Environmental Physics for Freshman Geography Students

Professor David Faiman. Lecture 10, v.1.4 (January 13, 2004)

1. Liquids

In the previous lecture we saw how the addition of heat energy to a solid may cause its molecules to vibrate with such large amplitudes that the solid loses the geometrical shape that was imposed by the inter-molecular forces. In other words: the solid becomes a liquid.

The temperature at which a solid turns into a liquid depends on the kind of atoms it is made of and the strength of the forces that bind them together. At room temperature, lead (Pb) and gold (Au) are solids but bromine (Br) and mercury (Hg) are liquids. Lead can be easily melted by heating it to 327 °C. Gold, on the other hand, will not melt unless its temperature is raised above 1,064 °C. Rock in the earth's interior is so hot that it is permanently in liquid form. Occasionally this liquid escapes to the surface through fissures in the earth's crust and we see it in this form as a volcanic eruption. As the resultant lava cools off it solidifies again.

One can actually see evidence for liquids being made up of atoms (or molecules) in constant motion. More than a century ago it was discovered that if pollen grains are suspended in water and viewed under a microscope they execute a rapid motion in randomly changing directions, known as Brownian motion (after the botanist Robert Brown who first reported the phenomenon in 1828). Albert Einstein's first famous scientific paper was a quantitative explanation of Brownian motion, which he proved, was caused by the continuous bombardment of the pollen grains by the vibrating molecules of the liquid. Einstein actually used the phenomenon of Brownian motion to demonstrate (for the first time ever) that liquids are made of atoms (or molecules).

Now in order to contain a liquid it is only necessary for the vessel to have a bottom and sides: Not a lid. This is because the upward motion of most of the molecules in a liquid can not overcome a force called *surface tension* - that apparent "film" that seems to hold the surface together. We can actually use our molecular picture to understand the origin of surface tension. Molecules in the bulk of the liquid are acted upon, equally, by attractive forces from molecules above and below. But molecules on the surface can only be attracted by molecules below them: they are consequently pulled by a net downward attractive force. Occasionally, of course, a surface molecule happens to be given enough of a push, by the random collisions among the underlying molecules, to overcome surface tension and cause it to escape. In this manner the liquid gradually evaporates.

In the 3rd Century BCE Archimedes discovered an important property of liquids that remains true to the present day. He observed that *a solid object sinks in water until the upward force it experiences is precisely equal to the weight of water displaced by it.* Let us look at **Fig. 1** to see what this means.

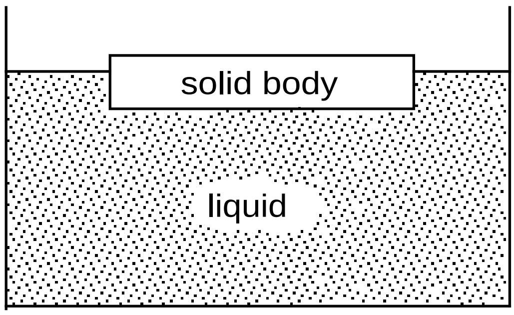

Figure 1: A solid body can float on the surface of a liquid

Fig. 1 shows a slab of solid material, e.g. wood, floating in a liquid, e.g. water. Suppose the contact area between the slab and the water is A and the body sinks to a depth d. Then the volume of water displaced must be $V = A \times d$. Now what has happened to this displaced water? Clearly it must have squeezed upward around the solid body. It would "like" to go back to its original level but it can not do so because the wooden slab is keeping it out. What is the force with which this displaced water is pushing down in order to try and return? Clearly, with a force equal to its weight = $A \times d \times \rho \times g$, where ρ is its density and g is the acceleration due to gravity. But this force must be precisely canceled by the weight of the slab = $M \times g$. This then is the meaning of *Archimedes' Principle*.

Now Archimedes' Principle illustrates another interesting and important property of fluids: The weight of the displaced water acts downward but pushes the slab upward. How is that possible?

With fluids it is convenient to talk about *pressure*, which is *force per unit area*. The weight Mg of the slab acts on an area A thus exerting a pressure Mg/A on the water. This is the pressure with which the water - which has been squeezed up and around the slab - must push back in order to hold the slab in a stable position.

This shows us that unlike force, which acts in a definite direction (force is a vector quantity), the pressure in a liquid acts uniformly in all directions. Again, from the atomic viewpoint, this is sensible because the pressure is caused by colliding molecules moving very rapidly in random directions.

2. Melting icebergs.

Let us now use these ideas in order to calculate something that the newspapers tell us has profound consequences for the “environment”. I refer to the effect on sea level that may be caused if all the icebergs melt. We may simplify the calculation by making an iceberg of cylindrical shape, 10 cm long and 1 cm^2 in cross-sectional area. It thus has a volume of 10 cm^3. Let us place this iceberg in a glass of water at 0^oC (so that it will not melt until we are ready).

Now the density of ice is 0.92 gm cm^{-3}. Our iceberg accordingly has a mass of 9.2 gm. However, according to Archimedes’ Principle, the iceberg will sink until it has displaced 9.2 gm of water - which, being water ($\rho = 1$), occupies precisely 9.2 cm^3 of volume. If we mark the water level on the side of the glass, *after* we have floated our mini iceberg, our apparatus may be regarded as representing a simulation of the present level of the oceans with all of their floating icebergs. We are now ready to perform a global warming experiment.

