(\omega)S_2(\omega)\frac{d\omega}{2\pi}$$

Rate of energy transfer [FPE version]:

$$A(x) = \partial_x D + (\beta_1 - \beta_2)D$$

For canonical preparation:

$$\langle A(x)\rangle = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\langle D(x)\rangle$$



The reaction coordinate is x:
$x =$ refers to subsystem 1
$E-x =$ refers to subsystem 2
For a Bose-Hubbard system,
the reaction coordinate might be
the occupation of the subsystem.

Hurowitz, DC (EPL 2011) - MEQ version

Tikhonenkov, Vardi, Anglin, DC (PRL 2013)] - FPE version

Bunin, Kafri (JPA 2013) - NFT version

Khripkov, Vardi, DC (NJP 2015) - Resistor network calculation of $D(x)$

Question: Do we have $\xi = g(x)D(x)$?

The Bose Hubbard Hamiltonian

The system consists of N bosons in L sites.
Optionally we can add a gauge-field Φ.

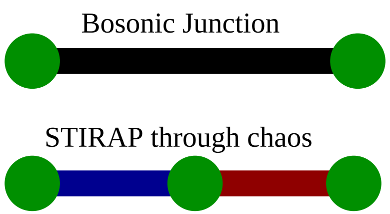
$$\mathcal{H}_{\rm BHH} \;=\; \frac{U}{2}\sum_{j=1}^{L} a_j^{\dagger} a_j^{\dagger} a_j a_j \;-\; \sum_{\langle i,j\rangle} \frac{K_{ij}}{2} a_i^{\dagger} a_j$$

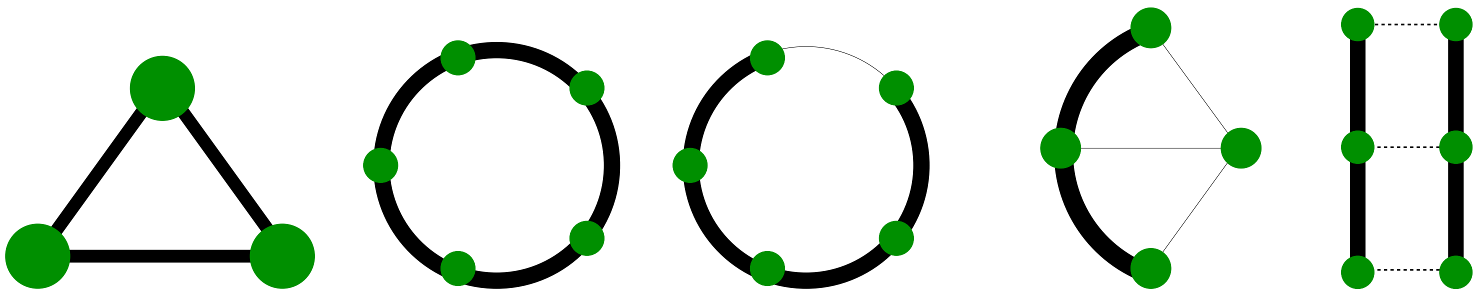
$$u_L \;\equiv\; L\frac{NU}{K} \qquad \text{[classical, stability, supefluidity, self-trapping]}$$

$$\gamma_L \;\equiv\; \frac{LU}{NK} \qquad \text{[quantum, Mott-regime]} \quad \propto \frac{1}{N^2}$$

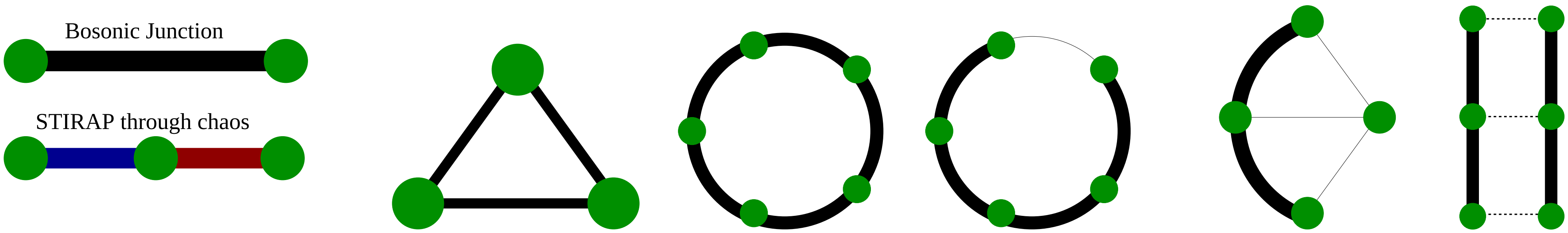
The two dimensionless parameters have a well defined value also in the GP limit.

$L = 2, 3, 4, 5, 6$





Minimal configurations



Dimer (L=2): Bosonic Josephson junction; Pendulum physics [1a].

Driven dimer: Landau-Zener dynamics [1b]; Kapitza effect [1c]; Zeno effect [1d]; Scars [1e].

Rings (L>2): Superfluidity [2a]; SF-Mott transition [2b].

Driven trimer: Many body STIRAP [3a]; Hamiltonian Hysteresis [3b]; Quasistatic transfer protocols [3c].

