Environmental Physics for Freshman Geography Students

Professor David Faiman. Lecture 6, v.2.3 (December 9, 2003)

**1. Motion of the earth-sun system**

A *day* is most simply defined as the time it takes the earth to rotate once about its own axis. We can observe this rotation by looking at the stars at night. They appear to rotate about a fixed point in the sky (located very close to the position of the star *Polaris*, if you are in the Northern Hemisphere. The corresponding point about which the stars in the southern sky appear to rotate is a bit more complicated to find as it does not lie close to any of the easily visible stars). Now, the earth also revolves around the sun once in approximately 365 days. This motion can be charted, by studying the gradual night-to-night change in the apparent positions of the stars.

The earth is, of course, held in its orbit by the gravitational attraction between it and the sun. Now here is an interesting situation. In 24 hours the earth will have advanced in its orbit around the sun by 1 day out of a full orbit of 365 days, i.e. by approximately 1º. This means that if we set our clocks to read "noon" the moment when the sun appears directly overhead, we shall have to wait until the earth has rotated approximately 361º before we see the sun in the same position the next day. This is because the orbital motion of the earth around the sun makes the sun appear to be displaced eastwards by about 1º each day. Now this is confusing because it is more convenient to measure time using the sun than the distant stars (the latter being the only "fixed" reference frame relative to which the rotation of the earth can be measured).

In order to overcome this confusion, *two* distinct types of day are defined. The *sidereal day* is a day measured relative to the fixed stars: i.e. a day based on an axial rotation of 360º. Alternatively, the *mean solar day* is one based on the average length of time between the sun being overhead on two successive occasions. Clearly the mean solar day is about 4 minutes longer than a sidereal day.

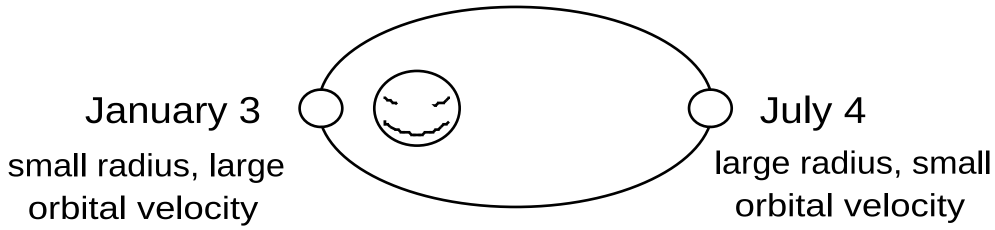


**Figure 1**: Elliptical orbit of the earth (with exaggerated ellipticity), with the sun at one focus

You will, no doubt, have noticed the adjective "mean" attached to the definition of a solar day. To see why this is necessary let us examine the shape of the earth's orbit a bit more closely. Actually it is not precisely true that the earth rotates around the sun. It is more correct to say that the earth and the sun both rotate about their common center of gravity. However, the mass of the sun is so much greater than that of the earth (333,000 times!) that their common center of gravity lies very close to the sun. The net effect is that we may regard the sun as being essentially stationary and the earth as moving around it in a *slightly* elliptical orbit. We are in fact closest to the sun on January 3rd and farthest away on July 4th (but the difference is only about 3 %). In **Figure 1** I have exaggerated the ellipticity of the orbit in order to make this point clearer.

However, conservation of angular momentum, as we learnt last lecture, implies that on January 3rd the orbital velocity of the earth must be greatest, and on July 4th it must be least. To see this more clearly, if m is the mass of the earth, v is its orbital speed and r is its distance from the sun then, conservation of angular momentum requires that the earth's orbital angular momentum in January is equal to its value at all other times, for example, in July. Hence:

$$m\, v_{Jan}\, r_{Jan} = m\, v_{Jul}\, r_{Jul} \qquad (6.1)$$

In January, r is small; therefore v must be correspondingly large in order for their product to remain constant.

