Environmental Physics for Freshman Geography Students

Professor David Faiman. Lecture 3, v. 1.7 (November 18, 2003)

1. Force and Newton's Laws of Motion

Let us return, yet again, to eq (**2.9**) from the last lecture, (which is the same as eq (**1.6**) from the first lecture but with all terms having the correct dimensions):

$$E = mc^2 + p^2/2m \qquad (\mathbf{3.1})$$

It will be slightly more helpful, in the following discussion, to write the kinetic energy term using velocity instead of momentum:

$$E = mc^2 + mv^2/2 \qquad (\mathbf{3.2})$$

The kinetic energy term is not an intrinsic property of the particle since it depends upon how *we* measure its velocity. This may seem strange, however, if we run along side the particle at the same speed then we shall observe that its velocity seems to be zero. Hence, it will not have any kinetic energy. It will still have its built-in "rest-mass energy" since that part does not depend upon its motion.

Now all the time that the particle remains in uniform motion, i.e. v = constant, no real meaning can be attached to its velocity. If we were observing it in free space we could not tell whether the particle or we were moving. You may have noticed similar effects on a boat at sea, or even while sitting in a bus and waiting for it to move. As the bus next to us moves forward we get that momentary strange feeling that we are moving backward? This is an example of the **Principle of Relativity** which states: *There is no experiment you can perform that will enable you to know the absolute velocity of a uniformly moving object.*

Similarly, *a uniformly moving object will continue that way forever unless acted on by some external force that changes its velocity* (i.e. speeds it up, slows it down or changes its direction). This is a statement of what is known as **Newton's First Law of Motion.**

But what do we mean by "force"? In order to understand this concept, in a quantitative manner; let us return to the definition of velocity as the *rate of change of* position. It is convenient to visualize this graphically.

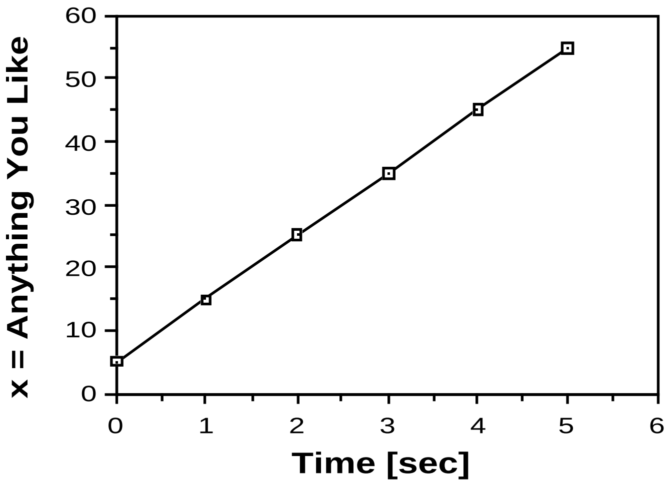

Figure 1: Graph of "anything" vs. time. If "anything" represents *position*, the slope of the graph gives the object's velocity. If "anything" represents *velocity*, the slope of the graph gives the object's acceleration.

Let x indicate the position of a particle, and t the time. If the particle moves at constant velocity then equal changes in x will occur during equal intervals of time. In the particular example shown on the graph in **Fig. 1** the particle is at x = 5 m (or cm or km or anything you like) at t = 0. After 1 sec it has moved to x = 15 m. After another 1 sec it is at x = 25 m, etc., etc. We see that its velocity is 10 m s^{-1}, i.e. its position is changing uniformly at the rate of 10 m each second. This uniform velocity is represented on the graph by a straight line of a definite slope angle. If the velocity of the particle were larger - e.g., 20 m s^{-1} - the slope of the graph would be steeper. (Plot it and see!). On the other hand, for a smaller velocity the slope would be gentler. In fact, the slope would be zero if the particle were stationary. Why? Because at times t = 0, 1, 2, ..., etc., it would always have the same value of x.

What would happen to the graph if the particle's velocity were not constant? If the particle accelerates its velocity increases. This means that the slope of the graph would increase and it would no longer be a straight line. But let us stay with straight-line graphs for the time being.

I labeled the vertical axis of the graph in **Fig. 1** "x = Anything You Like" because x does not have to represent the position of the particle. We could equally well use such a graph to represent the way velocity changes with time. Thus, in the graph in **Fig. 1**, where x now represents velocity, at time t = 0 the velocity is 5 m s^{-1} (or any other units you may prefer), at t = 1 sec the velocity has increased to 15 m s^{-1}, at t = 2 sec the velocity is now 25 m s^{-1}, etc., etc. We see that the velocity is increasing at a steady rate of 10 m s^{-1} each second - or, expressed mathematically, at the rate of 10 m s^{-2}. In this case, the slope of **Fig. 1** represents the *acceleration* of the particle.

These straight-line graphs, when something is plotted as a function of time, are very helpful for visualizing quite abstract concepts. Velocity is an everyday phenomenon so it is reassuring to find that a constant velocity is represented by a straight line on a position-time graph. Acceleration is a slightly more abstract concept. We can "feel" it but the velocity-time graph gives us a quantitative measure for acceleration, and if the latter is constant then the linearity of the plot enables us to see, at a glance, precisely what the velocity is at every instant of time.

If the acceleration were zero, then the velocity-time graph would be a horizontal straight line. I.e. at times t = 0, t = 1, t = 2, ..., etc. the velocity would always have the same value.

Now, if we multiply the particle's velocity by its mass we obtain its momentum. So the same horizontal graph of momentum plotted against time would tell us that the particle was moving with constant momentum.