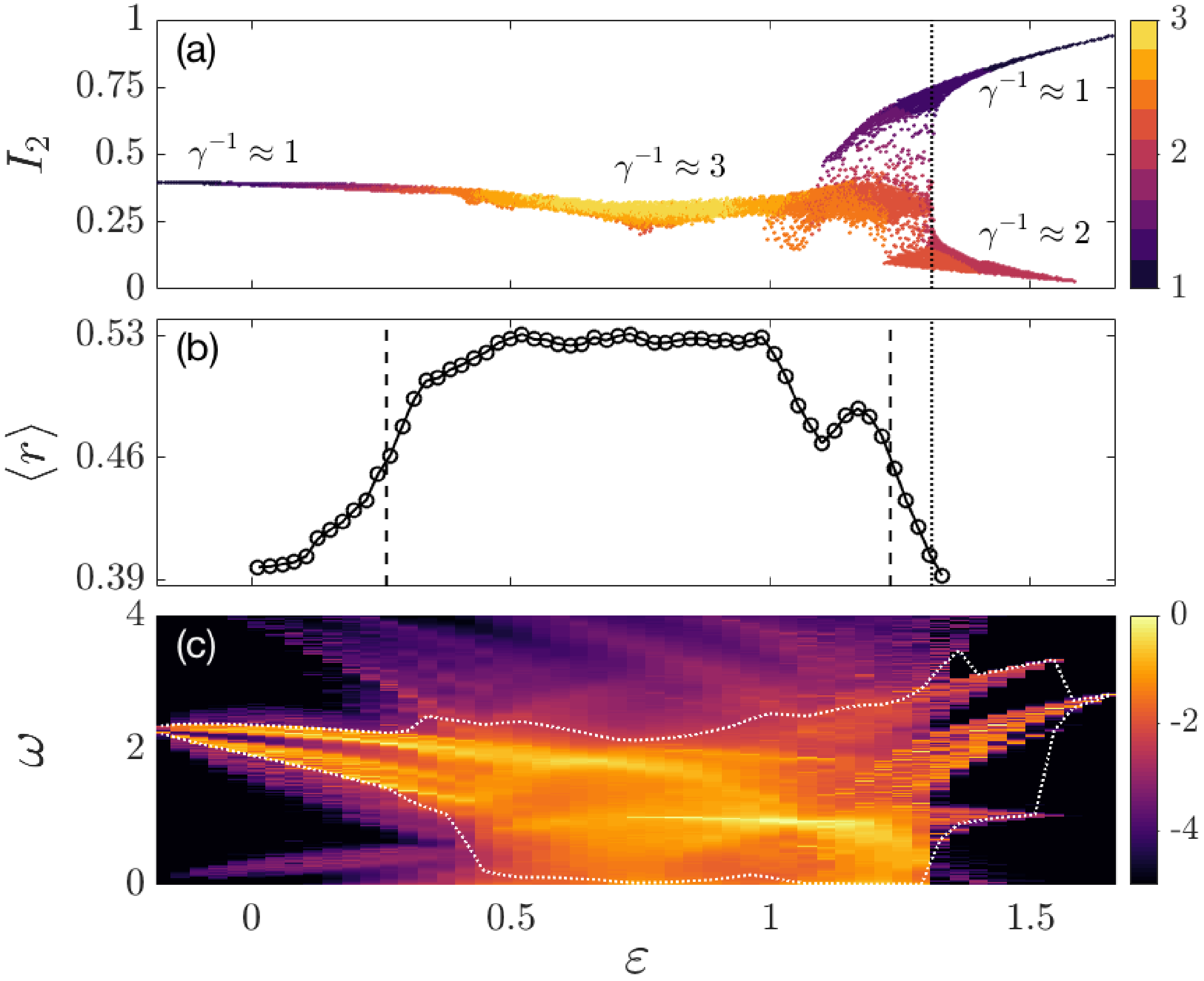
Coupled subsystems (L>3): Minimal model for Thermalization [4a,4b].

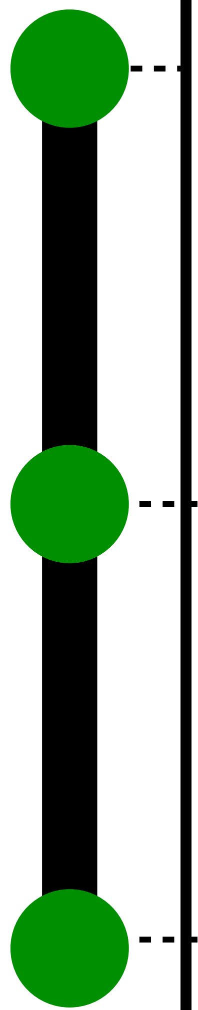
[1a] Chuchem, Smith-Mannschott, Hiller, Kottos, Vardi, DC (PRA 2010).
[1b] Smith-Mannschott, Chuchem, Hiller, Kottos, DC (PRL 2009).
[1c] Boukobza, Moore, DC, Vardi (PRL 2010).
[1d] Khripkov, Vardi, DC (PRA 2012)
[1e] Khripkov, DC, Vardi (JPA 2013, PRE 2013).
[2a] Arwas, DC (SREP 2015, NJP 2016, PRB 2017, PRA 2019).
[2b] Arwas, DC, Hekking, Minguzzi (PRA 2017).
[3a] Dey, DC, Vardi (PRL 2018, PRA 2019).
[3b] Burkle, Vardi, DC, Anglin (PRL 2019).
[3c] Winsten, DC (SREP 2021).
[4a] Tikhonenkov, Vardi, Anglin, DC, (PRL 2013).
[4b] Khripkov, Vardi, DC (NJP 2015, PRE 2018, PRA 2020).

