

e-01-05-012.5

$$\vec{v}_2 = 7\hat{i} + 4\hat{j} + 3\hat{k} \quad \vec{v}_1 = 6\hat{i} + 2\hat{k} \quad \underline{-1111}$$

$$\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = 0 \quad \text{-e } \gamma \quad \vec{v}_3 = ? \text{ (k)}$$

$$\vec{v}_1 + \vec{v}_2 = -\vec{v}_3$$

$$7\hat{i} + 4\hat{j} + 3\hat{k} + 6\hat{i} + 2\hat{k} = -\vec{v}_3$$

$$-7\hat{i} - 4\hat{j} - 5\hat{k} = \vec{v}_3$$

$$\vec{v}_3 = (-7, -4, -5)$$

> 1121 or 1120

$$\vec{v}_1 - \vec{v}_2 + \vec{v}_4 = 0$$

$$\text{-e } \gamma \quad \vec{v}_4 = ? \text{ (k)}$$

$$\vec{v}_2 - \vec{v}_1 = \vec{v}_4$$

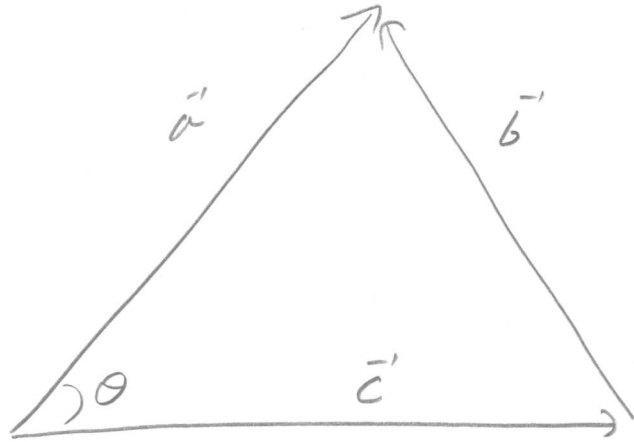
$$7\hat{i} + 4\hat{j} + 3\hat{k} - 6\hat{i} - 2\hat{k} = \vec{v}_4$$

$$-5\hat{i} + 4\hat{j} + \hat{k} = \vec{v}_4$$

$$\vec{v}_4 = (-5, 4, 1)$$

-1111c 9111 > 11c

e-01-2-008



$$\vec{a} \cdot \vec{c} = ac \cos \theta$$

מכאן נקודת!

האם זה אכן
א

$$\vec{a} = \vec{c} + \vec{b}$$

$$\vec{b} = \vec{a} - \vec{c}$$

אם נציב

$$\vec{b} \cdot \vec{b} = (\vec{a} - \vec{c}) \cdot (\vec{a} - \vec{c}) = \vec{a} \cdot \vec{a} + \vec{c} \cdot \vec{c} - 2\vec{a} \cdot \vec{c}$$

אם נציב ונקודת

$$\vec{a} \cdot \vec{a} = |\vec{a}| |\vec{a}| \cos 0 = |\vec{a}|^2$$

$$b^2 = a^2 + c^2 - 2\vec{a} \cdot \vec{c}$$

אם

$$b^2 = a^2 + c^2 - 2ac \cos \theta$$

exercise 1_1405

a.

$$\vec{A} \cdot \vec{B} = 3 \cdot 6 + 4 \cdot (-8) = -14 \quad (1)$$

$$\alpha = \arccos\left(\frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \cdot \|\vec{B}\|}\right) = \arccos\left(\frac{-14}{\sqrt{3^2 + 4^2} \cdot \sqrt{6^2 + (-8)^2}}\right) = \arccos\left(-\frac{7}{25}\right) \approx 106.26^\circ \quad (2)$$

b.

$$\vec{A} \times \vec{B} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 6 & -8 & 0 \end{pmatrix} = -48\hat{k} \quad (3)$$

$$\|\vec{A} \times \vec{B}\| = 48 \quad (4)$$

c.

$$\vec{C} \times \vec{D} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ 2 & 1 & 3 \end{pmatrix} = (3 \cdot 3 - 1 \cdot 3)\hat{i} + (3 \cdot 2 - 3 \cdot 3)\hat{j} + (3 \cdot 1 - 2 \cdot 3)\hat{k} = (6, -3, -3)$$

$$\|\vec{C} \times \vec{D}\| = \sqrt{6^2 + (-3)^2 + (-3)^2} \approx 7.34 \quad (5)$$

vectors

given: $\vec{a} = 2\hat{i} - 2\hat{j} - \hat{k}$, $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$, $\vec{c} = 4\hat{i} - 1\hat{j} + 3\hat{k}$

A. $|\vec{a}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = 3$;

$|\vec{b}| = \sqrt{6^2 + (-3)^2 + 2^2} = 7$;