But what if the momentum graph were to have a positive slope? This would tell us that the momentum was increasing at a constant rate. This, in turn, would indicate that the particle was accelerating. What would be the cause of such an increase in momentum or acceleration? The answer is that some external force must be acting on the object. **Newton's Second Law of Motion** states that *force is the rate of change of momentum*. This law actually provides a quantitative definition of the concept of force. Alternatively, since momentum = mass x velocity, it is also correct to define force as the mass of a body multiplied by its acceleration:

$$F = ma \qquad (\mathbf{3.3})$$

If mass is measured in kg units and acceleration is measured in units of m s^{-2}, then force is measured in "newtons", denoted by the symbol N.

Eq. (**3.3**) is only true if the object has a constant mass. This assumption would not be true for a rocket that is burning fuel as it accelerates and therefore has a decreasing mass. In that kind of situation one must calculate the rate of change of momentum where both the velocity and mass are changing. But, in this course we shall not encounter such complicated problems.

Now, unlike velocity, acceleration is *not* relative. If you are in free space and looking at an accelerating object its velocity will appear to be increasing relative to you. But how do you know that it is not you who are accelerating? Simple - take a bucket of water with you. If the water level remains smooth you are not accelerating. Did you ever try drinking some hot tea in a bus, at precisely the moment the driver decides to move out of the bus station? You are the one who accelerates: not the girl sitting in the other bus and laughing!

Newton's Third Law of Motion is the observation that *active and reactive forces are equal and opposite.* What does this mean? Simply, that if you stand on the ground the ground pushes up on you with a “reactive force” that is of exactly the same magnitude as the force with which you push down on the ground. If this were not the case then you would either move upward or downward. An elevator floor, for example, is capable of pushing up on you with a greater force than you push down on it. This enables it to raise you to whichever floor you would like to reach. Other buttons enable you to exert a greater force on the elevator than it exerts on you and, hence, to go down.

Newton’s three laws of motion are very general, in that they apply in all situations one encounters in everyday life. They constitute a complete formulation of the science of “mechanics”. There are also several other alternative formulations – that physics students have to learn – but because Newton’s laws are intuitively rather clear, they are pedagogically the most useful.

2. Gravity

Newton's three famous laws of motion are also very general in that they apply no matter what kind of forces are at work (with certain very specific kinds of exception that are outside the scope of this course since they are of no relevance to geography). But Newton also discovered a very specific force law for the case of gravitational forces. By observing the motions of the sun, moon and planets he realized that their varied and complicated motions could all be explained if the force with which any two bodies attract one another has two simple properties:

(1) It is proportional to the product of their masses.

I.e. double the mass of one of them (either one - it does not matter which) and the gravitational force doubles; double the mass of one and triple the mass of the other then the gravitational force becomes six-times as great; etc., etc.

(2) It is inversely proportional to the square of the distance between them.

I.e. double the distance between them and the gravitational force decreases to a quarter of its original value; triple the distance between them and the gravitational force decreases by a factor of nine; etc., etc.

Newton's Law of Gravity may be written:

$$F = (G\, m_1\, m_2) / r^2 \qquad \textbf{(3.4)}$$

where, m_1 and m_2 are the masses of the two bodies, r is the distance between them, and G is a constant that must be fixed according to the units one is using. If the masses are measured in kg and the distance between them is measured in m, then Newton’s so-called **gravitational constant** takes the value $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$.

For terrestrial purposes, i.e. the gravitational attraction of the Earth to objects on its surface (like you and me), it is more convenient to make use of another constant, g **the acceleration due to gravity**. In order to see the relationship between G and g it is convenient to consider the gravitational attraction between the Earth and an object on its surface. We have:

$$F = G M m / R^2 \qquad (\mathbf{3.5})$$

where M is the mass of the Earth (5.97×10^{24} kg), m is the mass of the object on its surface, and R is the mean radius of the Earth (6370 km). On the other hand this force of attraction is simply equal to the weight of the object, which, in the last lecture, we defined as:

$$F = m g \qquad (\mathbf{3.6})$$

By equating these two expressions we see that the weight cancels out, and:

$$g = GM/R^2 = 9.81 \text{ m s}^{-2} \qquad (\mathbf{3.7})$$

The constant g is equal to the acceleration with which a particle falls if you release it near the Earth's surface. Acceleration is, of course, *the rate of increase with time* of velocity. If the object is released at rest, then after 1 sec. its velocity will be 9.81 m s^{-1}. After 2 sec. its velocity will have increased to 19.62 ms^{-1}. After 3 sec. its velocity will have increased to 29.43 ms^{-1}, etc.

In a later lecture we shall see that because the Earth is rotating and not uniformly spherical in shape, the precise value of g will vary very slightly from place to place. Therefore, for purposes of calculations in this course, the value 9.8 m s^{-2} will be of adequate accuracy.

Problem set No.3: **(forces, etc.)**

1. In **Figure 1**, suppose that **"Anything You Like"** represents the velocity of an automobile in units of km per hour. What is its acceleration in m s^{-2}.

2. If the mass of the automobile in the previous question is 750 kg, what force is its engine exerting on it in order to produce such an acceleration?

3. What is the "weight" of the above-mentioned automobile, in units of: (a) kg; (b) N ?

4. The planet Mars has a mass which is 10.7% that of the Earth, and a radius that is 53.2% that of the Earth. Calculate the value of g, the acceleration due to gravity, on the surface of Mars.

5. If a person weighs 90 kg on Earth, how much would he weigh on Mars?