Reminder about the trimer

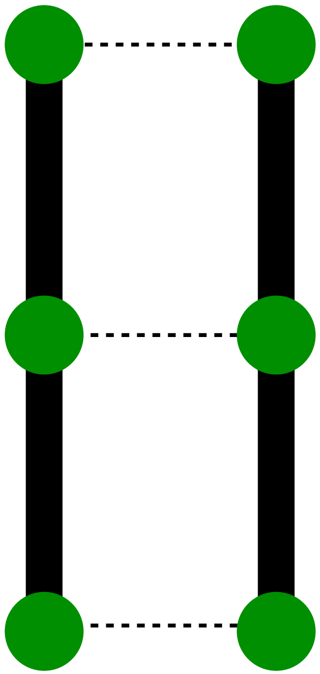
The trimer with $u_L = 18$ qualifies as a chaotic subsystem in the range $0.26 < \varepsilon < 1.23$

- Normalized occupation of the second site (I_2), and number of participating orbitals (γ^{-1})
- Level spacing statistics (r)
- Power spectrum $S(\omega)$ of the occupation coordinate





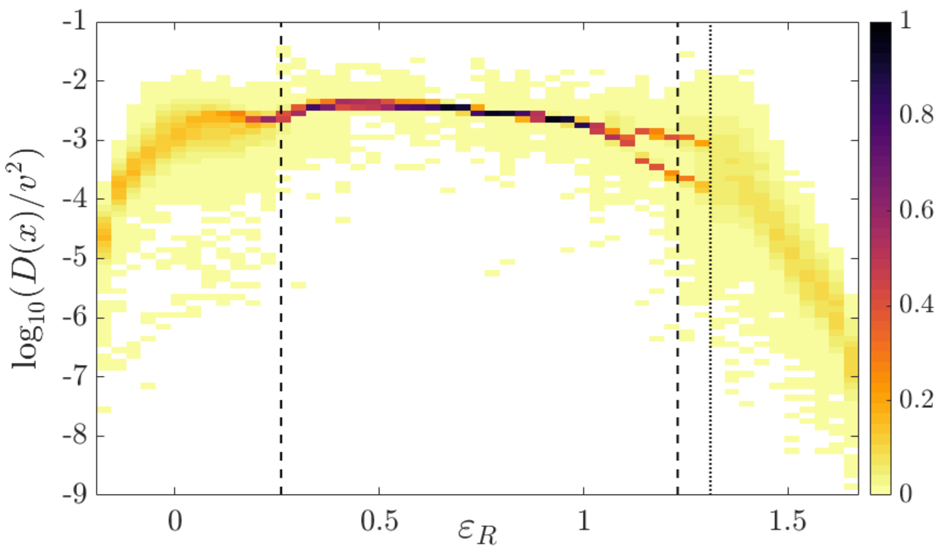
Expected diffusion coefficient



$x = \varepsilon_R - \varepsilon_L$

$$S_{jk}(\omega;\varepsilon) \quad = \quad \mathrm{FT}\left\{\langle I_j(t+\tau) I_k(t)\rangle\right\}$$

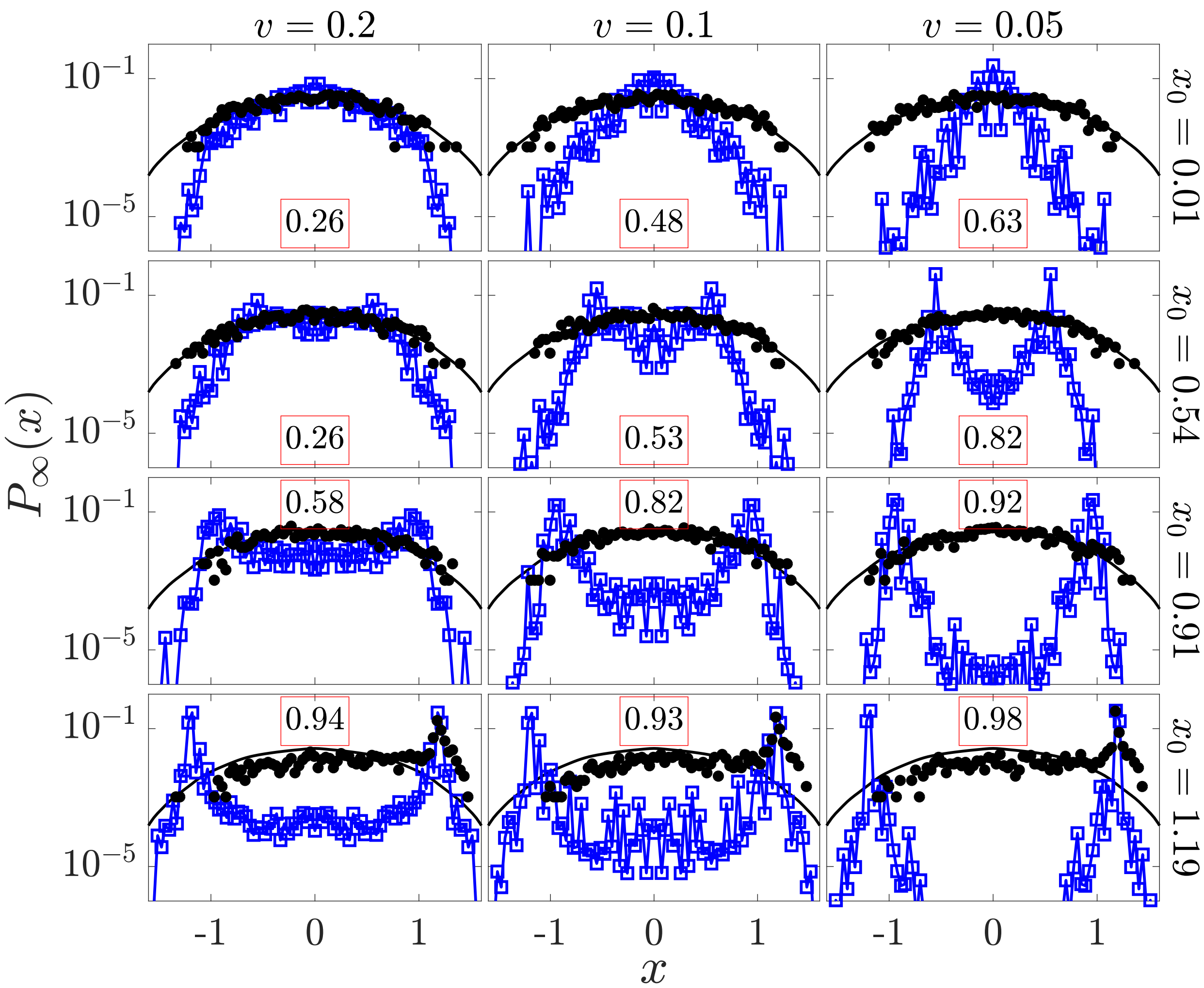
$$D(x) \quad = \quad \frac{v^2}{8} \sum_{j,k=1}^{3} \int_{-\infty}^{\infty} \omega^2 S_{jk}(\omega,\varepsilon_L) S_{kj}(\omega,\varepsilon_R) \frac{d\omega}{2\pi}$$



Agreement with FPE is not always good; Nevertheless D is a way to characterize the coupling.