This speeding up and slowing down of the earth in its orbit implies that one solar day will not be precisely the same length of time as the next. This is the reason we have to define a *mean solar day*. It also implies that simple sundials can not be accurate clocks! This is because if we mark off hourly intervals from noon (i.e. the moment when the sun appears highest in the sky) on one day we shall discover that the next day, our solar clock will be slightly slow or fast depending upon which time of year we calibrated it.

Now there is also another effect that contributes to the difference between one solar day and the next. But before telling you what it is, let me discuss how we describe the location of points on the surface of the earth.

**2. Longitude and Latitude**

The millimeter graph paper we talked about in the first lecture is useful for plotting graphs on flat surfaces. In fact we plotted such a graph in Lecture 3 in order to discuss velocity, acceleration, etc. But square graph paper is not very useful for plotting graphs on curved surfaces because on such a surface you can walk 1 km south followed by 1 km east and then 1 km north and end up at the same place: Something that is impossible on a plane surface. So, how do we specify locations on a curved surface? In the case of the earth, we draw a set of so-called *great circles*, i.e. circles of maximum possible diameter, that all pass through the north and south poles. *Four* such great circles are shown in **Figure 2**. Can you see them?

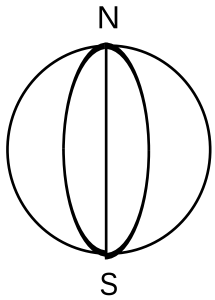


**Figure 2**: Schematic diagram of the earth, showing *four* great circles

By arranging a grid of evenly spaced circles of this kind we define a set of so-called *lines of longitude*. We measure the angular separation between lines of longitude in degrees. By convention, and for historic reasons, the 0º line passes through Greenwich, near London, UK. This line is sometimes called the *Greenwich meridian*. Similarly, we talk about locations being east of the Greenwich meridian or west of it. For example, Beersheva has longitude 34.6º E, New York City has longitude 74º W. Note that as you get closer to the poles the lines of longitude all bunch together whereas they are at their maximum separation around the equator.

Exercise: If the earth's equatorial radius is 6,378 km, what is the separation between each degree of longitude around the equator?

Now just as we need both x and y axes on a flat surface we need another set of reference circles on the earth's surface - otherwise we could not distinguish between Mitzpe Ramon, Sede Boqer, Beersheva and Tel Aviv, which all happen to have the same longitude.

By convention, the second set of circles is all chosen to be parallel to the equator. They are called *lines of latitude* and are measured in degrees north of the equator and degrees south of the equator, where the angular measurement is performed relative to the center of the earth. **Figure 3** shows three lines of latitude: the equator (0º by convention), 30º N and 60º S. The figure on the left is the way these lines would appear on a map. The figure on the right is a cross-section through the earth, showing how the angles are measured.

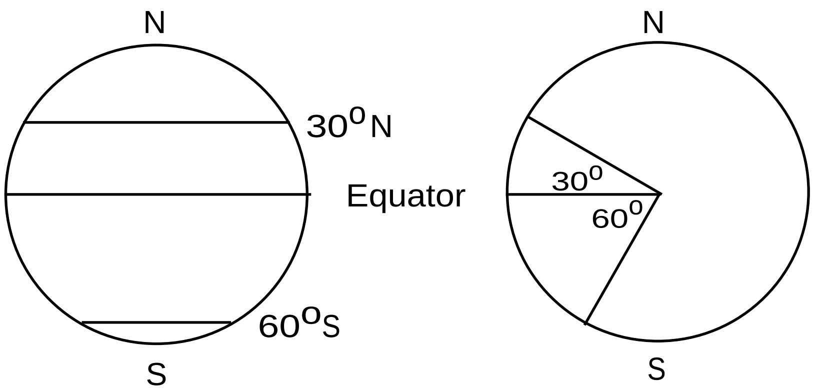


**Figure 3**: Schematic diagram of the earth showing 3 lines of latitude. Left-hand-side shows the lines as they might appear on a map. Right-hand-side is a cross-section through the earth showing how the angles are defined

Note that, unlike lines of longitude, the lines of latitude do not bunch together. We can now distinguish Mitzpe Ramon as having latitude 30.6º N, Sede Boqer as having latitude 30.9º N, Beersheva as having latitude 31.2º N and Tel Aviv as having latitude 32.1º N. When greater accuracy is required, each degree is subdivided into 60 minutes and each minute is further subdivided into 60 seconds. These are not minutes and seconds of time but of *arc*.

