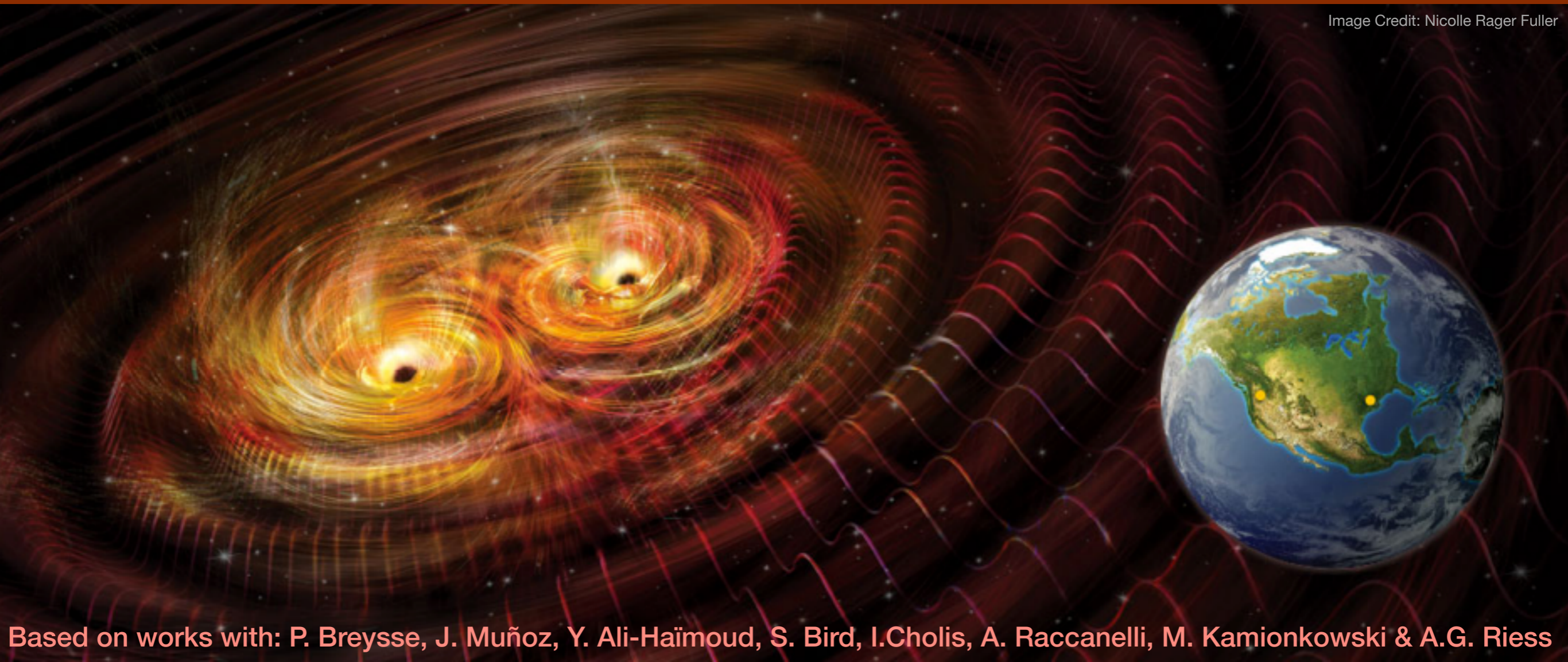


# Gravitational Waves from Merging Black Holes as a Probe of Astrophysics and Cosmology

**Ely D. Kovetz**

Johns Hopkins University

Image Credit: Nicolle Rager Fuller



Based on works with: P. Breysse, J. Muñoz, Y. Ali-Haïmoud, S. Bird, I. Cholis, A. Raccanelli, M. Kamionkowski & A.G. Riess

