

Environmental Physics for Freshman Geography Students

Professor David Faiman. Lecture 7, v. 3.2 (December 16, 2004)

1. Electrostatic Forces

In Lecture 4 we studied Newton's law of gravity. You will recall that it involves force with the mathematical form:

$$F = G m_1 m_2 / r^2 \quad (7.1)$$

where m_1 and m_2 are the masses of the two bodies which attract one another (measured in kg), r is the distance between them (measured in m), and G is Newton's gravitational constant ($= 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$). Note that G takes the same value whether the masses are small, as is the case of molecules of air in the earth's atmosphere, or large, as in the case of the earth and the sun.

Another kind of force that occurs in nature was discovered by Coulomb. It is the force between two electrically charged objects. Remarkably, Coulomb's law for electric charges has a similar mathematical form to Newton's law of gravity. It is:

$$F = K q_1 q_2 / r^2 \quad (7.2)$$

where q_1 and q_2 are the amounts of electric charge (measured in special units called "coulombs", C), r is the distance between the charges (measured in m), and K is Coulomb's electrostatic constant ($= 8.99 \times 10^9 \text{ kg m}^3 \text{ s}^{-2} \text{ C}^{-2}$). The introduction of electric charges into the simple world of mechanics requires the use of a new "dimension" in addition to mass, length and time. That dimension is electric charge. However, if you check the overall dimensions on the right hand side of eq, (7.2), you will see that the C's cancel out and we are left with kg m s^{-2} , which are the correct dimensions for force.

There is, however, one important difference between Newton's and Coulomb's laws. Because masses can only be positive numbers, Newton's force is always an *attractive force*. However, electric charges can be positive or negative. Therefore, Coulomb's law can be *attractive or repulsive*. If both charges have the same sign, the force is repulsive; if they have opposite signs the force is attractive. For example, two electrons repel one another, whereas an electron and a proton, which have equal but opposite charges, attract one another. This latter fact allows the hydrogen atom to exist. For, the hydrogen atom is something like the earth and moon, in that a lightweight electron "orbits" around a massive proton. In fact, an early model of the hydrogen atom, constructed by Niels Bohr, pictured the electron as circling in an orbit exactly as the moon circles the earth. Bohr equated the coulomb attraction between the two particles to the centripetal force on the electron.

2. Bohr's Model of the Hydrogen Atom

In Lecture 4 we equated the gravitational force between a planet and the sun to the planet's centripetal force. If we do the corresponding thing for the electron and proton in a hydrogen atom, we have:

$$K q^2 / r^2 = m v^2 / r \quad (7.3)$$

where q is the magnitude of the charge on either the electron or the proton = 1.60×10^{-19} C (they have identical charges - just opposite signs), r is the distance between them, m is the mass of the orbiting electron = 9.11×10^{-31} kg, v is the speed of the electron along its orbital path. This leads, as in the case of gravity, to a fixed relationship between v and r , no matter how far from the proton the electron orbit happens to be. Namely:

$$r v^2 = K q^2 / m \quad (7.4)$$

In the case of gravity, that is as far as we can go. Namely, *whatever value is taken for r* , the value of v will be given by the gravitational equivalent of eq. (7.4). However, Bohr discovered that there is another restriction for atomic systems - and it is an extremely peculiar one. Bohr realized that the orbital angular momentum of the electron could *only* take one of the *discrete* values:

$$L = m v r = n L_0, \quad n = 1, 2, 3, \dots \quad (7.5)$$

where $L_0 = 1.05 \times 10^{-34}$ J s is a constant known as “Planck’s constant divided by 2π ”. This is peculiar because we would have expected that L could take any value whatsoever. But nature has decided otherwise. L can be $L_0, 2L_0, 3L_0$, etc., but nothing in between.

Planck’s constant is the underlying numerical constant behind so-called *quantum mechanics*; that is the mechanics that describes the way molecules, atoms and other small-scale systems interact with one another. One of the unfortunate aspects of modern physics is that there is no known simple way to picture quantum mechanics - but it is one of the most accurate known computational tools in all of physics. Even Bohr’s model, useful as it certainly is, gives nonsense if we try to dig too deeply into it in terms of a physical picture. For example, if we eliminate v between equations (7.5) and (7.4), we find that the distance between the electron and proton can *only* take one of the *discrete* values:

$$r = n^2 L_0^2 / (K m q^2), \quad n = 1, 2, 3 \quad (7.6)$$

where *all* symbols on the right hand side of this equation are constants. This implies that the electron can not be at any arbitrary distance from the proton; only at discrete allowed distances, given by the values of the so-called *quantum number* n . However, what happens when an electron jumps from one orbit to another (which it *can* do)? Surely, our senses tell us, it must pass through the forbidden region. But if it does, then the region is not forbidden!!! This is just one example of the conceptual problems we get into if we try and picture in our imagination what is actually happening in the quantum world.

Similarly, the kinetic energy of the electron can also *only* take discrete values:

$$(1/2) m v^2 = m K^2 q^4 / (2 n^2 L_0^2) \quad (7.7)$$

where, again, everything on the right is constant.

And the electrostatic potential energy is also *quantized* (i.e. it takes only discrete values):

$$-K q^2 / r = -m K^2 q^4 / (n^2 L_0^2) \quad (7.8)$$

You will notice that the potential energy is negative, and twice the magnitude of the kinetic energy. Therefore, the total energy of the electron, being simply the sum of the kinetic and potential energies, is also quantized:

$$E = - m K^2 q^4 / (2 n^2 L_0^2) \quad (7.9)$$

Note that the *sum* of the kinetic and potential energies of the electron in a hydrogen atom is *negative*. This means that the mass of a hydrogen atom is *slightly less* than the sum of the masses of a proton and an electron. What do I mean by “slightly”? The rest mass energy of a proton is 938 MeV (where MeV is a convenient unit of energy), the rest mass energy of an electron is 0.511 MeV, but the largest value of E, in eq. (7.9) is only 13.6×10^{-6} MeV.