**3. Time Zones**

In principle, *noon* is the time of day when the sun reaches its highest point in the sky (called "the zenith"). Since this moment will not be the same for two observers located relative to one another along an east-west line, it is necessary to define so-called *Time Zones* for which, by agreement, noon occurs at the same time for all people within the zone. By universal agreement, the 360 degrees of longitude are divided into 24 times zones of 15 degrees (i.e. 1

hour in time) each. Wintertime in London, England, is referred to “Universal Time” (UT). As we proceed eastward in 15-degree steps from London we encounter, successively, the time zones: UT + 1 hour, UT + 2 hours, UT + 3 hours, etc. On the other hand, as we proceed westward from London we pass through the time zones UT - 1 hour, UT - 2 hours, UT - 3 hours, etc. Because the earth is round, the times zones UT +12 and UT - 12 have the same local time. How broad is a time zone? In theory it is precisely 15 degrees but in practice it is a matter of local policy. For example, most of continental Europe, including Spain, has chosen to be in time zone UT - 1 even though in the case of Spain, the UT time zone would be more natural.

There is one further practical complication associated with time zones. It concerns what day of the week it is. Suppose it is midnight between Saturday night and Sunday morning in London. Then in time zone UT + 12 it should be noon on Sunday. On the other hand, in time zone UT - 12 it should be noon on Saturday. But it cannot be Saturday and Sunday at the same time! Correct. In order to overcome this problem an *International Date Line* is defined. This coincides mainly with the longitude = 180$^o$ line, which passes mainly over water in the Pacific Ocean. In those few places where the 180$^o$ line passes over land, the International Date Line is made to deviate from it so that the latter passes only over water along its entire length. Then, by definition, as you pass across this line from east to west, you *lose* one day. For example, if it is Saturday just east of the line, then it is the same time Sunday just west of the line. On the other hand, if you cross the line from west to east, you gain a day: Sunday becomes Saturday which you can live all over again!

Two final points regarding the local definition of time: (1) Some countries move their clocks forward by one hour (and sometimes more) in summer time in order to save fuel. This is sometimes called “daylight saving time”, or simply “summer time”. (2) Some countries (e.g. India) subdivide the universal time zones into half-hour sub-divisions. Thus, for example, when it is 10 am in Tel Aviv, it is 13:30 pm in New Delhi.

**4. Tilt of the Earth’s Axis**

Let me now return to that other effect I warned you about which contributes to variation of the length of a solar day. Although the earth rotates about its own axis (once every 24 hours) in the same general direction that it orbits the sun (in approximately 365 days) - i.e., both rotations are anti-clockwise when viewed from above the North Pole - these two rotations are not precisely parallel to each other: the earth’s spin axis is tilted at approximately 23.5$^o$ to the axis of its rotation about the sun.

This tilt is, of course, responsible for the seasons of the year: It is summer over that part of the globe that is tilted towards the sun and winter over the part that is tilted away from the sun. In the Northern Hemisphere mid-summer occurs around June 21 and mid-winter around December 21. In the Southern Hemisphere the opposite holds true.