# Motivation: Detection of Gravitational Waves

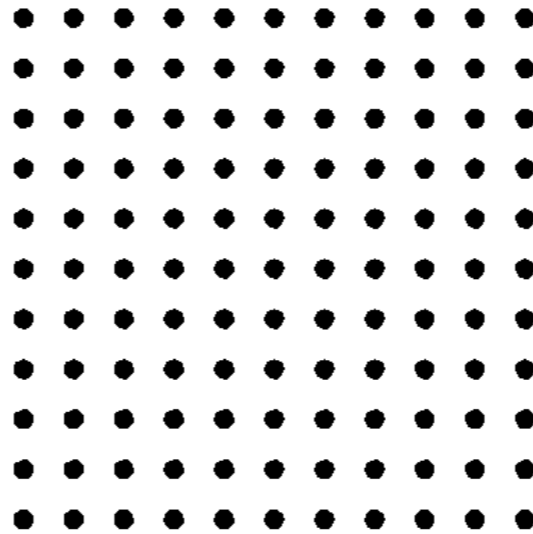
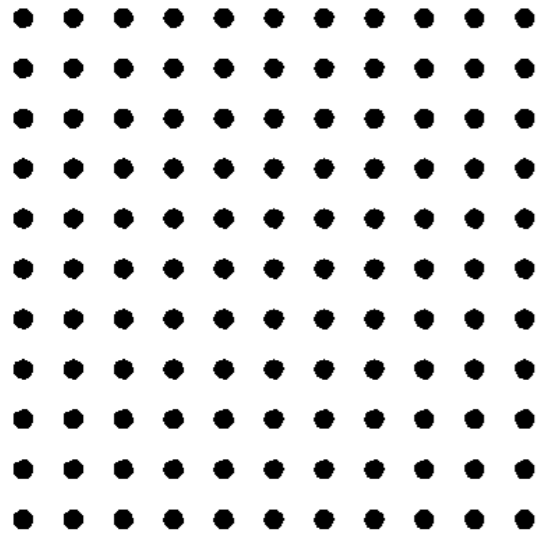
---

Credit: S. Larson

# Motivation: Detection of Gravitational Waves

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What do we measure?

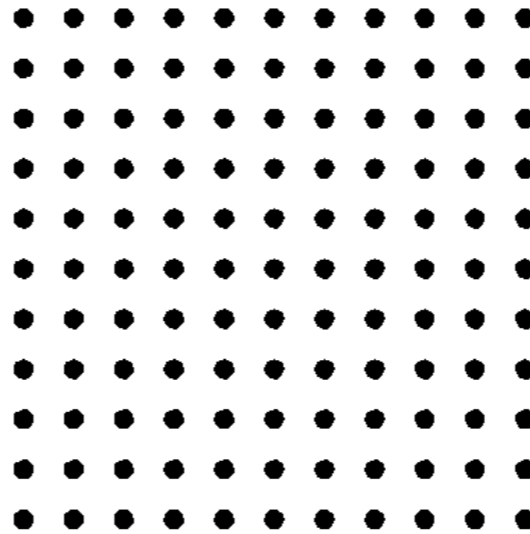
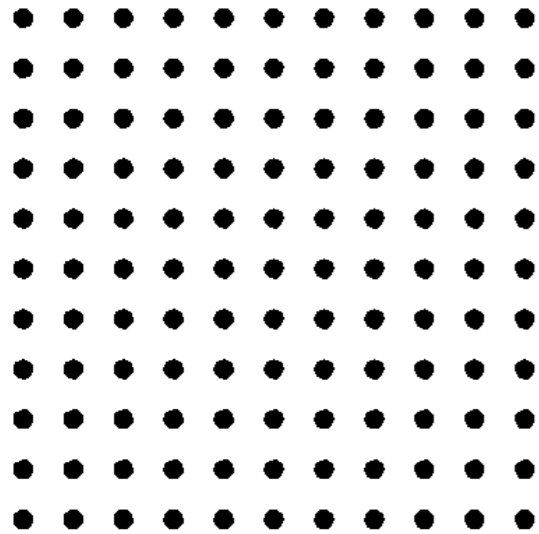


