Environmental Physics for Freshman Geography Students

Professor David Faiman. Lecture 2, v. 3.3 (November 4, 2003)

1. Momentum

In lecture 1 we encountered a number of terms that are probably semi-familiar to some of you but, perhaps, not yet fully understood. Others among you may have merely heard the words "mass", "momentum" and "energy", but never encountered them in a scientific context. One of the aspects of science that often confuses non-scientists is that it tends to take commonly used words and give them an extremely specialized definition. This is the case for these three words, so let me now begin to try and clarify their meaning as used in the context of physics.

Last week we found that, by applying Pythagoras' theorem to one of the equation's that Einstein discovered, a slowly-moving, free particle has two components to its total energy. One component is denoted by the symbol m. This component is always present, no matter how quickly or slowly the particle may be moving: even when it is completely at rest. For this reason it is known as the *rest-mass energy.* However, if the particle is moving slowly, it has an additional energy component due to its motion. This component is called the *kinetic energy* and is equal to the quantity $K = p^2/2m$, where p is the so-called *momentum* of the particle.

Now you may be more familiar with another formula for kinetic energy:

$$K = mv^2/2 \qquad \textbf{(2.1)}$$

where v is the *velocity* of the particle. Velocity is a more familiar everyday quantity because it is something one can actually see. Velocity is the *rate at which the position of a particle is changing.* One moment it is here; a second later it is 5 m away; another second later it is 10 m away; still another second later it is 15 m away: it's position is therefore changing at the rate of 5 m each second - we say that its velocity is 5 m/sec or 5 m sec^{-1}.

Momentum, on the other hand, is less familiar as an everyday phenomenon, so it will require some further discussion. But before we start, it is important to realize that since kinetic energy can be described either in terms of the velocity v, as in formula (**2.1**) or, alternatively, in terms of the momentum p, via the expression:

$$K = p^2/2m \qquad \textbf{(2.2)}$$

Then, it must be that:

$$p = mv \qquad \textbf{(2.3)}$$

We may take equation (**2.3**) as a working definition of momentum: *momentum equals mass times velocity*. But it is rather a "dry" formula so let us now try and put some life into it.

If you fire a gun you notice that the gun jumps backwards as the bullet flies forwards. Now you could make a movie of this phenomenon in slow motion, and use the film frames to calculate the velocity of the bullet in the forward direction and the velocity of the gun in the backward direction - at the moment it fires. You would then find something very interesting: the forward velocity of the bullet multiplied by the mass of the bullet would equal the backward velocity of the gun multiplied by the mass of the gun. In other words, *the forward momentum of the bullet equals the backward momentum of the gun*.

Experiments, performed on all kinds of objects, with ever increasing accuracy, show that this is always true: If a stationary object breaks up into a number of parts then the sum of all their momenta - when due account is taken of their various directions - will be zero, i.e. the momentum of the object before it broke up. On the other hand, if a moving object breaks up while in motion, the sum of the momenta of its parts will equal the momentum of the object before it broke up. This law of nature is called the *Law of conservation of momentum*. It is the reason that guns are heavy and bullets are light: we want the bullet to move rapidly and the gun to move slowly! If we want to use a heavy bullet (say for piercing armor plate) then we must use an even heavier gun.

The Law of conservation of momentum is a universal law that applies to everything: exploding stars, exploding bombs, exploding atoms and even exploding sub-atomic particles.

Two out of the many known sub-atomic particles are the *rho-meson* and the *pi-meson*. A rho-meson does not last very long after it is created - about10^{-24} sec in all. It then breaks up into two pi-mesons. If the break-up occurs when the rho-meson is stationary then the resulting two pi-mesons fly off in exactly opposite directions with exactly equal, but opposite, velocities. This is because momentum must be conserved. The reaction may be written like a chemical reaction:

$$\rho^o \to \pi^+ + \pi^- \qquad \textbf{(2.4)}$$

where the superscripts in equation (**2.4**) indicate the electric charge that each pi-meson carries.