But there is another interesting effect of the earth's axis being tilted. In both mid summer and mid winter the two motions responsible for the difference between sidereal and solar time (i.e. orbital motion of the earth around the sun and the apparent motion of the sun from one line of longitude to the next) are precisely parallel to each other. On the other hand, at the so-called *equinoxes* (approximately March 21 and September 21), these two motions have an angle of 23.5$^o$ between them. Therefore, just as a change in ocean current will cause a swimmer, swimming at constant velocity relative to the water, to experience a change in his velocity relative to the shore, so too will the difference between sidereal and solar time vary between these pairs of seasons.

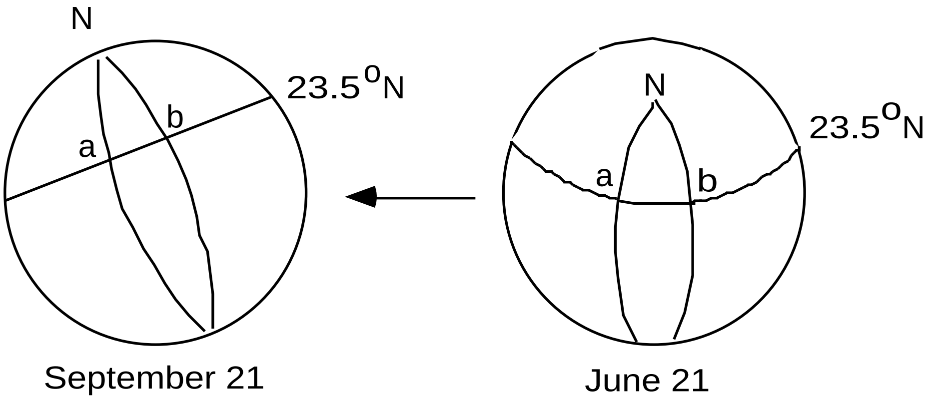


**Figure 4**: Two schematic diagrams of the earth as viewed from the sun. (left, September 21; right, June 21). The arrow indicates the direction of the earth's orbital motion. The longitude lines passing through points a and b on the earth's surface are separated by 15 degrees

**Figure 4** illustrates this situation on June 21 and September 21. In summer a rotation of the earth from a to b represents 1 hr of sidereal time. During this time the earth moves in the direction of the arrow. An observer at position a would see the apparent motion of the sun as being the sum of the earth's orbital and rotational motions. A precisely similar situation would occur in mid winter (i.e. December 21). In autumn however, the sidereal motion of the earth - by precisely the same amount as in summer - is now at an angle of 23.5$^o$ relative to the orbital motion and, hence, their combined effect will be slightly less than in summer and winter.

These two effects: that of the ellipticity of the earth's orbit with its annual cycle, and that of the tilt of the earth's axis with its 6-month cycle, together, account for the daily difference between sidereal and solar time. This difference is sometimes referred to as the *equation of time*. The difference between sidereal time and solar time varies from approximately -15 minutes in mid February to approximately +16 minutes at the end of October.

## 5. Precession of the Earth's Axis

You will probably have noticed that most times you spin a coin the axis of rotation does not remain fixed in a vertical direction. Instead, it moves in a conical manner that we call *precession.* Those toy gyroscopes do the same thing when placed on the miniature Eiffel Towers they come with. The phenomenon of precession is well understood but requires a fairly elaborate mathematical formalism for its quantitative description. Since such formalism lies outside the scope of this course I'll tell you in words what is happening:

If you consider a coin that is standing on its edge but leaning over slightly, it will obviously fall over. Why? Because a gravitational force pulls its center of gravity downward while the table pushes upward on the lowermost part of the coin. Since these two opposing forces do not act along the same line they will obviously *rotate* the coin onto one of its sides, as in the right hand side of **Figure 5**.