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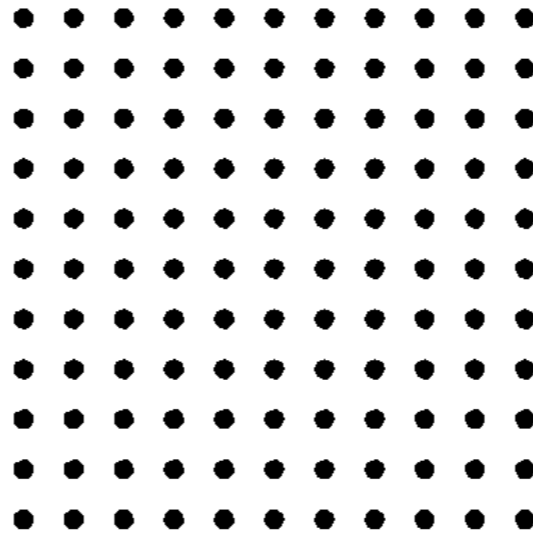
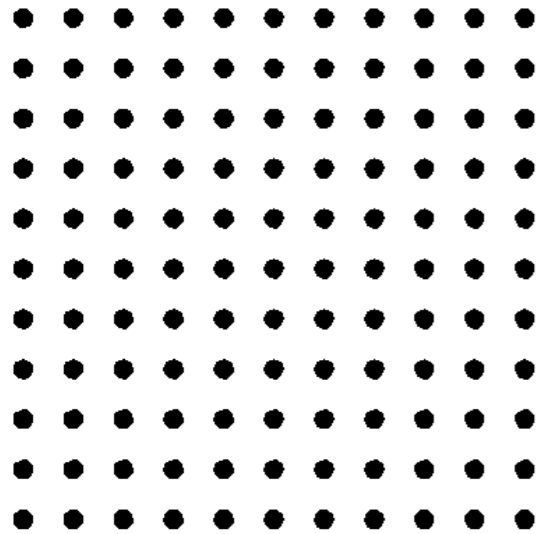


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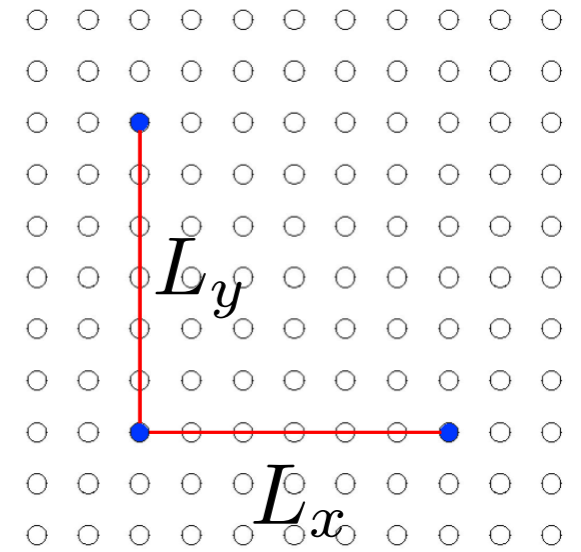
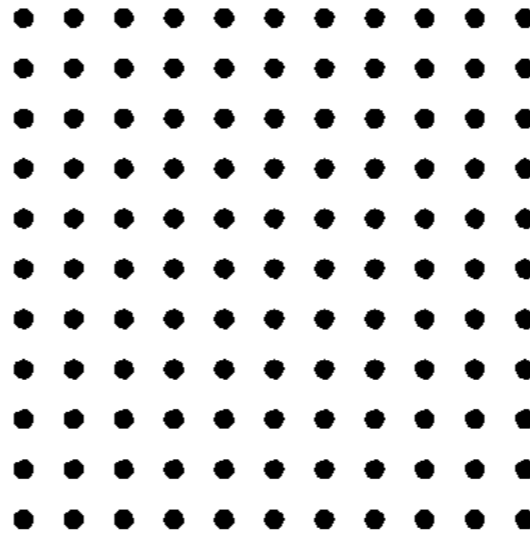
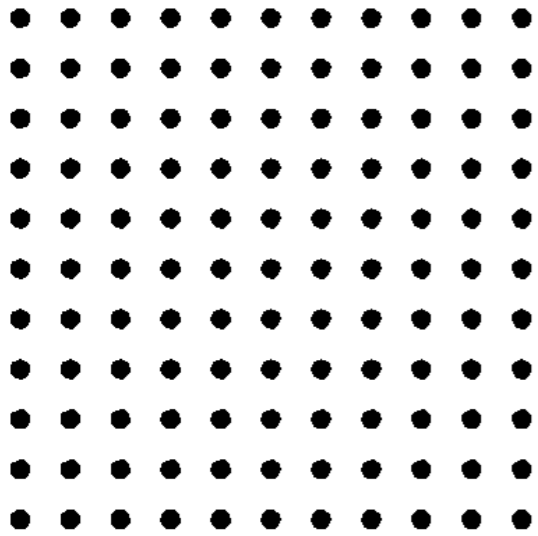