Saturation profiles

- The inter-trimer coupling is v. Each column is for different v.
- The energy difference is x. Each row is for different x_0.



Black line - ergodic distribution (DOS)

Black dots - based on classical simulation

Blue squares - based on quantum eigenstates

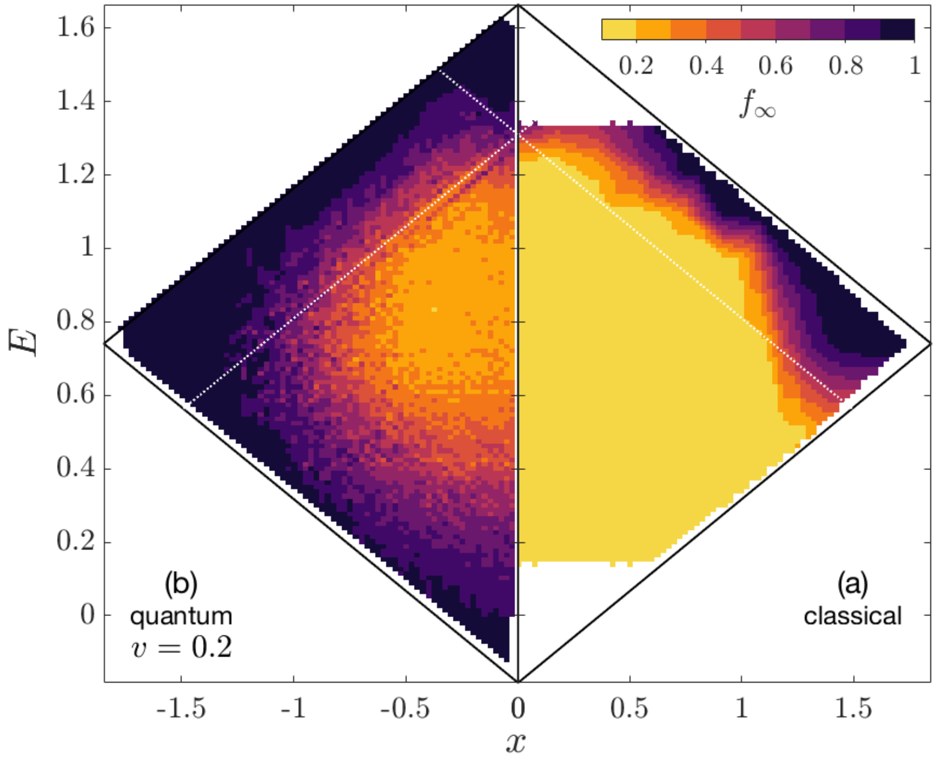
$$P_\infty(x) = \sum_n |\langle E, x|\mathcal{E}_n\rangle|^2 \, |\langle \mathcal{E}_n|E_0, x_0\rangle|^2$$

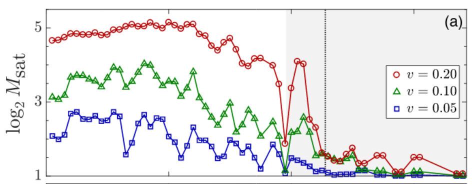
Note mirror symmetry "by construction"

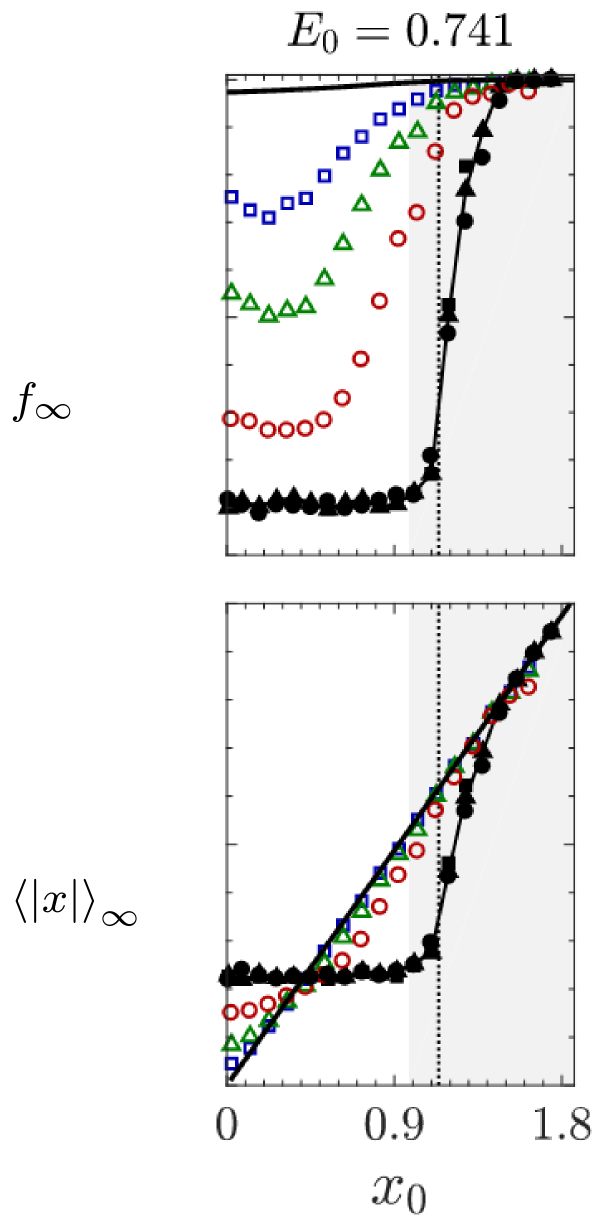
Boxed number $= f_\infty$

Ergodicity and localization regimes

Classically we have localization in the self-trapping region.
Quantum mechanically - in the periphery of the chaotic sea.







