Selected Topics – Solid State

Problem Set # 3

(n=1)

1. To second order in perturbation theory, show that the recursion relation for the coefficient U_6 of $\sum_i S_i^6$ in the Hamiltonian has the form

$$U_6^1 = b^{\lambda_6^0} \quad (U_6 - A_6\, U_4\, U_6 - B_6\, U_6^2)$$

a. Identify λ_6^0, A_6 and B_6.

b. Linearize the recursion relation near $U_6^* = 0$, $U_4^* = \varepsilon/36K_4$, and find the exponent λ_6 in $\Delta U_6' = b^{\lambda_6}\, \Delta U_6$.

c. What happens when $U_4 \equiv 0$? Show that a new non trivial fixed point appears in $d = 3 - \varepsilon$ dimensions and find U_6^*.

d. How does the existence of U_6 affect the recursion relations for t and U_4 ? What are the exponents describing the rescaling of t and U_4 near the new fixed point $t^* = U_4^* = 0$, $U_6^* \neq 0$?

e. Similarly, write down the recursion relation for U_8, and describe qualitatively the analogs of steps (b) - (d) above (without explicitly finding the numerical values of the coefficients).

2. Long range interactions can be described by and exchange $J(r) \sim 1/r^{d+\sigma}$.

a. Show that the Hamiltonian of an Ising system with such interactions can be written in the form

$$\overline{\partial \mathcal{L}} = -\tfrac{1}{2} \int_{\vec{q}} (r + c_2\, q^2 + c_\sigma\, q^\sigma + \ldots)\, \sigma(\vec{q})\, \sigma(-\vec{q})$$

$$-u \iiint \sigma(\vec{q}_1)\, \sigma\, \vec{q}_2)\, \sigma\, (\vec{q}_3)\, \sigma(-\vec{q}_1 - \vec{q}_2 - \vec{q}_3).$$