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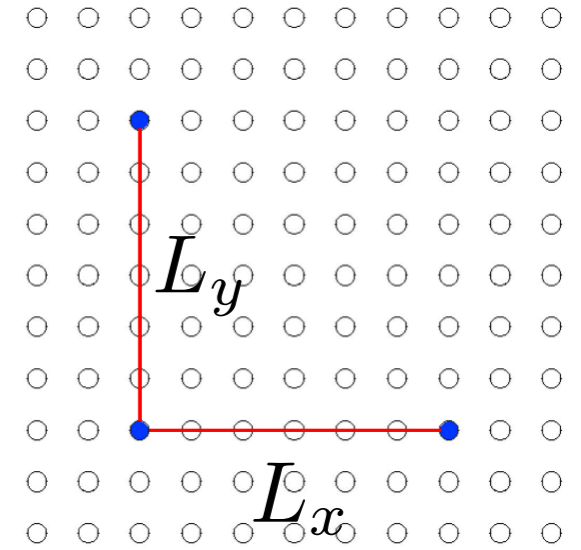
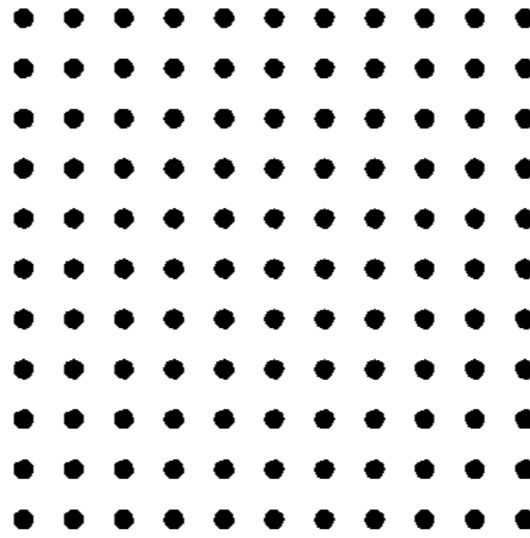
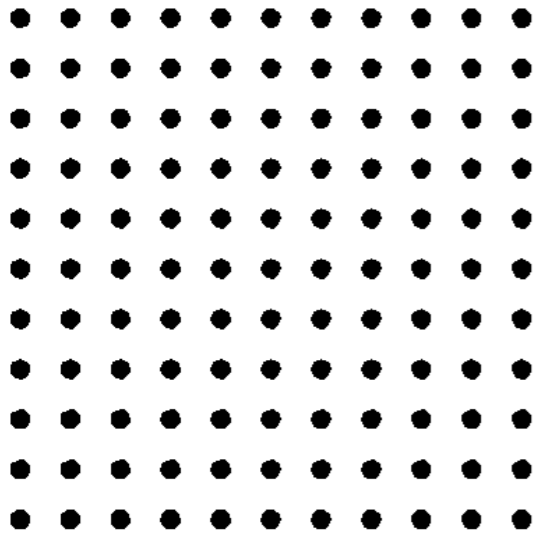


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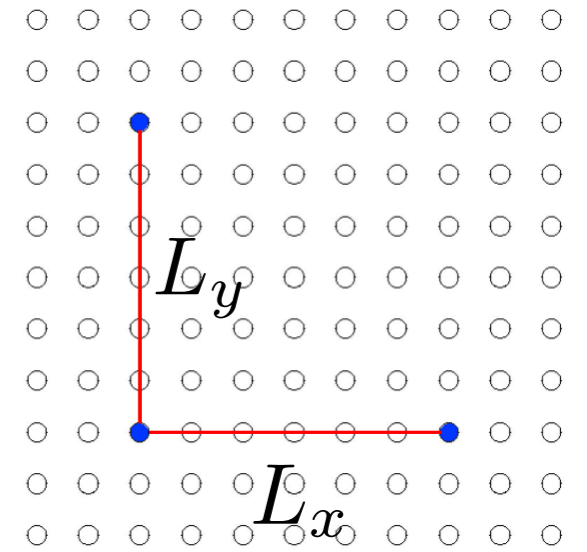
Credit: S. Larson