(gray region - both trimers are not chaotic)

Classical exploration of space

Inspired by Montroll and Weiss (1965): random walk on a lattice.

Spreading of a cloud:

$$\Omega(t) \quad = \quad \left\{ \sum_r \left[\rho_t(r)\right]^2 \right\}^{-1} \quad \equiv \quad \mathrm{PN}\left\{\rho_t\right\}$$

Exploration by a single trajectory:

$$\mathcal{N}(t) \quad = \quad \mathrm{PN}\left\{\overline{\rho}_t\right\}$$

where

$$\overline{\rho}_t \;\equiv\; \frac{1}{t}\int_0^t \rho_{t'}\,dt'$$

Classical exploration for random walk on a lattice

$$\mathcal{N}(t) \;\sim\; \sqrt{D_0 t} \qquad \text{for } d=1$$

$$\mathcal{N}(t) \;\sim\; \frac{v_0 t}{\log(t)} \qquad \text{for } d=2$$

$$\mathcal{N}(t) \;\sim\; v_0 t \qquad \text{for } d>2$$

Example:

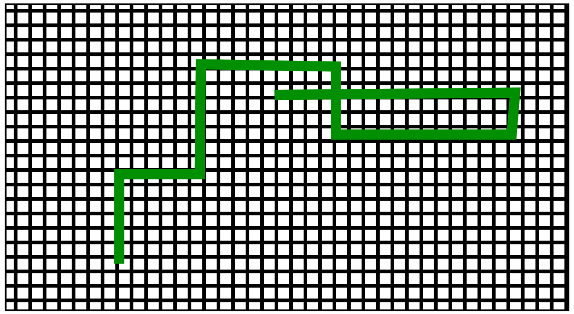
Random walk in 3D

$t = 100$ steps

explored volume ~ 99

spreading radius ~ 10

spreading volume ~ 1000

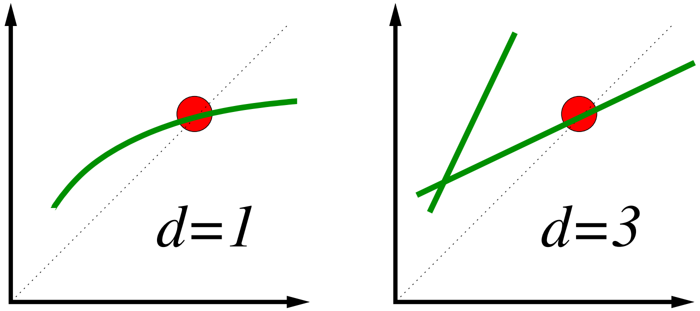


The breaktime concept

- Stationary view of strong localization: interference of trajectories.
- Scaling theory of localization: the importance of dimensionality.
- Dynamical view of strong localization: breakdown of quantum-classical correspondence.

$$t_H[\text{volume}] \;=\; \frac{2\pi}{\Delta_0} \;\propto\; \text{volume}$$

$$t \;\ll\; t_H[\mathcal{N}(t)] \qquad \rightsquigarrow \qquad t^*$$



$$\mathcal{N}(t) \;=\; \sqrt{D_0 t} \qquad \text{for } d=1 \qquad \rightsquigarrow \qquad \text{always localization}$$

$$\mathcal{N}(t) \;=\; c_0 + v_0 t \qquad \text{for } d>2 \qquad \rightsquigarrow \qquad \text{mobility edge}$$

For diffusion in quasi-1D we get $\xi = gD$, where g is the DOS per length.

Chirikov, Izrailev, Shepelyansky [SovSciRevC 1981]; Shepelyansky [PhysicaD 1987];
Dittrich, *Spectral statistics for 1D disordered systems* [Phys Rep 1996];
DC, *Periodic Orbits Breaktime and Localization* [JPA 1998].

Phase space exploration

We propose a generalized QCC condition for the purpose of breaktime determination:

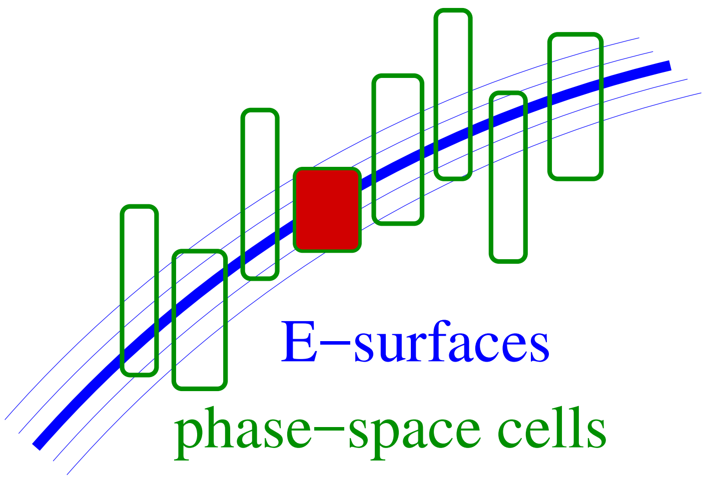
$$\mathcal{N}^{\text{cl}}(t) \quad = \quad \mathcal{F}^{\text{qm}}_{\text{erg}} \left[\frac{\mathcal{N}_E}{\Omega_E}\right] \Omega^{\text{cl}}(t)$$

$\mathcal{N}_E$ = total number of states within the energy shell (r_0 dependent)

$\mathcal{F}^{\text{qm}}_{\text{erg}}$ = filling fraction for a quantum ergodic state, say $= 1/3$

Ω_E = number of cells that intersect an energy-surface

Ω^{cl}_t = explored phase-space volume during time t (starting at r_0)



$$\left|\langle r_j | E_\alpha \rangle\right|^2$$

It is unavoidable to use in the semiclassical analysis improper Planck cells. Namely, a chaotic eigenstate is represented by a microcanonical energy-shell of thickness $\propto \hbar^d$ and radius $\propto \hbar^0$. For some preparations it is implied that $\mathcal{N}_E \ll \Omega_E$ rather than $\mathcal{N}_E \sim \Omega_E$.

