**Environmental Physics for Freshman Geography Students**

**Professor David Faiman. Lecture 1, v.3.4 (October 28, 2003)**

## 1. From Atoms to Planets: Scales of Size

Imagine a large sheet of millimeter graph paper, of size 1 m x 1 m, glued to the wall. Stare at it and, if you have reasonably good vision, you will be able to see, at one time, a million little squares. One thousand in row No.1, one thousand in the next row – that's two thousand, so far. Then, one thousand more little squares in each of the other rows, until we reach the last thousand squares in row No.1000. A thousand thousands = one million little squares!

This may well be the largest single number that it is possible to see at a single glance. Why? Because although a larger sheet of graph paper - say, 3 m x 3 m - would contain more squares (How many?), one would have to step back in order to see the entire sheet in focus but, in so doing, one would probably not be able to resolve the individual 1 mm squares.

But let's go back to our 1 m x 1 m sheet, and cut it into 1000 strips, each 1 mm wide and 1 m long. If we place these 1000 strips end to end we will have a strip 1 km in length.

We can thus visualize the relationship between 1 mm and 1 km: There are 1,000,000 mm in 1 km, therefore, 1 km is 1,000,000 times larger than 1 mm. It is convenient to write the number 1,000,000 as $10^6$, where the superscript "6" indicates the number of times that 10 has been multiplied by itself. (Alternatively, it represents the number of zeros after the 1). We can also see that 1 mm is 1,000,000 times *smaller* than 1 km, or, 1/1000,000 times as large as 1 km. We may write the fraction 1/1,000,000 as $1/10^6 = 10^{-6}$, where the superscript "-6" indicates how many zeros there are in the denominator.

If we wanted to measure the world in millimeters we would have to refer to 1 m as $10^3$ mm and 1 km would be $10^6$ mm. This is clearly inconvenient and it is the reason why different scales of length have been invented (i.e., purely out of convenience) for measuring different scales of distance. We use kilometers to measure the distance between cities, meters to measure the size of buildings and millimeters to measure small, visible, objects in a machine shop. Of course, astronomers and atomic scientists naturally find other scales of length more convenient for their work. Let's see what kinds of distance they need to measure.

Since we now feel perfectly comfortable with the concept of one million or $10^6$ (i.e., 1 km is simply a million times larger than 1 mm, or 1 km = $10^6$ mm), let us apply it to the kilometer, used as a starting point and increase this unit by a factor of one million.

**Q**. What use do you think might be made of a length scale of $10^6$ km, or 1 *Giga*meter = 1 Gm?

**A**. The mean distance between the earth and the moon is 0.3844 Gm. So the Gm could be a convenient scale for measuring the distance between planets and their satellites.

We could also increase the Gigameter by another factor of a million. This would give us one *Peta*meter, or 1 Pm = $10^6$ Gm = $10^{12}$ km = $10^{18}$ mm. The Petameter is a very large distance indeed. It is approximately 1/10 of a *light-year*, i.e. the distance that a beam of light would travel in approximately 40 days.

Now let us go in the opposite direction.

**Q**. What is one million times smaller than 1 mm, or $10^{-6}$ mm?

**A**. If we could see such small distances with our eyes we would be able to see the individual atoms laid out in regular arrays within all kinds of crystals. For this length scale, also called 1 *nano*meter or 1 nm, typifies the distances between atoms. For example, in table salt, the atoms of Na and Cl are arranged in a series of perfect interlocking cubes - just like the crystals themselves - the distance between adjacent atoms of the same kind being 0.563 nm. Today, so-called "nano-technology" is a buzzword. It refers to engineering devices at the scale of the interatomic distances in crystals.

But we needn't stop there. If we decrease the nanometer by another factor of a million we arrive at the *femto*meter. 1 fm = $10^{-6}$ nm = $10^{-12}$ mm. This is approximately the size of the protons and neutrons that reside in the nuclei of atoms.

Summary

We see that "one million", which is the ratio between the two familiar length scales of 1 mm and 1 km, is also the ratio of 1 km to the start of astronomical distances and, conversely, the ratio of inter-atomic distances to 1 mm. The good news is that, in this course, we shall not require distance scales smaller than nanometers, or larger than Gigameters!

**2. The role of atoms in a geography course?**

The purpose of this course is to introduce you to the physics of the environment. Now, consider an automobile. Anyone can learn to drive and enjoy the benefits of an automobile without having any idea how one works. In fact, this is precisely what most of us do! On the other hand, a garage mechanic does understand how an automobile works because he understands the function of the parts out of which it is built. Well, we have a similar situation in geography. We can all enjoy mountains, lakes, fresh air, etc., without having any idea "how they work", but by understanding something about the atoms out of which they are made we shall come to appreciate the physics of the environment in which we live.

In each lecture I shall, therefore, endeavor to pick at least one geographical phenomenon and relate it to what the underlying atoms are doing.

Now usually this would require a considerable mastery of mathematics, which few of you are likely to have. So I shall simplify matters by developing any necessary mathematical techniques as we go along.

**3. Pythagoras' theorem and square roots**

You are all familiar with the famous theorem about right-angled triangles: "The square on the hypotenuse equals the sum of the squares on the other two sides". Or, in symbols:

$$A^2 + B^2 = C^2 \qquad \textbf{(1.1)}$$

For example, if A = 3 and B = 4, then $A^2 + B^2 = 9 + 16 = 25$ and hence, C = 5. Such a triangle obeys Pythagoras' theorem and is consequently a right-angle triangle.