$$\text{Strain: } h(t) \equiv \Delta L/L = (\delta L_x - \delta L_y)/L$$

# Motivation: Detection of Gravitational Waves

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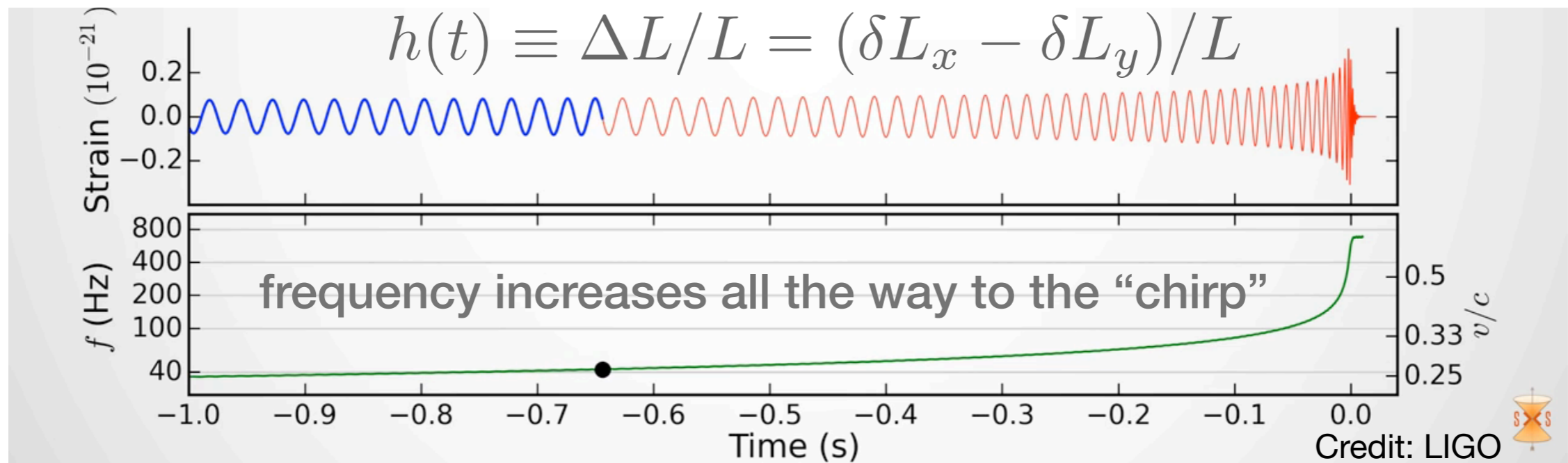
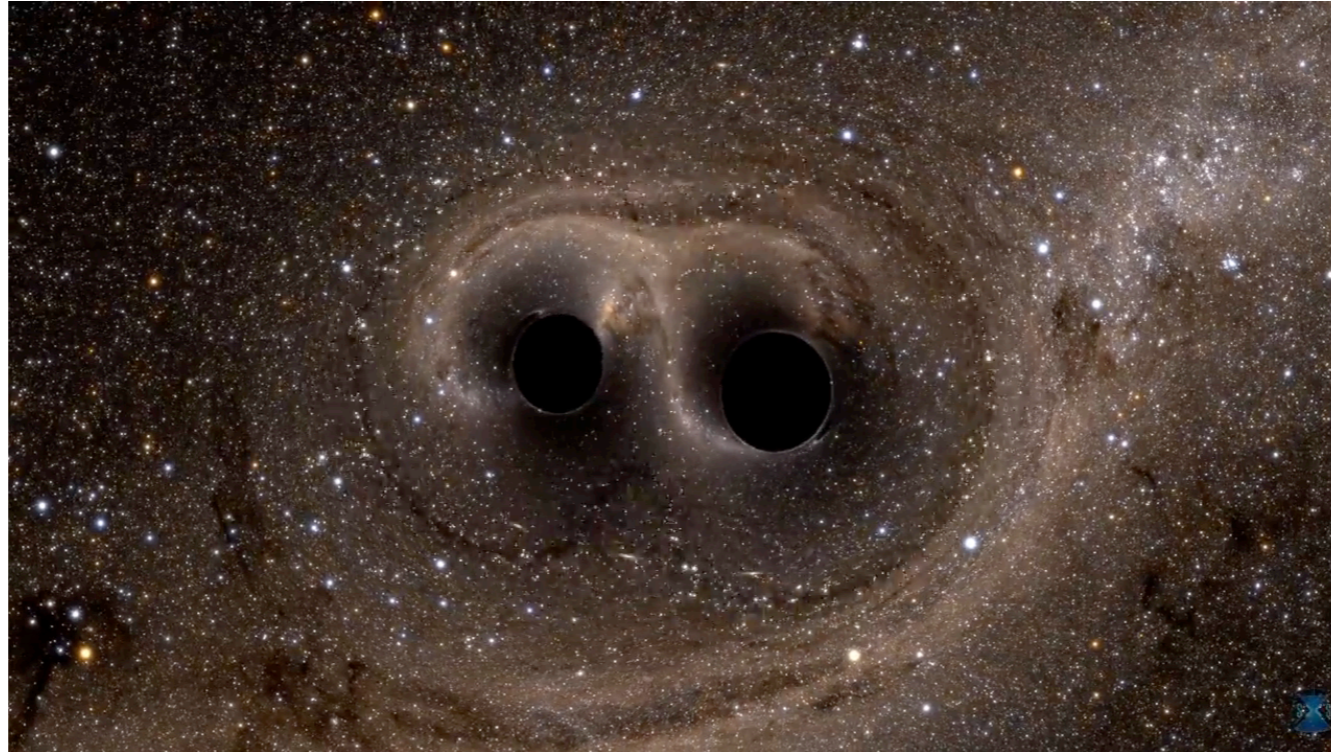
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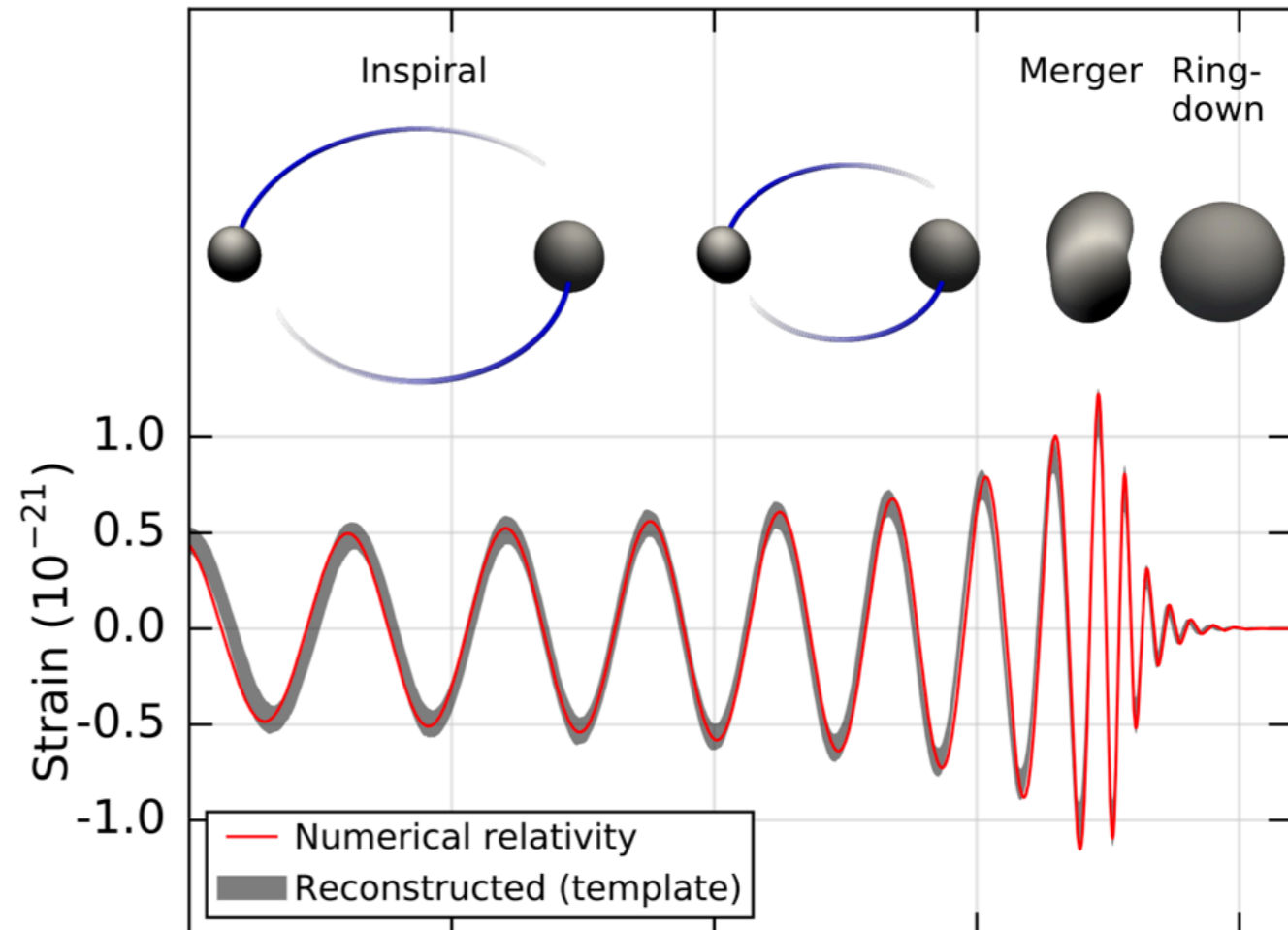