Cartoon: $\Omega_E = 8$, while $\mathcal{N}_E = 5$.

Proper Planck cell: $\Delta Q \Delta P > \hbar/2$ for each coordinate.

Phase space formulation of the breaktime phenomenology

Quantum spreading follows classical spreading as long as

$$\mathcal{N}^{\mathrm{qm}}(t) \quad < \quad \mathcal{N}^{\mathrm{cl}}(t)$$

where

$$\mathcal{N}(t) \quad \equiv \quad \left\{ \mathrm{trace}\left[\overline{\rho}(t)^2\right] \right\}^{-1}$$

[converted to Planck-cell units in the classical case]

$$\overline{\rho}(t) \; \equiv \; \frac{1}{t}\int_0^t \rho(t')dt'$$

Definition of $\mathcal{N}^{\mathrm{cl}}(t)$ inspired by Montroll and Weiss (1965)

Definition of $\mathcal{N}^{\mathrm{qm}}(t)$ follows Heller (1986)

Heller's observation: the quantum exploration can be determined semiclassically

$$\mathcal{N}^{\mathrm{qm}}(t) \; \equiv \; \left\{ \mathrm{trace}\left[\overline{\rho}(t)^2\right] \right\}^{-1} \; = \; \left[\frac{2}{t}\int_0^t \left(1-\frac{\tau}{t}\right)\mathcal{P}(\tau)d\tau\right]^{-1} \; \propto t \quad \text{[for short times]}$$

Demonstration for trimer-monomer system

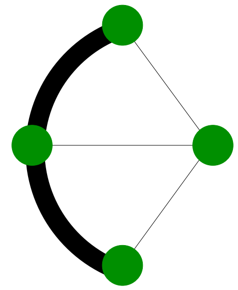
Red line - Semiclassical spreading

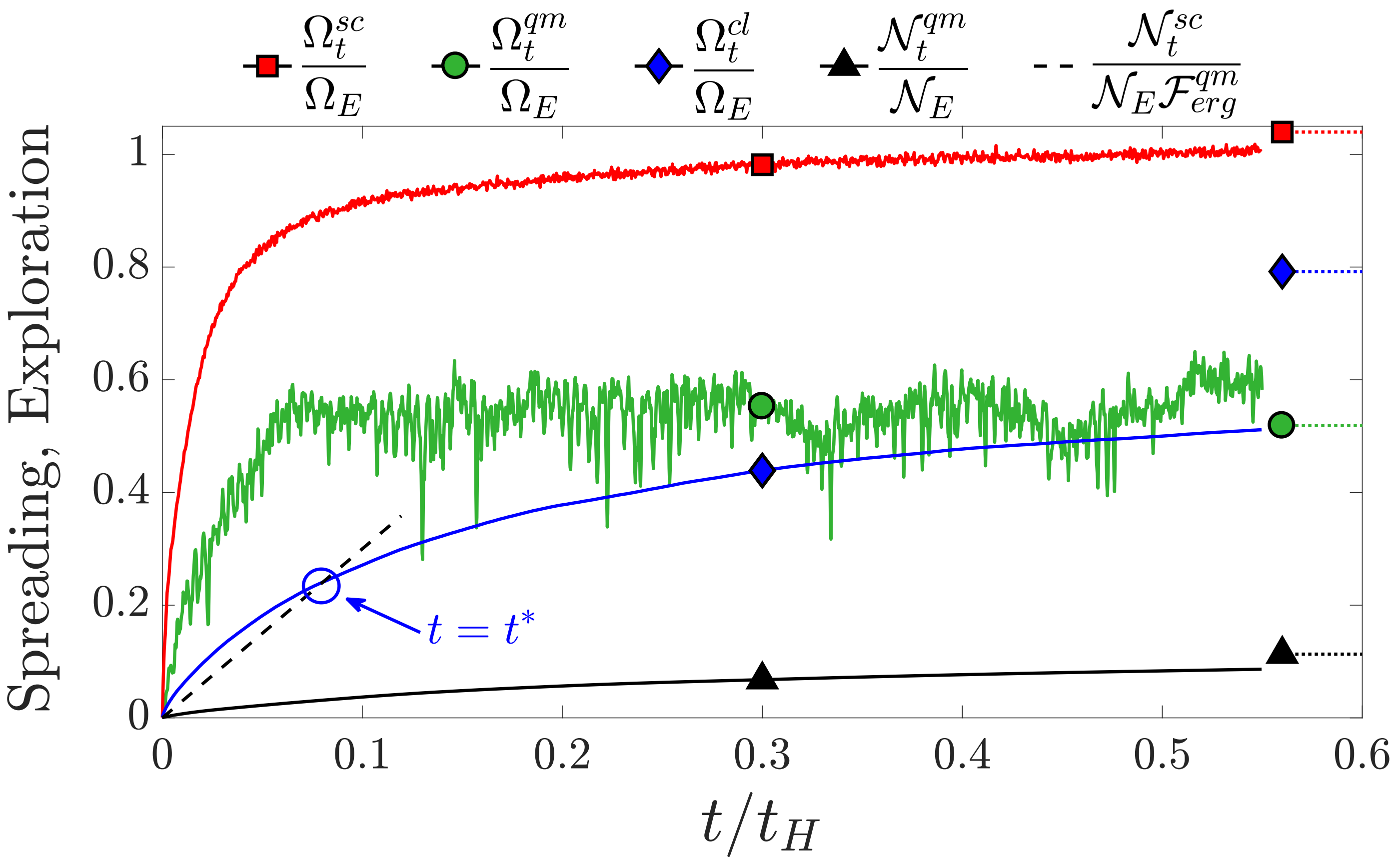
Green line - Quantum spreading

Blue line - classical exploration

Black line (dashed) - quantum exploration (scaled)