$|\vec{c}| = \sqrt{4^2 + (-1)^2 + 3^2} = \sqrt{26}$;

B. $\vec{a} + \vec{b} + \vec{c}$ angles : $\alpha = \text{between } \vec{a}$; $\beta = \text{between } \vec{b}$; $\gamma = \text{between } \vec{c}$;

$\vec{a} + \vec{b} + \vec{c} = (2 + 6 + 4) \cdot \hat{i} + (-2 - 3 - 1) \cdot \hat{j} + (-1 + 2 + 3) \cdot \hat{k} = (12, -6, 4)$;

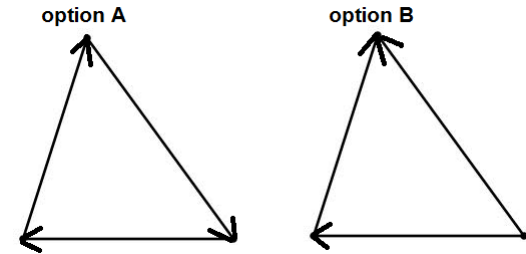
$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{12^2 + (-6)^2 + 4^2} = 14$;

$\cos(\alpha) = \frac{12 \cdot 2 + (-6) \cdot (-2) + 4 \cdot (-1)}{14 \cdot 3} = \frac{16}{21}$; $\alpha = 40.37^\circ$;

$\cos(\beta) = \frac{12 \cdot 6 + (-6) \cdot (-3) + 4 \cdot 2}{14 \cdot 7} = 1$; $\beta = 0^\circ$;

$\cos(\gamma) = \frac{12 \cdot 4 + (-6) \cdot (-1) + 4 \cdot 3}{14 \cdot \sqrt{26}} = 0.92$; $\gamma = 22.4^\circ$;

C. There are 2 options for vector triangle:



option A : head to tail : $\vec{a} + \vec{b} + \vec{c} = (0, 0, 0)$.

and also: $(180^\circ - \theta_{ab}) + (180^\circ - \theta_{bc}) + (180^\circ - \theta_{ac}) = 180^\circ$.

not our case ($\vec{a} + \vec{b} + \vec{c} = (12, -6, 4)$) ;

option B: 2 vector addition gives the third vector $\vec{a} + \vec{c} = \vec{b}$.

and also $\theta_{ab} + \theta_{bc} + (180^\circ - \theta_{ac}) = 180^\circ$

$\vec{a} + \vec{c} = (2 + 4) \cdot \hat{i} + (-2 - 1) \cdot \hat{j} + (-1 + 3) \cdot \hat{k} = (6, -3, 2) = \vec{b}$

\vec{b} is in the same direction as $\vec{a} + \vec{b} + \vec{c}$.

therefore: 1. angle between \vec{a} and \vec{b} equal 40.37° ; 2. angle between \vec{c} and \vec{b} equal 22.4° .

calculate angle between \vec{a} and \vec{c} :

$\cos(\theta) = \frac{2 \cdot 4 + (-2) \cdot (-1) + (-1) \cdot 3}{3 \cdot \sqrt{26}} = 0.458$; $\theta = 62.767^\circ$

$40.37^\circ + 22.4^\circ + (180 - 62.767^\circ) = 180^\circ$

triangle!!!

שאלה 10
אם C קיים α ו β כך

$$(3, 2, \beta) = C(\alpha, -5, 2) \quad ?$$

מטרת השאלה היא למצוא את C

$$3 = C \cdot \alpha$$

$$2 = -5C$$

$$\beta = 2C$$

$$\begin{cases} \alpha = -\frac{15}{2} \\ \beta = -\frac{4}{5} \\ C = -\frac{2}{5} \end{cases}$$

תשובה

שאלה

$$\vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) = 3(3, 3, 1) - 2(2, -2, -1) = \textcircled{16}$$

$$= (5, 13, 5)$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (a_y(\vec{b} \times \vec{c})_z - a_z(\vec{b} \times \vec{c})_y, a_z(\vec{b} \times \vec{c})_x - a_x(\vec{b} \times \vec{c})_z, a_x(\vec{b} \times \vec{c})_y - a_y(\vec{b} \times \vec{c})_x)$$

$$= (a_y(b_x c_y - b_y c_x) - a_z(b_z c_x - b_x c_z), a_z(b_y c_z - b_z c_y) - a_x(b_x c_y - b_y c_x), a_x(b_z c_x - b_x c_z) - a_y(b_y c_z - b_z c_y)) =$$

$$= (5, 13, 5)$$

$$\vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) = 5(-1, 1, 3) - 5(2, -1, 1) = \textcircled{5}$$

$$(-15, 10, 10)$$