Now there are actually three charge varieties of pi-meson in nature: the third is electrically neutral and also unstable but it has an extremely interesting (for our purposes) way of breaking up. In contrast to the rho-meson, the π^o meson "lives" an incredibly long time - about 10^{-16} sec! Like the rho-meson, the pi-meson also breaks up into two particles, but these are *photons*. The reaction may be written:

$$\pi^o \to \gamma + \gamma \qquad \textbf{(2.5)}$$

where, once again, the two photons fly off in opposite directions with equal velocities because of the law of conservation of momentum.

But in reaction (**2.5**) we encounter something almost unique: The photon is a particle that has momentum but no mass! How is that possible if p = mv? You are correct: it is a mathematical impossibility unless the photons move at infinite velocity - which they do not. Photons move at the *velocity of light*, (denoted by the symbol c, and equal to the universal constant: 299,792,458 m sec^{-1}) because they *are* light. We should therefore expect them to have zero momentum if they have zero mass.

The answer to this riddle is that p = mv is only strictly true for *slowly-moving* particles - you will remember that we derived eq. (**2.2**) for a slowly-moving particle, using our small-angle approximation to Pythagoras' theorem. For particles that move with velocities that are an appreciable fraction of the velocity of light, that approximation is no longer very accurate. Therefore, it should not be surprising that the formula p = mv would need to be modified. However, in geographical situations we shall never encounter a particle with mass that moves fast enough for us to have to worry about what the accurate formula is for momentum, so I shall not even write it down.

For our purposes the important things to know about photons are:

(**1**) They behave like any other sub-atomic particles.
(**2**) They happen not to have any mass (just as they happen not to have any electric charge).
(**3**) They always travel at the speed of light. (This turns out to be a consequence of their not having any mass, but that is the *theory of relativity*, which we shall not need to learn for purposes of the present course).
(**4**) They are the constituent particles out of which light and all other forms of electromagnetic radiation (e.g. gamma rays, X-rays, UV radiation, IR radiation, Radar, Microwaves, Radio

waves, etc.) are made.

(**5**) In addition to behaving like particles (e.g. they can move freely through empty space), they also have wavelike properties. (It turns out that so too do all of the other sub-atomic particles that do have mass. But an understanding of this fact – *quantum theory* - also lies outside the requirements of the present course).

We shall return to properties (**4**) and (**5**) in later lectures because the light and heat energy that reach us from the sun arrive in the form of streams of photons.

2. Mass

We have talked a lot about "mass" without saying what it is. All we know so far is that it is a property of particles, small and large (although some subatomic particles, like photons, evidently do not have any mass). But mass is easy to measure because it is also a property of large objects. It is a measure of how much of a particular substance we have. If we place two identical gold coins on the pans of a scale, they remain balanced because they both have equal amounts of mass. If we replace one of the coins by a smaller gold coin, that scale pan goes down because the smaller coin has less mass than the large coin. One may define a convenient unit for measuring the mass of coins, e.g. the gram. One can then use this unit, for example, to quantify how much mass there should be in a particular kind of gold coin (e.g. 10 gm), and, thereby, to help detect forgeries.

For more massive objects, the gram is inconveniently small, so, in like manner for measuring distances, we can define the kilogram, the megagram (better known as a tonne), the gigagram, etc. … Similarly, for assessing the mass of extremely light objects we may use, milligrams, micrograms, nanograms, etc.

For most purposes in this course (i.e. unless stated to the contrary) we shall employ the kilogram (kg) as our unit of mass, the meter (m) as our unit of length, and the second (s) as our unit of time. This combination is known as "*S.I. units*". All other mechanical quantities, such as force, energy, power, etc. will then be defined in terms of appropriate combinations of these three basic units

3. Weight

In daily life, we also use an expression "weight" which is easy to confuse with "mass" because the same units are commonly used for both. But, as we shall see in a moment, the two concepts are very different.

Recall that we measure mass by placing the object we wish to assess on one pan of a pair of scales, and placing a succession of standard "weights" on the other pan until the two pans are found to balance. In this way we discovered above that a particular kind of gold coin has a mass of 10 gm. But we could also use a different kind of measuring instrument, based on the concept of stretching a spring. This, so-called, "spring-balance" has a graduated scale attached to it, with grams marked out. Thus, if we place the same gold coin on the pan of a spring-balance, we shall find that the spring extends until the pointer ends up at the mark 10 gm.