The breaktime is determined by the intersection of $\mathcal{N}^{\mathrm{qm}}(t)$ with $\mathcal{N}^{\mathrm{cl}}(t)$





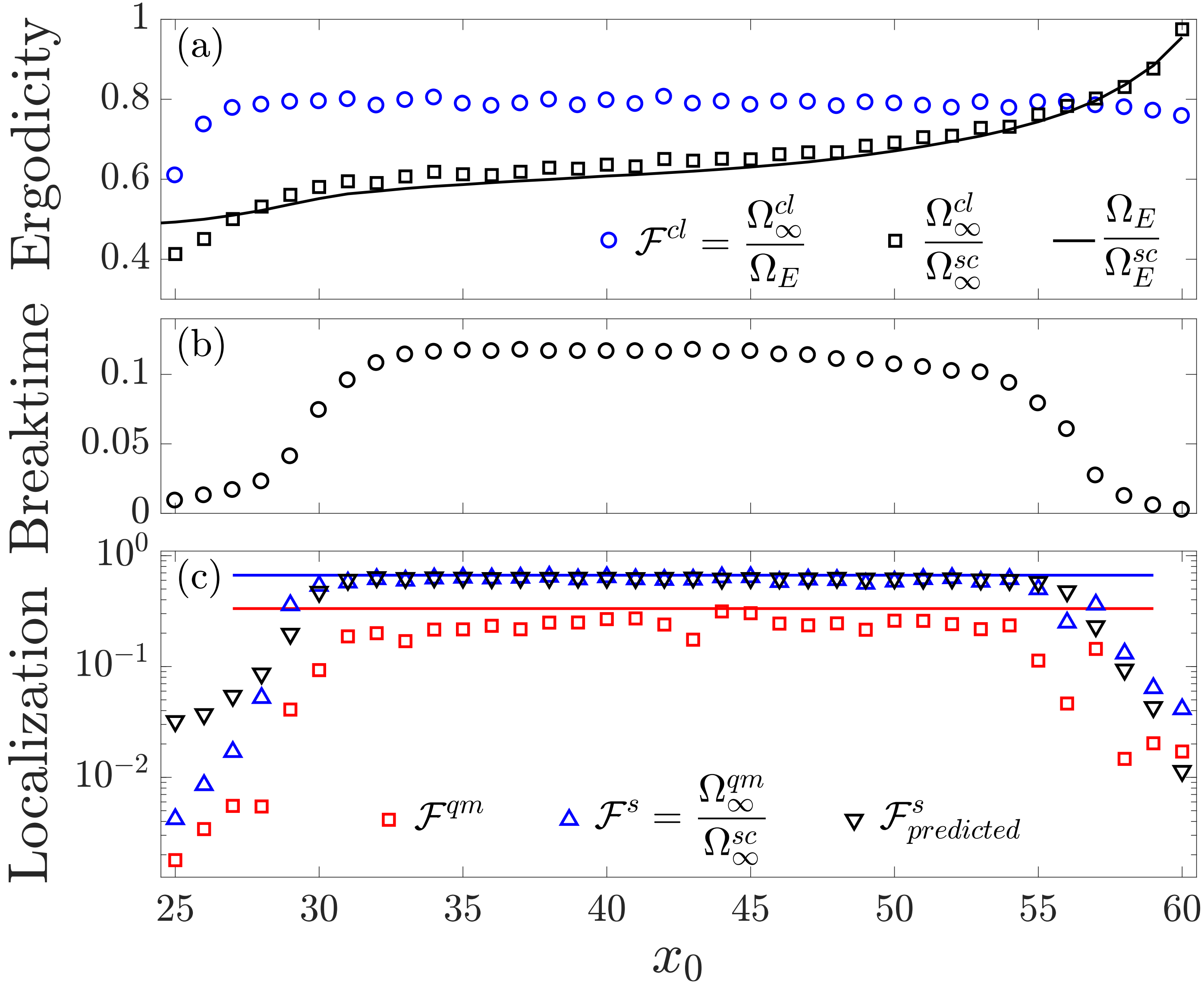
$$\mathcal{F}^{\mathrm{s}} \equiv \frac{\Omega_\infty^{\mathrm{qm}}}{\Omega_\infty^{\mathrm{sc}}}$$

$$\mathcal{F}_{\mathrm{erg}}^{s} \approx \frac{2}{3}$$

Prediction:

$$\Omega_\infty^{\mathrm{qm}} \approx \mathcal{F}_{\mathrm{erg}}^{s} \, \Omega_{t^*}^{\mathrm{sc}}$$

Localization measures



$$\mathcal{F}^{\mathrm{cl}} \equiv \frac{\Omega^{\mathrm{cl}}_{\infty}}{\Omega_E}$$

$$\mathcal{F}^{\mathrm{qm}} \equiv \frac{\mathcal{N}^{\mathrm{qm}}_{\infty}}{\mathcal{N}_E}$$

$$\mathcal{F}^{\mathrm{qm}}_{\mathrm{erg}} \approx \frac{1}{3}$$

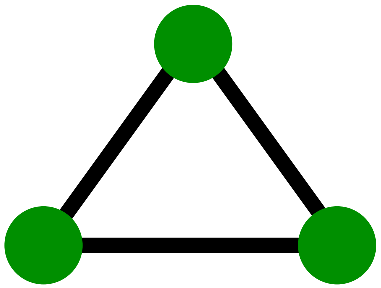
$$\mathcal{F}^{\mathrm{s}} \equiv \frac{\Omega^{\mathrm{qm}}_{\infty}}{\Omega^{\mathrm{sc}}_{\infty}}$$

