

1. Conservation Of Energy

In lecture 2 we encountered the *Law of Conservation of Momentum*. Although the concept of *momentum* is not as familiar in everyday life as is *velocity*, for the description of motion, its conservation - via the recoil of guns, etc. - is relatively easy to grasp. Velocity, on the other hand, is not conserved: That is why we need to introduce momentum in the first place.

Now, *energy* is also conserved but the *Law of Conservation of Energy* is much more subtle. The reason for this is that energy takes a large variety of forms so it is not always easy to see that it is conserved. Let us, as an example, consider a meteor, traveling through space. Consider all the changes in energy it undergoes.

Far from the earth (and from any other massive body) it has only kinetic energy, $p^2/2m$ and rest-mass energy, mc^2 . Or so we might at first imagine! But, as it approaches the earth something strange happens: It speeds up! The meteor's kinetic energy increases but its rest-mass energy does not decrease. So where is this extra energy coming from?

From Newton's Law of Gravitation we understand that the meteor and the earth are attracting one another, and that the nearer they get together the stronger will be the force of gravity. Remember: if we half the distance, we quadruple the force, etc. This force, by Newton's Second Law, produces an acceleration which causes the meteor's velocity to increase. Thus the source of the increasing kinetic energy of the meteor must be some form of gravitational energy.

We can understand this phenomenon in terms of energy, by assigning to the distant meteor a property called *potential energy*, which is simply proportional to its distance from the earth. As the meteor approaches the earth, it loses potential energy and gains kinetic energy at the same rate. Or, if you like, its potential energy is converted into kinetic energy. In this manner, energy is conserved.

Close to the earth's surface it is convenient to measure the potential energy (P.E.) of an object as the product of its *mass m*, *times the acceleration due to gravity g*, *times its height above the surface h*. Thus,

$$\text{P.E.} = m g h \quad (5.1)$$

If the object is released at rest from a height h , then, as it falls, it will lose potential energy and gain kinetic energy (K.E.). At the moment it reaches the ground, its potential energy will be zero and it will have a kinetic energy that is equal to the original value of the potential energy. Thus,

$$\text{K.E.} = m v^2 / 2 = \text{P.E.} = m g h \quad (5.2)$$

Eq. (5.2) enables us to calculate the velocity if the height is known. Notice that the mass cancels out, so any object, no matter how massive, will fall at the same rate and reach the ground with the same speed. This is what Galileo discovered in Pisa!

You may object that this seems a bit arbitrary. After all, we can understand all aspects of the meteor's motion from Newton's Law of Gravitation alone - without ever having to introduce the concept of potential energy. Your objection is correct. But there are three reasons for introducing the concept of potential energy. First: convenience. It is simpler, in many cases - including the present example - to calculate with energies rather than with forces. Second: generality. Not only gravitational forces but also many other types of force in physics (e.g. electrical forces) can be replaced by corresponding potential energies. In each case, greater simplicity is achieved in performing calculations.

The third reason is, perhaps the most amazing of all. In fact, it is so remarkable that it would not usually be introduced to physics students until they study for a more advanced degree. It goes under the title of *Noether's Theorem*. I want to talk a little about Noether's Theorem for two reasons. First, it gives a deeper insight into why these various mechanical concepts I keep throwing at you - in a seemingly arbitrary manner - are not arbitrary. Second, it will be a convenient manner for me to introduce the third and final conservation law you will need to know about.

2. A Philosophical Digression into the Relationship between Conservation Laws and Symmetries in Nature

Before I state what Noether's *theorem* is, I must emphasize that it is very much like a theorem in geometry, in that it can be approached from either side. For example, once you grasp the meaning of Pythagoras' theorem, you realize that not only do right-angle triangles possess a certain, well-known, relationship among the squares of their sides but also, conversely, if any triangle is found to obey this relationship then it *must* contain a right angle. Now for Noether's theorem, which may be simply stated: *Any symmetry of nature implies and is implied by a conservation law.*

In the case of the laws of conservation of momentum and energy the corresponding symmetries turn out to be simple but fundamental properties of space and time - this is why I said that Noether's theorem is "amazing". What fundamental symmetry can we assign to *time*? That it flows forward and backward at equal rates? Nonsense: Time only flows forward! So, to what property of something as vague as time can we assign a symmetry? The answer, if we think about it, is the rate at which time proceeds. Have you ever looked at a clock and wondered how we know that all 5 sec. intervals are truly equal to one another? They don't always "feel" similar! Noether's theorem provides an answer: If time is *homogeneous* (i.e. 5 sec. now is exactly the same as 5 sec. last week) then something must be conserved, and that something turns out to be *energy*. Conversely, if energy is conserved then time must be homogeneous. Proof of this requires a graduate course in Mechanics because it is highly mathematical. But it is such an amazing and simple relationship that I wanted you to know about it, even if you cannot prove it. Now, just as it is possible to grasp Pythagoras' theorem without knowing how to prove it, let me move on to what Noether's theorem implies for the symmetries of space.