3. Waves

Everyone is familiar with the phenomenon of waves in water. If you drop a stone into a pond of still water, a circular wave spreads out from the point where the stone fell. Now, the stone originally contains potential energy before it is dropped. That energy is converted to kinetic energy as the stone reaches the pond, and most of this kinetic energy is transferred to the water molecules. It is this energy that causes the wave to move.

Now a water wave is an interesting example of what is known as *collective motion*. As the wave spreads out from the stone’s point of impact, all that happens is that water molecules move up and down locally. No water actually travels with the wave: it only looks that way. The wave, however, carries energy with it. As it spreads out, each part of the wave contains less and less energy. Hence, as time passes, the wave is less and less able to move water molecules up and down. The wave thus, eventually, dies out.

Suppose now, instead of dropping a single stone, we arrange for a motor-driven hammer to strike the water repeatedly. This time a series of waves spread out from the hammerhead. If the hammer strikes the water at a constant frequency, ν times per second, the waves will spread out with a fixed separation λ between successive wave crests. It is easy to see that the speed v with which the waves move through the water is:

$$v = \nu \lambda \quad (7.10)$$

We refer to λ as the *wavelength* of the wave, and ν as the *frequency* of the wave. For example, if $\nu = 5$ crests per second (which is denoted 5 Hz, or simply 5 s^{-1}), and $\lambda = 10 \text{ cm}$, then from eq. (7.10), the wave spreads out at a speed $v = 50 \text{ cm s}^{-1}$.

In the case of water waves, wave motion is easy to understand, but there is another kind of wave that was discovered in the 19th century, the *electromagnetic wave*, that was much more difficult to understand. The reason is that electromagnetic waves can travel through a vacuum!

Electromagnetic waves have wavelength and frequency, just like any other kind of wave. Those with wavelengths of hundred of meters are familiar as *AM radio* waves. Those with wavelengths of about 1 m are familiar as *FM radio* and *Television* waves. Those with wavelengths of about 10 cm are familiar as *microwaves*. Those with wavelengths of about 10 μm are familiar as *infrared radiation*. Those with wavelengths in the range 400 - 700 nm are familiar as *light waves*. Those with wavelengths in the range 250 - 400 nm are familiar as

ultraviolet radiation. At shorter wavelengths still, we encounter first *X-rays*, then *gamma rays*. All of these are examples of electromagnetic radiation.

Strangely, it was quantum mechanics that helped clear up the mystery of how electromagnetic waves can travel in empty space. The reason for this is that although in the large-scale world around us, particles and waves are completely different objects, at the atomic scale this is no longer the case. At the atomic scale, particles can behave like waves and waves can behave like particles. Richard Feynman, one of the pioneers of our modern understanding of quantum mechanics called them all *wavicles*. How does this help us understand the ability of electromagnetic waves to travel through a vacuum? Well, if an electron can travel through a vacuum, so too can a *photon*. A photon is the particle manifestation of electromagnetic waves. But then, does an electron also have wave properties? It certainly does - otherwise we could not have electron microscopes.

Quantum mechanics gives us simple relationships between the particle-like and wave-like properties of these wavicles: Wavelength of the wave is related to momentum of the particle by de Broglie's equation:

$$p = h / \lambda \quad (7.11)$$

and frequency is related to energy by Einstein's equation:

$$E = h \nu \quad (7.12)$$

where h is a constant known as Planck's constant ($h = 2\pi L_0 = 6.63 \times 10^{-34} \text{ J s}$).

Notice that if we divide eq. (7.12) by eq. (7.11) we obtain:

$$E/p = \nu \lambda = v \quad (7.13)$$

i.e. the speed of the wave is the ratio of the corresponding particle's energy to its momentum. In Lecture 2 we learned that, being a massless particle, the photon has $E = pc$, where c is a constant equal to the speed of light in a vacuum. Eq. (7.13) thus tells us that all photons (no matter what their wavelength) travel at the speed of light.

And what about electron waves? The situation is slightly more complicated here. Eq. (7.11) remains true, but eq. (7.12) must be modified slightly. If you plug eq. (7.11) into the energy-momentum-mass relationship in Lecture 1, you will discover that instead of E , the left hand side of eq. (7.12) now becomes $\sqrt{(E^2 - m^2c^4)}$. As a result, the velocity of electron waves also turns out to be c , even though the electrons themselves move at slower speeds. But here we are departing rather far from "environmental physics" so it is time to stop.

Problem set 7 (hydrogen atoms and electromagnetic waves)

1. (a) How many atoms are there in 1 kg of hydrogen gas? (b) What is the total mass of all the protons? (c) What is the total mass of all the electrons [1 molecule of hydrogen contains 2 atoms; the proton mass = 1.67×10^{-27} kg, the electron mass = 9.11×10^{-31} kg].

2. Insert numbers into eq. (7.6) in order to calculate the radii of the first Bohr orbit ($n = 1$).

3. If the hydrogen atom were bound by a gravitational force rather than an electrostatic force, how large would its first Bohr orbit be?

4. A hydrogen atom in an excited state $n = 2$, returns to its ground state ($n = 1$) by emitting a photon. Assuming that energy is conserved, calculate the wavelength of the photon.