But the sides do not have to be expressible as simple integers in order for them to obey the theorem. For example, if A = 2 and B = 3 and the angle between them is $90^o$ then, from Pythagoras' theorem, $C^2 = 4 + 9 = 13$ and consequently $C = \sqrt{13}$.

In order to convert this to a usable number you must use your calculators. The result is then 3.6 or 3.605 or 3.60555, etc., depending upon how accurately you need to know the answer.

But the rules of mathematics sometimes enable us to evaluate even quite complicated numbers rather simply. In particular, Pythagoras' theorem presents us with a method for evaluating certain square roots without the need for a calculator – specifically, ones in which B is small compared with A. For such situations the theorem becomes:

$\sqrt{(A^2 + B^2)} \approx A + B^2/2A$ (**1.2**)

where the wiggly symbol means, "approximately equal to".

This expression is relatively easy to evaluate without a calculator, but it only works when B is "small". How small is small? We shall need to examine a few cases in order to find out.

*First case*: B = 0.5, A = 1.
Then C = $\sqrt{1.25}$
Our calculator tells us that C = 1.118, whereas eq. (**1.2**) gives 1.125 (but the error is < 1%)

*Second case*: B = 0.1, A = 1.
Then C = $\sqrt{1.01}$
Our calculator tells us that C = 1.0049875, whereas eq. (**1.2**) gives 1.005

*Third case*: B = 0.05, A = 1.
Then C = $\sqrt{1.0025}$
Our calculator tells us that C = 1.0012492, whereas eq. (**1.2**) gives 1.00125

*Fourth case*: B = 0.01, A = 1.
Then C = $\sqrt{1.0001}$
Our calculator tells us that C = 1.0000499, whereas eq. (**1.2**) gives 1.00005.

I have not gone any further because, at this stage, the approximate formula is now as accurate as the 8-digit calculator I am using!

I have drawn your attention to this particular mathematical trick because it turns out to be enormously useful in many areas of physics – including one we shall now examine.

## 4. Pythagoras' Theorem and Some Basic Physics

From time to time I will have to "throw" a physics formula at you, since this is a physics course and since there will not be enough time for me to derive the formula in a manner that would enable you to understand where it came from. However, whenever I do this, I shall try to explain the formula as well as I can.

One of the most remarkable physics formulae to have emerged in the 20th century has a mathematical form that is identical to Pythagoras' theorem:

$$m^2 + p^2 = E^2 \qquad (\mathbf{1.3})$$

This formula applies to any freely-moving particle, i.e. one that is not moving under the influence of any external forces. This formula relates the *energy* E of a particle to its *mass* m and its *momentum* p. (For the time being, let us not worry about what these italicized words mean. I'll get back to this later). Now the formula need not scare us because, mathematically, it is no more difficult than Pythagoras' theorem. But it contains a wealth of information about the physical world!

Basically, the formula in equation (**1.3**) makes 3 strong statements:

(a) If a particle is not moving it has no momentum (i.e. p = 0), then:

$$E^2 = m^2 \qquad (\mathbf{1.4})$$

(b) If a particle has no mass - amazingly, there are such particles (i.e. m = 0) - then:

$$E^2 = p^2 \quad \textbf{(1.5)}$$

(c) A particle can never have zero energy (otherwise $m^2 = - p^2$, which is a mathematical impossibility, since the square of any real number can not be negative).

Now what happens if we have a slowly moving particle? I.e. if p is very small and, hence, very much less than m. Then, from formula (**1.2**) we expect:

$$E = m + p^2/2m \quad \textbf{(1.6)}$$

Eq. (**1.6**) tells us that for a slowly moving particle the energy equals the sum of a term that depends on its momentum (we call this term "kinetic energy") and another term which is apparently simply the mass of the particle (high energy physicists call this term the “rest mass energy”). We shall look more closely at the meaning of energy, momentum and mass in the next lecture.

**Problem set No.1 (numbers, dimensions and units)**

1. A particular kind of solar collector is rectangular with dimensions 1 m x 2 m. What is the maximum number of solar collectors that could be laid out horizontally in a square field of dimensions 1 km x 1 km, without the collectors shading one another?

2. Suppose that a crystal has the shape of a cube with dimensions 1 cm x 1 cm x 1 cm. If the individual atoms from which the crystal is made are represented by hard spheres of diameter 0.1 nm, and they are packed tightly together so that they touch one another, how many atoms would the crystal contain?

3. Use Pythagoras’ theorem in order to estimate the value of $\sqrt{110}$ *without* using a calculator. What is the percentage error in your estimation? You may use a calculator for the second part.

4. In British units, 1 mile = 63,360 inches. 1 inch is defined as 25.4 mm exactly. There are precisely 3600 seconds in one hour. How large is the speed of 100 miles per hour (mph) in meters per second ($ms^{-1}$)?

5. The so-called “fine structure constant” is a dimensionless number defined as:
$\alpha = (e^2/4\pi\varepsilon_0)/(h/2\pi)/c$, where

$e = 1.602 \times 10^{-19}$ C is the amount of electric charge on an electron,
$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ kg^{-1}\ m^{-2}\ s^2$ is a certain electrical property of the vacuum,
$h = 6.626 \times 10^{-34}$ kg m $s^{-1}$ is a constant of nature known as the “Planck constant”_
$c = 2.998 \times 10^8$ m $s^{-1}$ is the speed of light in a vacuum,
$\pi = 3.142$ is the ratio of the circumference to the diameter of a circle.

Calculate the value of $\alpha$, and check that your result is dimensionless.