Empty space is also *homogeneous*: i.e. 1 m near the earth has the same length as 1 m near Mars. As you may now have guessed, Noether's theorem relates the *homogeneity of space* to

another conservation law. This one is the law of *conservation of momentum*. Once again, the proof is highly mathematical but the concept is simple: If space is homogeneous then momentum must be conserved; conversely, if momentum is conserved then space *must* be homogeneous.

Thanks to Noether's theorem, the mechanical laws of conservation of momentum and energy, may be understood as being simple *requirements* of the fact that space and time are homogeneous. In similar manner, the conservation of electric charge may be understood as being a requirement of a certain kind of symmetry possessed by electromagnetic fields. Physicists use Noether's theorem for other, more puzzling, conserved quantities in order to try and probe the, largely unknown, field equations responsible for the various elementary particles (e.g. quarks and neutrinos) but here we are beginning to stray too far from the needs of a geography course. So let us return to empty space.

A moment's thought will convince you that space, unlike time, has another symmetry property in addition to homogeneity: It is also *isotropic* - i.e. the same in all directions. Noether's theorem therefore requires something to be conserved. That something is called *angular momentum*, which we shall return to below.

At this stage you may well be asking whether there is no limit to the mechanical concepts and jargon that I can throw at you. But, thanks to Emmy Noether, there is such a limit and we have now reached it. Space and time have only three symmetries among them and these imply three and only three conservation laws. Physicists have, as I have already mentioned, found other quantities to be conserved by nature - e.g. electric charge and other properties of the fundamental particles of nature - but these conservation laws have nothing to do with the properties of space and time, and hence lie outside the realm of mechanics. We can thus begin to sense a deep unity that exists among the laws of physics. They are not simply individual expressions of the outcome of unrelated experiments but, rather, a unified description of the way objects *must* behave because of the fundamental properties of the universe they occupy: Truly environmental physics at the deepest level!

3. Back To Energy

After this philosophical digression, let us return to our meteor as it speeds towards the earth. It is losing potential energy and gaining kinetic energy. But then it encounters the earth's atmosphere and a new phenomenon occurs. It begins to encounter molecules of nitrogen and oxygen. To begin with, the effect is barely noticeable since the atmosphere is so thin - it simply pushes these molecules aside. Of course, in the process of transferring some energy to those molecules the meteor will lose some energy. But, as the atmosphere becomes ever denser, the kinetic energy lost by the meteor becomes more and more significant: We say that *friction* is slowing it down. Where is the lost kinetic energy going? It is heating up the surrounding air - i.e. increasing the kinetic energy of the gas molecules. But friction is a two-way process: the surrounding air is also heating the meteor which, at a certain temperature, gets so hot that it starts to emit light and we can see it moving across the sky. It thus starts to lose *light energy*. It becomes what we call a *meteorite*. Let us see what is happening at the atomic level.

Atoms consist of a dense positively charged nucleus of tightly-bound protons and neutrons, surrounded by a lightweight, "cloudlike" distribution of negatively charged electrons. I placed quotation marks around the word cloudlike because for a massive atom, such as lead,

it is not difficult to imagine the 82 electrons behaving like a cloud. But even for hydrogen, it's one and only electron also behaves like a cloud in that it is not possible to know where the electron is at any given moment. This is one of the peculiar lessons of *quantum theory* which describes all phenomena at the atomic scale and smaller. Furthermore, quantum theory tells us that, although we can not know where precisely the individual electrons are located in an atom, they must have one of a discrete set of allowed values for their energy. In the normal state of an atom, all electrons sit in their minimum allowed energy states, called "the ground state energy". However, atoms can become excited to higher states of energy by the absorption of energy from outside. When this happens, some of the electrons have their energies raised to higher excited states among the allowed energy states of that atom. However, such excited states are not stable: each excited electron falls back to its ground state by the emission of photons - i.e. quanta of light. In the case of our meteor, the impacts of the rapidly moving nitrogen and oxygen molecules, as they fly past, impart energy to the electrons of the atoms comprising the meteor and raise them to excited energy states. As those electrons fall back to their ground states light is emitted. That is the light that we see as the meteorite streaks across the sky. In fact, all light is generated this way, whether from a match, an electric lamp, or the sun. In the case of a match, frictional forces excite its electrons; in the case of a lamp, electrical energy does the job; and in the case of the sun, nuclear reactions are ultimately (but indirectly) responsible for the light we see.

It is possible to hit an electron so hard that it leaves its parent atom altogether and the atom remains positively charged. But not for long! It soon grabs an electron from some nearby convenient source. In the case of our hot meteorite the nearby source are the oxygen atoms of the surrounding air. The meteorite thus catches fire and burns. In the process, it loses a certain amount of *chemical energy*.