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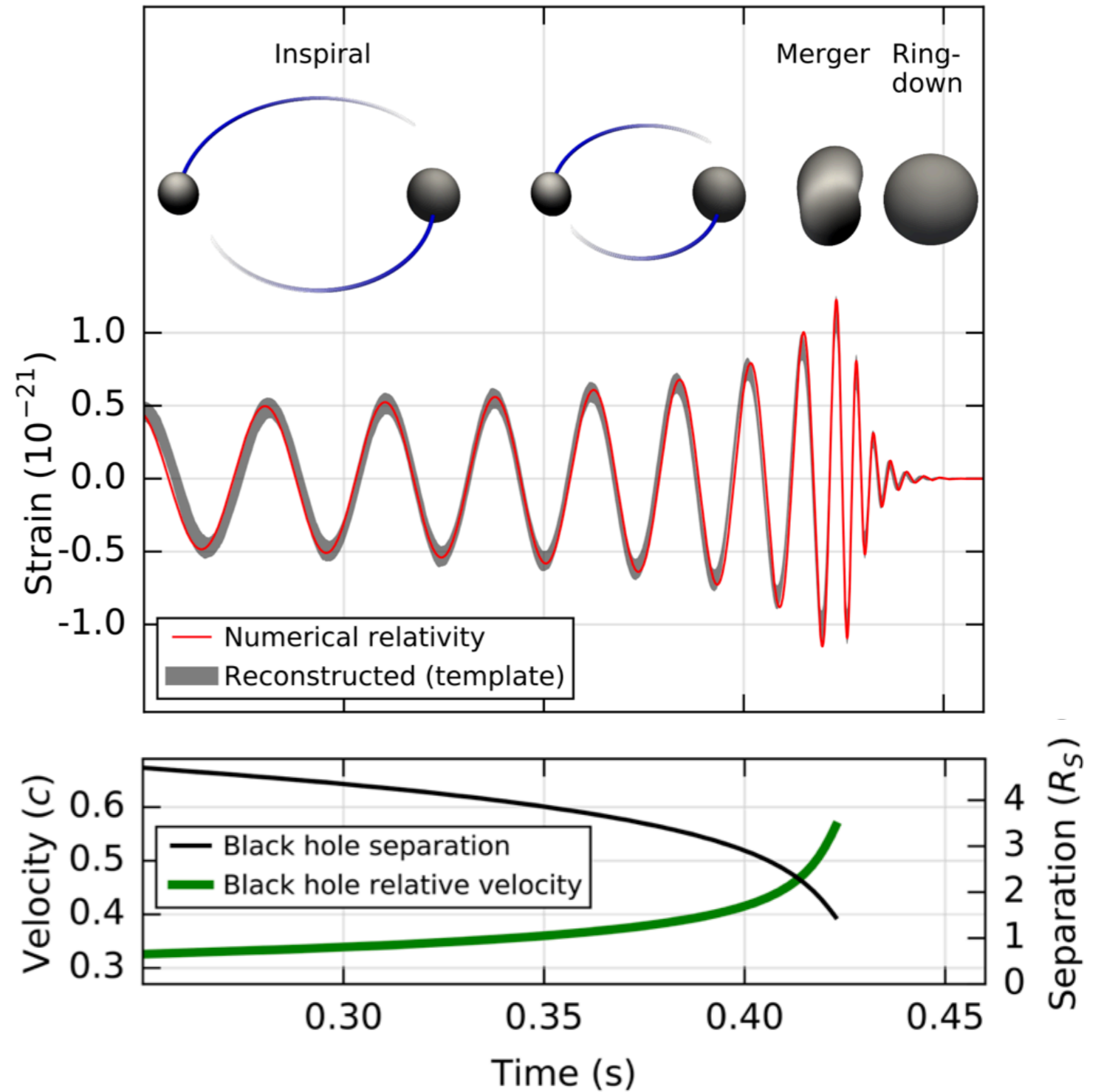
