Theoretical questions for an exam in theory of radiation

1. Derive Plank's equation for the energy density of thermal radiation.

2. Derive Wien displacement low: show that λ_m corresponding to the maximum intensity of radiation is given by $\lambda_m \approx 0.2hc/kT$.

3. Show that the pressure of thermal radiation p is connected with the energy density U by relation: $p = U/3$.

4. Using phenomenological approach of Einstein find relations between the coefficients of absorption (B_{lu}), spontaneous (A_{lu}) and stimulated (B_{ul}) emission.

5. In the dipole approximation find an absorption probability per unit time (dW_{abs}) of the photon of frequency ω propagating in definite direction inside the solid angle $d\Omega$ and having definite polarization. The number of photons in the initial state before absorption is n_λ, the matrix element of the dipole moment is d.

6. Using the model of damped harmonic oscillator find the natural line shape.

7. Show that in the case of collisional broadening in the impact approximation the line profile is Lorentzian (Lorentz theory) (Corney)

8. Derive an expression for the line profile in the case of Doppler broadening.

9. Derive an expression for the line profile in the gas if the Doppler and Lorentz line-widths are of the same order of magnitude (Voigt function). (Doppler line profile is given by $G(\omega) = (4 \ln 2/\pi\Delta\omega_D)^{1/2}\exp[-4\ln 2(\omega-\omega_0)^2/\Delta\omega_D^2]$.

10. Show that in the wings of the line ($\omega - \omega_0 >> \Delta\omega_D, \Delta\omega_L$) the line profile is always Lorentzian.

11. Derive an expression for absorption coefficient k in terms of the wavelength λ, the spontaneous coefficient A, lineshape finctiom $g(\nu)$ and the number densities of the particles in the lower and upper level, N_l and N_u, respectively.

12. Derive an expression for the flux of radiation energy of frequency ω emitted by the layer of depth l. Assume that the layer consists of the two-level molecules with the energy of transition $h\omega$. The temperature is T. What is the flux in the limiting cases of the optically thick and thin layer.

13. Estimate the linewidth of the radiation emitted by the optically thick body in the cases of Lorentzian and Doppler broadening. The product of the

absorption coefficient in the center of line k_0 and the dimension of the body l is much greater than unity.

14. Find dependencies of the population inversion on the pumping rate for 4- and 3- level systems using rate equations. For what system (3- or 4- level) is it easier to get population inversion and why?

15. Derive the homogeneous saturation law: find the gain at frequency ω, when a strong saturating signal of intensity I is applied at the same frequency ω in the case of homogeneous (Lorentzian) broadening. Assume that the small signal gain in the line center (frequency ω_0) is g_0, the life time of the upper level τ and stimulated emission cross section σ.

16. Derive the inhomogeneous saturation law: find the gain at frequency ω, when a strong saturating signal of intensity I is applied at the same frequency ω in the case of homogeneous (Doppler) broadening ($\Delta\omega_D >> \Delta\omega_L$). Assume that the small signal gain in the line center (frequency ω_0) is g_0 and the saturation parameter is $I_{s,res}$.

17. Line narrowing in the amplifier. Find the linewidth after passing through the laser amplifier with the unsaturated small signal gain α_0 and length l; assume that before the amplifier the line has Lorentzian profile with width $\Delta\nu$. Neglect saturation effects.

18. Homogeneous saturation in laser amplifier. Laser light with intensity I_{in} is passing through the amplifier with unsaturated small signal gain α_0 and length l. Find the output intensity I_{out} assuming homogeneous saturation with saturation parameter I_S. Find maximum available power which can be extracted from the amplifier; for which I_{in} this power can be extracted.

19. Laser oscillation. Find the output power of the laser with small signal gain α_0 and the gain length l. The transmission of the output mirror is t, the absorption/scattering losses of this mirror are a. Find optimal transmission of the mirror, assume that intensity is uniform inside the resonator.

20. Find maximum power and equation for minimum value of the population inversion for Q-switched pulse. The values of the initial and threshold population inversions are N_i and N_{th}, respectively.