We have a 9.2 gm iceberg, which has displaced 9.2 cm^3 of water. If we raise the water temperature in the glass, the iceberg will melt. How much volume does a 9.2 gm fully melted iceberg occupy? The same volume as 9.2 gm of water, i.e. 9.2 cm^3. But this is *precisely* the volume in the glass that was previously occupied by the submerged part of the solid iceberg.

We conclude, therefore, that if global warming causes all the icebergs to melt there will be no change in sea level!

That is the good news. The bad news is that the Antarctic contains vast quantities of ice that are not floating: this ice is attached to solid ground. If it melts and pours into the sea, it will certainly cause sea level to rise.

3. Tides

Before leaving the topic of liquids I’d like to discuss the phenomenon of tides and why there are two of them every 24 hours. In ancient times it was realized that the moon was related to the phenomenon of tides but it was not until people (a) realized that the world was round and (b) knew about gravitation, that they were able to address the question of how many high tides there should be in 24 hours - one or two. After all, as the earth rotates on its axis once every 24 hours, the moon should cause an attractive force on the ocean, and we might expect there to be one high tide each day. But there are two! How come?

There was much confusion on this issue until Newton showed how to perform the calculation correctly. The problem was: Given that the moon attracts the sea, why is there also a high tide on the *other side* of the world?

Newton's law of gravitation states that two masses attract each other with a certain force that we discussed in a previous lecture. However, if - as is the case for the earth and the moon - the two masses also rotate around each other, there will also be a centrifugal force trying to separate them. In equilibrium, both bodies will revolve around their mutual center of gravity in such a manner that the gravitational attraction exactly balances the centrifugal repulsion. Let us look at a schematic diagram of the earth-moon system, in which the ocean is included.

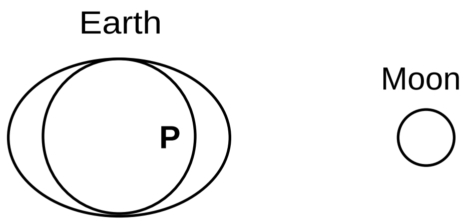

Figure 2: The earth and moon revolve around their mutual center-of-gravity at point **P**. The gravitational force of the moon is greater to the right of **P** and causes a high tide there. The centrifugal force is greater to the left of **P** and causes a high tide there

In **Fig. 2**, the point **P** marks the center of gravity of the revolving earth-moon system. Point **P** is the center around which the entire system revolves and at which the gravitational force precisely balances the centrifugal force. Now the ocean on the side of the earth nearest the moon will, on average, "feel" the gravitational attraction of the moon more strongly than the centrifugal force of rotation - because it is nearer to the moon than the other water. There will therefore be a high tide on the side of the earth facing the moon. This was obvious even without Newton's help! Around the other side of the earth, however, the situation is the opposite. The water here will, on average, feel the centrifugal force more strongly than the gravitational force of the moon. There is accordingly a high tide around the back of the earth, caused, not by an excessive gravitational force but by an excessive centrifugal force. This is what Newton explained.

4. Gases

If we add yet more kinetic energy to the molecules of a liquid the surface tension film breaks apart and the molecules all escape. Boiling has occurred and we shall lose the material completely unless it is contained within a suitable vessel.

A suitable vessel for containment might be a gas jar with a lid. But we can also use the force of gravity (as we shall show later in a more quantitative fashion). If a planet is not very massive then any atmosphere it might have had when it was formed will rapidly have evaporated into space. However, if the planet is massive enough (e.g. the earth) then the force of its gravity is sufficient to trap the atmosphere and stop all but the lightest gas molecules from escaping.

Now, just as some materials can be solid and others liquid at room temperatures, some can be gases. Thus, oxygen, nitrogen and carbon dioxide are gases, water and oil are liquids and coal, iron and silicon are solids.

Quantitative experiments performed on gases, some 300 years ago, revealed a number of interesting regularities between the temperature, pressure and volume of a gas.

First, experiments were performed on gases held at *fixed temperature.* It was discovered that if the pressure is increased the volume decreases and vice-versa. This is almost obvious but what was remarkable about the measurements was that the product PV remained constant. (This is known as *Boyle's Law*).

Another interesting result was that if the *pressure is kept constant* then heating causes the temperature and volume to increase in simple proportion to one another. provided that temperature is measured on the *absolute temperature scale*, i.e. in degrees Kelvin. (This is known as *Charles' Law*). For example, increasing the temperature by any given factor would cause the volume to increase by that same factor.

Finally, if the *volume* of gas in an enclosed container was *kept constant*, it was discovered that the addition of heat to the container caused the temperature and pressure to rise in simple proportion to one another - again, provided that the Kelvin scale of measurement is used for temperature. (This law was discovered by *Gay-Lussac*).

