

Problem Set # 3 Continued

- b. Show that c_σ is irrelevant for $\sigma > 2$.
- c. For $\sigma < 2$, choose the spin rescaling factor so that $c'_\sigma = c_\sigma$. Explain this choice, and show that c_2 becomes irrelevant.
- d. For the case (c), find recursion relations for u and t . For $d > 2\sigma$, show that the model has generalized Gaussian exponents, $\nu = 1/\sigma$, $\eta = 2 - \sigma$, $\gamma = 1$.
- e. For $d < 2\sigma$, find the critical exponents to leading order in $\epsilon = 2\sigma - d$.
- f. What is the critical behavior of the Ising model with $J(r) \sim e^{-r/a}$? Explain.
3. Add to the Ginzburg-Landau-Wilson n -vector model Hamiltonian a cubic term, $\partial \mathcal{L} = v \sum_i (S_i^\alpha)^3$, where S_i^α is the α 'th component of the spin at site i ($\alpha = 1, 2, \dots, n$).
- a. Show that the recursion relations for u and v are
- $$u' = b^\epsilon \left\{ u - 4k \ln b \left[(n+8)u^2 + 6uv \right] + \dots \right\}$$
- $$v' = b^\epsilon \left\{ v - 4k \ln b \left[12 uv + 9 v^2 \right] + \dots \right\}.$$
- b. Find all the fixed points in the u - v plane, and draw the flow lines for $n > 4$ and $n < 4$. Which fixed point describes the critical behavior in each case?
- c. Find the recursion relations for t , and calculate the critical exponent ν to order $\epsilon = 4-d$ in each case.
- d. What happens when $n \rightarrow 0$?