Now suppose that we were to repeat these two measurements on the Moon. We would find the (perhaps) surprising result that, whereas the 2-pan pair of scales would still indicate the mass of the gold coin as being 10 gm, the spring-balance would indicate only 1.65 gm!

Why would we receive such a surprising result? The reason is that a spring-balance actually measures not the mass of the gold coin but, rather, the *force* that *gravity* exerts on it. Force and gravity are two more of those common words that are used by physicists in very specific ways. We shall define them more precisely in later lectures. As we shall also learn, because the force due to gravity is smaller on the Moon than it is on Earth, the spring balance would indicate a smaller reading than the 2-pan scales. If this is so, why would there be no change in the 2-pan scales? Because, the lower force of lunar gravity would act in precisely the same way on the masses in both pans. Therefore, they would remain balanced.

We may say, therefore, that a 2-pan pair of scales measures true mass, whereas a spring-balance measures the force of gravity on that mass. This force we call *weight*. It is traditional to use the *same units* for mass and weight, which is why there may be some confusion. This tradition has grown up because in the past, nobody had ever left the Earth. But one day, when inter-planetary travel becomes common, it will be necessary to make a careful distinction between mass and weight – particularly, if you are the buyer!

Strictly speaking, *weight equals mass times the acceleration due to gravity*. In symbols we may write this as:

$$W = m\,g \qquad (\mathbf{2.6})$$

where g is the acceleration caused by gravity (which we shall define and discuss in a later lecture). The number g is approximately constant for all parts of the earth's surface, and equal to 9.8 m s^{-2}. Therefore, in a system of units in which a certain object has a mass of 10 kg, its weight should really be expressed as 98 N, where the Newton, N, is a unit of force in S.I. units, equal to 1 kg m s^{-2}. However, because g is essentially a constant everywhere on earth, it is more convenient to treat the mass of an object as its weight. This is equivalent to assuming that g = 1, but may sometimes cause confusion if you forget that mass is the intrinsic property of a body and that weight is really the force that gravity exerts on it.

4. Mass and Energy

If we return to eq (**1.6**) from the first lecture:

$$E = m + p^2/2m \qquad (\mathbf{2.7})$$

and check the dimensions and units, we see that the kinetic energy term has dimensions $m\ l^2\ t^{-2}$, i.e. mass times velocity-squared, which in SI units, would be kg m^2 s^{-2}. This unit of energy is called the *Joule*. However, one term in eq. (**2.7**) apparently has the wrong units. Mass again! This time we are not guilty of dropping a g because that constant would give mass the dimensions of a force. In order for it to have the dimensions of an energy it must be multiplied by a constant with the dimensions of a velocity squared. Although this will seem highly arbitrary in a geography course, the correct constant happens to be the square of the velocity of light in free space. This number has been measured with great accuracy and is now defined as being precisely the number given above for c. For our purposes c = 3 x 10^8 m s^{-1} is an excellent approximation. Thus, eq.(**2.7**) should actually read:

$$E = mc^2 + p^2/2m \qquad (\mathbf{2.8})$$

This is precisely the same as eq. (**2.7**) if we are using units in which c =1. Similarly, eq.(**1.3**) from lecture 1 should really have been written as:

$$E^2 = m^2c^4 + p^2c^2 \qquad (\mathbf{2.9})$$

Eq.(**2.9**) is correct for all possible particle velocities, fast as well as slow, and the dimensions are now such that m is indeed a mass, p is a momentum and E is an energy. However, physicists who actually deal with "high energy" particles every day prefer to use "energy units" in which c = 1. Similarly, we, who deal with masses of kilograms every day, prefer to use units of weight in which g = 1. It is simply a matter of convenience.

Returning now to eq. (**2.8**), having now unified the dimensions of all the terms and settled upon units of Joules for energy, it is instructive to calculate the relative amounts of kinetic energy and rest-mass energy in, say, a 10 g bullet moving at 1000 m sec^{-1}. First, the kinetic energy of the bullet is:

$$mv^2/2 = 0.01 \text{ kg x } (1000 \text{ m s}^{-1})^2 / 2$$

$$= 5{,}000 \text{ Joules} \qquad (\mathbf{2.10})$$

On the other hand the non-kinetic, rest-mass term takes the value:

$$mc^2 = 0.01 \text{ kg x } (300{,}000{,}000)^2$$

$$= 9 \times 10^{14} \text{ Joules} = 900{,}000 \text{ GJ} \qquad (\mathbf{2.11})$$

The quantity in (**2.11**) is a huge amount of energy. If it were converted into electrical energy it would equal 250,000,000 kWh which is approximately the amount of electricity that could be generated by a 30 MW power station operating non-stop for an entire year – i.e., enough for the entire city of Eilat. And all of this energy resides inside the mass of a 10 g bullet, independent of what speed it is moving!