The results of these three sets of experimental observations have been combined into what is known as the *Ideal gas law*:

$$PV = RT \qquad (10.1)$$

where P is the gas pressure measured in N m^{-2}, V is volume measured in m^3, T is temperature measured in K (Remember: degrees Kelvin = degrees Celsius + 273.15) and R is a constant, called the *ideal gas constant*, which takes the value 8.31 J K^{-1}.

To see that eq. (10.1) gives us each of the proceeding three laws it is sufficient to hold one variable fixed and see how the remaining two depend upon one another:

First, by holding the temperature fixed at some value T_o, eq.(10.1) gives:

$$P\,V = (R\,T_o) \qquad (10.2)$$

If V_1 and V_2 are the volumes occupied by a gas at pressures P_1 and P_2, respectively, then, from eq.(10.2) we have:

$$P_1V_1 = (R\,T_o) \qquad (10.3)$$

and

$$P_2V_2 = (R\,T_o) \qquad (10.4)$$

Equating eq.(10.3) and eq.(10.4) then gives us:

$$P_1V_1 = P_2V_2 \qquad (10.5)$$

which is the usual form of Boyle's Law.

On the other hand, if we now hold the pressure constant at some value P_o then eq.(10.1) gives:

$$V = (R/P_o)\,T \qquad (10.6)$$

Again, by considering two states of a gas, in which V_1 and V_2 are the volumes it occupies at temperatures T_1 and T_2, respectively, we obtain:

$$V_1 / V_2 = T_1 / T_2 \qquad (10.7)$$

which is the usual form of Charles' Law.

Finally, if we hold the volume fixed at some value V_o then eq.(10.1) gives:

$$P = (R/V_o)\,T \qquad (10.8)$$

from which we may derive

$$P_1 / P_2 = T_1 / T_2 \qquad (10.9)$$

which is the usual form of Gay-Lussac's Law.

The ideal gas law (also Charles' Law and Gay-Lussac's Law) is actually only an approximate law of nature. The reason for this is that if we could reduce the temperature while holding the pressure fixed then the volume should decrease to zero. But this is impossible since all the molecules would need to be compressed to a mathematical point. This is the sense in which the gas laws (10.1) - (10.9) are said to be *ideal*. But they work remarkably accurately provided that the gas density is sufficiently large that there is no danger of the molecules getting into permanent contact with one another. But this is a reasonable requirement because, as we have seen, long before this happens our gas would have become a solid.

Although the ideal gas law was discovered empirically it has to do with the collisions among gas molecules. We should therefore not be surprised to discover that eq.(10.1) contains important information about the molecules themselves. For this reason it often re-written in the form

$$PV = NkT \qquad (10.10)$$

where, N is *Avogadro's Number* ($N = 6.02 \times 10^{23}$ mole^{-1}) and k is *Boltzmann's constant* ($k = 1.38 \times 10^{-23}$ J $^{o}K^{-1}$).

Boltzmann's constant is actually a scaled-down form of the ideal gas constant R = 8.31 J $^{o}K^{-1}$ in eq.(10.1). The scale factor - Avogadro's Number - represents the number of molecules contained in a so-called *mole* of any substance. A mole is the gram molecular mass of a substance, i.e. the number of grams of a substance that are numerically equal to its molecular weight. For example, a mole of solid carbon (C) weighs 12 gm because the molecular weight of carbon is 12: it contains $N = 6.02 \times 10^{23}$ carbon atoms. Similarly a mole of liquid water (H_2O) weighs 18 gm (because the atomic weight of oxygen is 16 and that of hydrogen is 1): it contains $N = 6.02 \times 10^{23}$ water molecules. Similarly, a mole of oxygen *gas* (O_2) weighs 32 gm: it contains $N = 6.02 \times 10^{23}$ oxygen molecules. Note that a mole of oxygen *atoms* would weigh 16 gm, but the atoms would quickly combine in pairs leaving us with only half-a-mole of stable oxygen gas.

But you are probably wondering what all this has to do with an ideal gas, for which no specific atoms have been mentioned. Well, the answer is that (3/2) kT turns out to be the average kinetic energy of a gas molecule at temperature T, irrespective of what kind of molecule it is. Hence, although we originally defined it in terms of an ideal gas, the right hand side of eq.(10.10) actually represents 2/3 of the kinetic energy of all N gas molecules in our volume V - whatever kind they may be. In this sense we see that temperature is proportional to the kinetic energy of the molecules in a gas.

Problem set 10 (liquids and gases)

1. The element mercury is a liquid at room temperature, with density = 13.55 gm cm^{-3}. A copper disk with density = 8.9 gm cm^{-3}, diameter = 2 cm and height = 1 cm is placed on the surface of a pool of mercury. To what depth will the copper disk sink? Draw a sketch diagram with all relevant dimensions labeled.

2. An iceberg is made of pure water ice (density = 0.92 gm cm^{-3}). It is floating in salty sea water (density = 1.03 gm cm^{-3}). If the iceberg melts, will the sea level rise or fall?

3. If the moon had the same mass as the earth, would there be 2 tides or 1 every 24 hours? Explain.

4. If dry air consists of 78% nitrogen, 21% oxygen and 1% argon, what is the mass of 1 mole of air? How many molecules of each type does it contain?