At this stage the meteorite is losing energy in the form of light and chemical reactions, not to mention pieces that are physically breaking off. Finally, if the original meteor was large enough to start off with, its remains will hit the ground. If you happen to be nearby you will hear the "whoosh" as it flies through the air and the "thud" as it hits the ground. These are examples of *sound energy* - pressure waves excited in the air.

After we have let the meteorite cool down we may well wonder what has happened to all the energy it started with. Well, we now know the answer. The meteor lost energy but that energy was ultimately absorbed by other bodies (both massive, and massless - i.e. photons): none of it disappeared from the universe.

4. Conservation of Angular Momentum

Let us now return to Noether's theorem and the three symmetries associated with space and time. We stated that the homogeneity of space and time lead, respectively to the laws of conservation of momentum and energy. Time is not isotropic (since it does not flow in both directions) so there is no additional conservation law here. But space *is* isotropic and the associated conservation law is the *Law of Conservation of Angular Momentum*.

Now, the isotropy of space means that if you stand still and look around you it looks the same in all directions. Angular momentum is therefore, not surprisingly, a quantity having to do with rotating objects. Its conservation has the effect of preserving the rate at which objects rotate. Moreover, since angular momentum is also a vector quantity, the direction in which their axis of rotation points is also conserved. When I say "the rate at which objects

rotate” I am not referring to their angular velocity because, like linear velocity, this quantity is not conserved. Angular momentum - which we shall discuss more quantitatively below - is the quantity that is conserved. A common example is the gyroscope, which can keep a ship’s compass horizontal even in heavy seas. Gyroscopes, of course, gradually slow down because of friction, but in empty space they would spin forever - as, for example, the earth does.

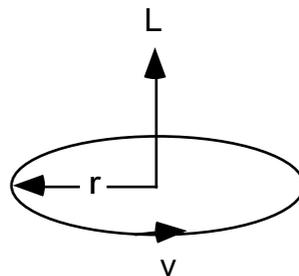
The earth itself is, of course, a giant gyroscope rotating in space at the rate of one rotation each 24 hours. If you imagine holding a mini globe of the earth in your right hand, and turning it in the direction in which your fingers are pointing, then your thumb points in the direction we call "north". This is the way the earth rotates: it is responsible for the sun appearing to rise in the east and set in the west. Don't try this with your left hand or you will get exactly the opposite result!

Now the earth also revolves around the sun, once in approximately 365 days. Interestingly, the rotation of the earth around the sun is also "right handed". So too are the rotation of the other planets of the solar system and the rotation of their moons about those planets that have them. [There are a small number of exceptions that we shall not discuss in detail]. This regularity, together with the law of conservation of angular momentum, suggests that the sun and solar system were formed from a single cloud of primeval matter which broke into smaller pieces each of which continued spinning in the same manner as the original dust cloud. The central piece became the sun whereas the smaller pieces became the planets and their moons.

Now it helps our understanding of angular momentum if we become slightly more quantitative. Suppose we have an object of mass m and velocity v (and hence momentum $p = mv$) rotating in a circle of radius r about an axis. We define the *magnitude* of the angular momentum as:

$$L = m v r = p r \quad (5.1)$$

and its *direction* as being along the axis of rotation in the direction of your right thumb! In the figure you see a particle of mass m moving with velocity v in a circular orbit of radius r . The angular momentum has magnitude mvr but its direction is as shown by the arrow L .



This has an interesting and important implication for the earth's motion about the sun, as we shall see in the next lecture.

Problem set No.5 (energy and angular momentum)

[1] A 5-storey building is 15 m tall. It has windows 2 m, 5 m, 8 m, 11 m and 14 m above ground level. Use the law of conservation of energy to calculate the velocity of an object as it passes each of the windows after falling off the roof.

[2] A railway car of mass m , moving with velocity v , strikes a stationary car of mass $3m$. If kinetic energy is conserved in this collision (i.e. none of the energy escapes from the collision), use the law of conservation of momentum in order to calculate the speed and direction of each car after the collision.

[3] The moon has a mass of 7.33×10^{22} kg. It orbits the earth at a mean distance of 384,000 km. The average time it takes to complete one orbit is 27.3 days. Calculate its mean velocity in orbit (in m s^{-1}) and the magnitude of its angular momentum (in $\text{J s} = \text{kg m}^2 \text{ s}^{-1}$).

[4] An early (over-simplified) description of the hydrogen atom pictured the electron as orbiting the proton in a similar manner to which the moon orbits the earth. If the electron has mass $m = 9.11 \times 10^{-31}$ kg, orbital radius $r = 2.12 \times 10^{-10}$ m and angular momentum $L = 2.11 \times 10^{-34}$ J s, calculate the magnitude of its orbital velocity v .