f(x)=cos(2x)
f(x)'=-2sin(2x)
f(x)''=-4cos(2x)
f(x)'''=8sin(2x)

f(x)=sin(x)
f(x)'=cos(x)
f(x)''=-sin(x)
f(x)'''=-cos(x)

f(x)=$\frac{1-x^3}{2+x^2}$
f(x)'=$\frac{-3x^2}{2+x^2} - \frac{1-x^3}{(2+x^2)^2}2x$

f(x)=e^{4x}
f(x)'=$4e^{4x}$

1)

$$f(x) = (2x+1)^{1/3}$$

$$f'(x) = (2x+1)^{-2/3} \cdot 2$$

$$f''(x) = (2x+1)^{-5/3} \cdot 4$$

2.) $$f(x) = \log(a^x \cdot b^{-x}) = \log(a^x) + \log(b^{-x}) = x\log a - x\log b$$

$$f'(x) = \log a - \log b = \log\frac{a}{b}$$

$$f''(x) = 0$$

3) $$f(x) = \frac{1}{\log x} = (\log(x))^{-1}$$

$$f'(x) = -(\log(x))^{-2} \cdot \frac{1}{x} = -\frac{1}{x(\log x)^2} = -x^{-1}(\log x)^{-2}$$

$$f''(x) = x^{-2}(\log x)^{-2} + 2x^{-1}(\log x)^{-3} \cdot x^{-1} = \frac{1}{(x\log x)^2} + \frac{2}{x^2(\log x)^3}$$

4.) $$f(x) = (\sin x)^{\tan x} \Rightarrow \log(f(x)) = \log((\sin x)^{\tan x}) \Rightarrow \log(f(x)) = \tan x \log(\sin x)$$

$$\frac{f'(x)}{f(x)} = \frac{\cos x}{\sin x} \cdot \underbrace{\frac{\sin x}{\cos x}}_{\tan x} + \frac{\log(\sin x)}{\cos^2 x} = 1 + \frac{\log(\sin x)}{\cos^2 x} \Rightarrow f'(x) = \left(1 + \frac{\log(\sin x)}{\cos^2 x}\right)(\sin x)^{\tan x}$$

$$\log(f'(x)) = \log\left(1 + \frac{\log(\sin x)}{\cos^2 x}\right) + \tan x \log \sin x$$

$$\frac{f''(x)}{f'(x)} = 1 + \frac{\log(\sin x)}{\cos^2 x} + \frac{1}{1 + \frac{\log(\sin x)}{\cos^2 x}} \cdot \left(\frac{1}{\sin x \cos x} - \frac{\sin x \log(\sin x)}{\cos^3 x}\right)$$

$$f''(x) = \left[1 + \frac{\log(\sin x)}{\cos^2 x} + \frac{1}{1 + \frac{\log(\sin x)}{\cos^2 x}}\left(\frac{1}{\sin x \cos x} - \frac{\sin x \log(\sin x)}{\cos^3 x}\right)\right] \cdot \left[1 + \frac{\log(\sin x)}{\cos^2 x}\right](\sin x)^{\tan x}$$

Function 1: $f(x) = \frac{x}{x^2+1}$**.**

$$f(0) = 0, \quad f(x > 0) > 0, \quad f(x < 0) < 0 \tag{1}$$

$$x \to \pm\infty \Rightarrow f(x) \to 0 \tag{2}$$

Maximum: $\frac{df}{dx} = 0$, $\frac{d^2 f}{dx^2} > 0$. Minimum: $\frac{df}{dx} = 0$, $\frac{d^2 f}{dx^2} < 0$.

$$\frac{df}{dx} = \frac{1 - x^2}{(1 + x^2)^2} = 0 \Rightarrow x = \pm 1 \tag{3}$$

$$f_{max} = \frac{1}{2}, \quad f_{min} = -\frac{1}{2} \tag{4}$$

Function 2: $f(x) = \frac{\sin x}{x}$**.**

$$f(0) = \lim_{x \to 0} \frac{\sin x}{x} = 1 \tag{5}$$

$$f(x \to \pm\infty) \to 0 \tag{6}$$

$$\frac{df}{dx} = \frac{x \cos x - \sin x}{x^2} \tag{7}$$

$$\frac{df}{dx} = 0 \Rightarrow \tan x = x \tag{8}$$

The last equation has infinite number of solutions but it is impossible to find analytical solutions.

Function 3: $f(x) = x \exp(-x^2)$**.**

$$f(0) = 0, \quad f(x > 0) > 0, \quad f(x < 0) < 0 \tag{9}$$

$$f(x \to \pm\infty) \to 0 \tag{10}$$

$$\frac{df}{dx} = (1 - 2x^2) e^{-x^2} \tag{11}$$

$$\frac{df}{dx} = 0 \Rightarrow x = \pm \frac{1}{\sqrt{2}} \tag{12}$$

$$f_{max} = \frac{1}{\sqrt{2}} e^{-1/2}, \quad f_{min} = -\frac{1}{\sqrt{2}} e^{-1/2} \tag{13}$$

Function 4: $f(x) = \frac{x^k}{1+x^k}$**,** $k >$ **integer.**

$$f(0) = 0 \tag{14}$$

$$f(x \to \pm\infty) \to = 1 \tag{15}$$

$$f(x) = 1 - \frac{1}{1 + x^k} \tag{16}$$

$$\frac{df}{dx} = \frac{k x^{k-1}}{(1 + x^k)^2} \tag{17}$$

$$\frac{df}{dx} = 0 \Rightarrow x = 0 \tag{18}$$

If $k = 2n$ then $f(x) > 0$ for $x \neq 0$. Then $x = 0$ is a minimum and $f_{min} = 0$. If $k = 2n + 1$ the $f(x) > 0$ for $x > 0$ and $f(x) < 0$ for $x < 0$. Then at $x = 0$ the function changes sign and there is no mininum of maximum.