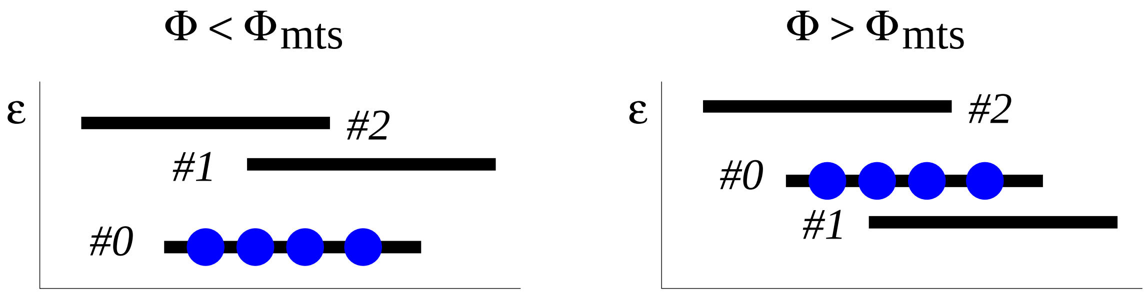
$$\mathcal{F}^{\mathrm{s}}_{\mathrm{erg}} \approx \frac{2}{3}$$

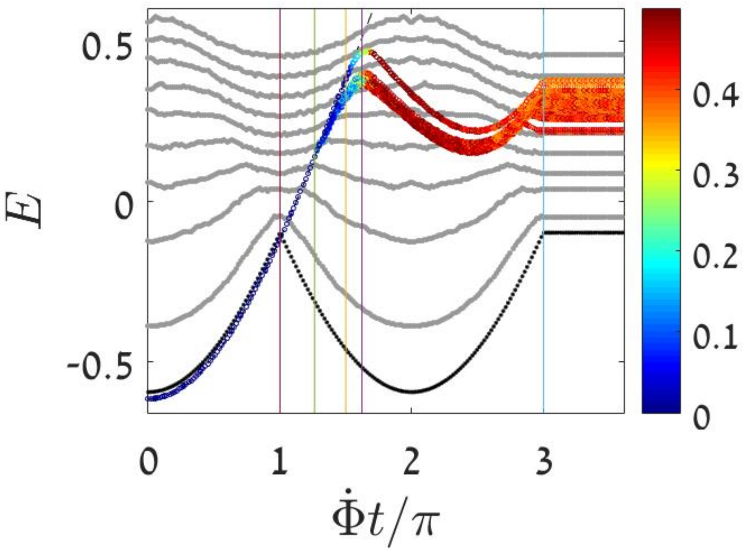
$$\mathcal{F}^{\mathrm{s}}_{\mathrm{predicted}} \equiv \mathcal{F}^{\mathrm{s}}_{\mathrm{erg}} \frac{\Omega^{\mathrm{sc}}_{t^*}}{\Omega^{\mathrm{sc}}_{\infty}}$$

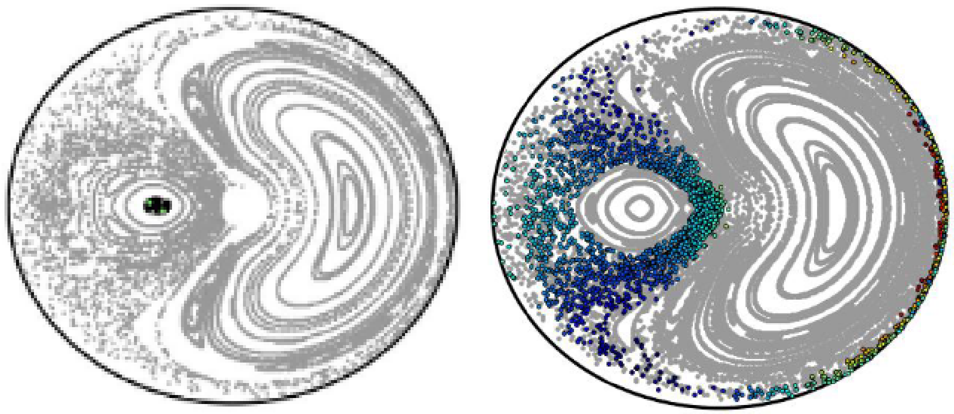
Chaos-assisted depletion

One should not under-estimate the importance of having mixed-chaotic phase-space...









Quasistatic transfer protocols for atomtronic superfluid circuits, Yehoshua Winsten, DC (SREP 2021).

Main messages

- Dynamical localization is a generic effect for Hamiltonians, due to the appearance of a chaotic sea. It manifest itself also for a many dof systems in regions of slow dynamics. Such regions are generic, because mixed chaotic phase-space (rather than hard-chaos) is the general case.
- The breaktime picture provides a way to relate localization to slow phase space exploration. We use the term "exploration" in the sense of Montroll and Weiss. Quantum dynamics can follow classical dynamics as long as $\mathcal{N}^{\rm qm}(t) < \mathcal{N}^{\rm cl}(t)$.
- Dynamical localization typically manifests itself in the periphery of the chaotic sea. Given Planck cell ("hbar"), we have demonstrated that the mobility border can be determined from classical simulations (no fitting parameters).
- Dynamical localization is relevant for the analysis of thermalization. It can be a strong effect (lack of thermalization), or a weak effect (memory of initial conditions).
- Dynamical localization is possibly relevant for the analysis of chaos-assisted depletion. The latter requires migration via a chaotic-corridor that is formed during the coalescence of separatrices.