Could the mass of a bullet actually be converted into a useful form of energy? The answer is: Yes. We call it *nuclear* energy, and we already saw an elementary example of the conversion of rest-mass energy to radiant energy in the reaction given in eq (**2.5**) where a "massive" pi-meson decays spontaneously into pure radiation - in this case, two photons.

Nuclear processes are responsible for the energy emitted by the sun and also for geothermal energy generated within the earth's interior. They are examples of mass being converted into radiation. We shall return to such matters in later lectures.

But why are we unaware of this massive amount of energy in everyday life? The answer is partly that it is locked up so strongly inside each piece of matter that it cannot be released unless we make an extraordinary amount of effort to get at it (i.e. unless we cause a thermo-nuclear reaction to occur). Secondly, in our everyday experience of energy we really only measure changes in energy. The bullet contains all of this rest-mass energy whether it is stationary in the gun or whether it is fired, but it is the kinetic energy of the bullet that is the quantity of practical use: a stationary bullet can not do any damage, but a rapidly moving bullet is another story.

In later lectures we shall encounter other forms of energy that are useful on a daily basis (e.g. potential energy, thermal energy, chemical energy, etc.) but in all cases we shall see that it is only *differences in energy* that are of interest: we shall rarely, if ever, have to calculate an absolute quantity of energy.

For this reason, after the present lecture, we shall simply forget about the existence of rest-mass energy. It will be of no further interest to us (except at examination time!).

Problem set No. 2 (momentum and energy)

1. Suppose that instead of taking mass (*m*), length (*l*) and time (*t*) as our fundamental dimensions, we were to adopt new ones based on : velocity (*v*), force (*f*) and energy (*e*). What would be the dimensions of mass length and time in terms of new dimensions?

2. A 1 km object has a velocity of 100 m s^{-1}, and a 1 tonne object has a velocity of 0.1 m $s^{\neq 1}$. Which object has:

   (a) the larger momentum?
   (b) the larger kinetic energy?
   (c) the larger total energy?

3. A 2 kg block of wood rests upon a frictionless horizontal surface. A 10 gm bullet, moving horizontally at 1000 m s^{-2}, embeds itself in the wooden block. With what velocity does the block begin to move after being struck by the bullet?

4. An astronaut has a mass of 90 kg when measured on earth. If the acceleration due to gravity on earth takes the value g_E = 9.81 m s^{-2}, and its value on the moon is g_M = 1.62 m s^{-2},

   (a) What is the weight of the astronaut on earth?
   (b) What is his mass on the moon?
   (c) What is his weight on the moon?

5. You are an astronaut, having a race with another astronaut. Your two space ships are travelling parallel to each other in space and both have the same speed of 1000 kph relative to the earth. Each of the space ships has a total mass (i.e. including crew and all on-board equipment) of 5 tonnes.

   You look sideways out of your window and see that the astronaut in the other space ship has a nasty smile on his face and is holding a 1 tonne object in his hand. He opens his window, slowly moves the heavy object outside, holds it between his ship and yours, and releases it!

   (a) Does the object appear to move forwards, backwards, or stand still ?
   (b) Why ?
   (c) Does the other space ship appear to move faster, slower or remain at the same speed?
   (d) Why?

   This gives you an idea! You now take hold of a 1 tonne object, open your window, place the object outside, but project it backwards at a speed of 10 m s^{-1} relative to your space ship.

   (e) Does the other space ship appear to move backwards, forwards, or remain stationary relative to your ship?
   (f) Why?
   (g) How does the 1 tonne weight that was ejected by the other astronaut now appear to move?
   (h) Why?
   (i) Calculate the values of the various speeds you have concluded.