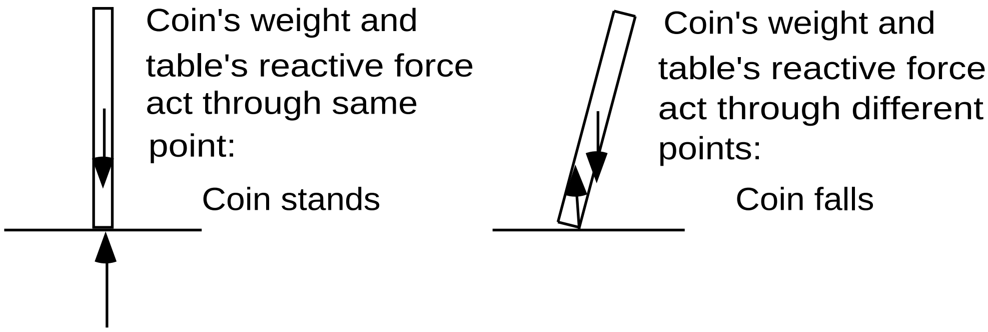


**Figure 5**: Sideways view of a coin standing on its edge. Left: The coin stands because its weight and the table's reactive force act through the same line. Right: The coin falls over because these two forces do not act through the same line - Their combined effect rotates the coin about a horizontal axis

Now spin the coin. A moment's thought will enable you to realize that there are now *two* rotation effects happening simultaneously: one trying to spin the coin about a vertical axis while the other tries to rotate it about a horizontal axis tangential to the bottom edge of the coin. Now we already encountered the law of conservation of angular momentum - which causes rotating objects to remain with the *direction* of their rotation axis fixed. But what happens when an object - like a coin - has two competing rotation axes? The answer is that these two rotation axes combine to form a third effective axis and it is the direction of this latter axis that is conserved by the law of conservation of angular momentum.

This is not as strange as it may seem. You are all familiar with having to allow for a side wind when throwing a ball, or having to allow for an ocean current while swimming or sailing. This allowance, mathematically, is referred to as the combination of *vectors*. There is one vector that describes your velocity relative to the water and another vector that describes the velocity of the water relative to the land. The vector that describes your velocity relative to the land is simply the combination of the first two vectors.

Well, rotations are also vectors: the combination of two of them (using an appropriate mathematical rule which we need not bother about in this course) produces a third effective rotation. Returning now to our spinning coin: If the coin is spinning fast and almost vertically then the dominant rotation will be its spin. In this case its precession will take the form of a very narrow cone. The more tilted it is, however, at the moment you let it go, the larger will be the rotation that is trying to topple it. In this case the resulting precession will be a much wider cone.

We can now talk about the earth. As we already saw, the earth's rotation about its own axis is not precisely perpendicular to the plane of its orbit around the sun - it is tilted at $23.5^{o}$. It therefore precesses. You might wonder *why* it precesses: After all, it is not standing on a table and the gravitational force of the sun seems to act through the earth's center of gravity. So where is the additional rotation coming from? The answer is that, unlike a coin, the earth does not have a perfectly regular geometric shape and its mass is not uniformly distributed. (We encountered this feature of the earth when we discussed the force of gravity and changes in the value of g over different parts of the globe). As a result, the gravitational attraction of the sun does not act precisely through the earth's geometrical center. This is a very small effect but it is enough to cause the earth's axis to precess on a time scale of one rotation in about 26,000 years!

**Problem set 6 (the earth and its motion)**

1. Where on earth would you be standing if you could walk 1 km southward, then 1 km westward, then 1 km northward, and end up where you started?

2. There is more than one answer to the above question. How many more can you find?

3. The planet Venus has an equatorial radius of 6,050 km. What is the distance between each successive degree of longitude around the equator of Venus?

4. Use maps with a scale 1:100,000 and estimate the latitude and longitude (in degrees and minutes) of the following sites in Israel: Kibbutz Malkiya (Upper Galilea), Kibbutz Giv’at Brenner (Central Plain), Kibbutz Ein Gedi (Northern Aravah), and Kibbutz Yotvatah (Southern Arava).

5. If it is 10 am Tuesday in Oakland, California (Approx. longitude = $122^{o}$ W), what time and day is it in Auckland, New Zealand (Approx. longitude = $175^{o}$ E)?