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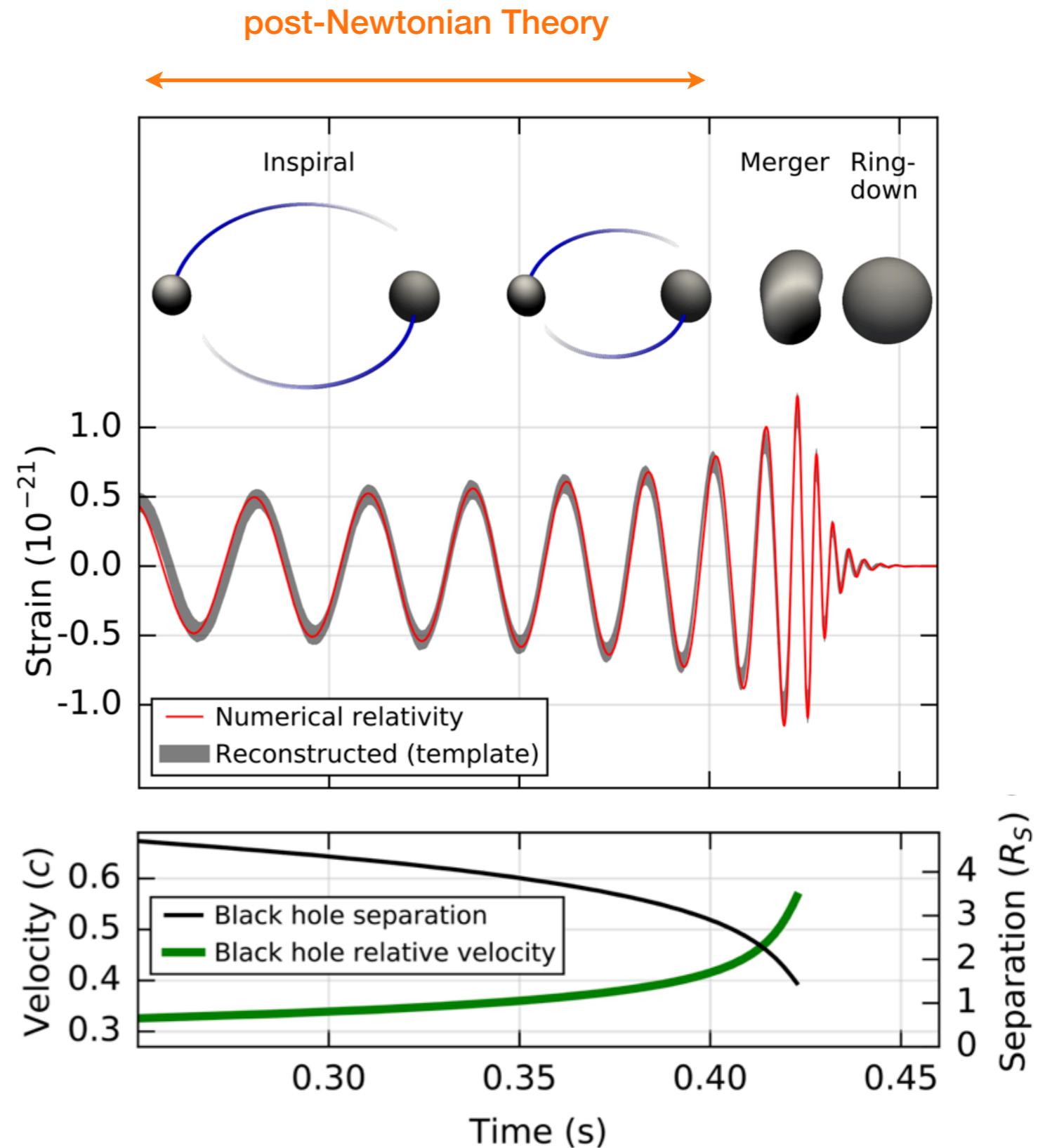
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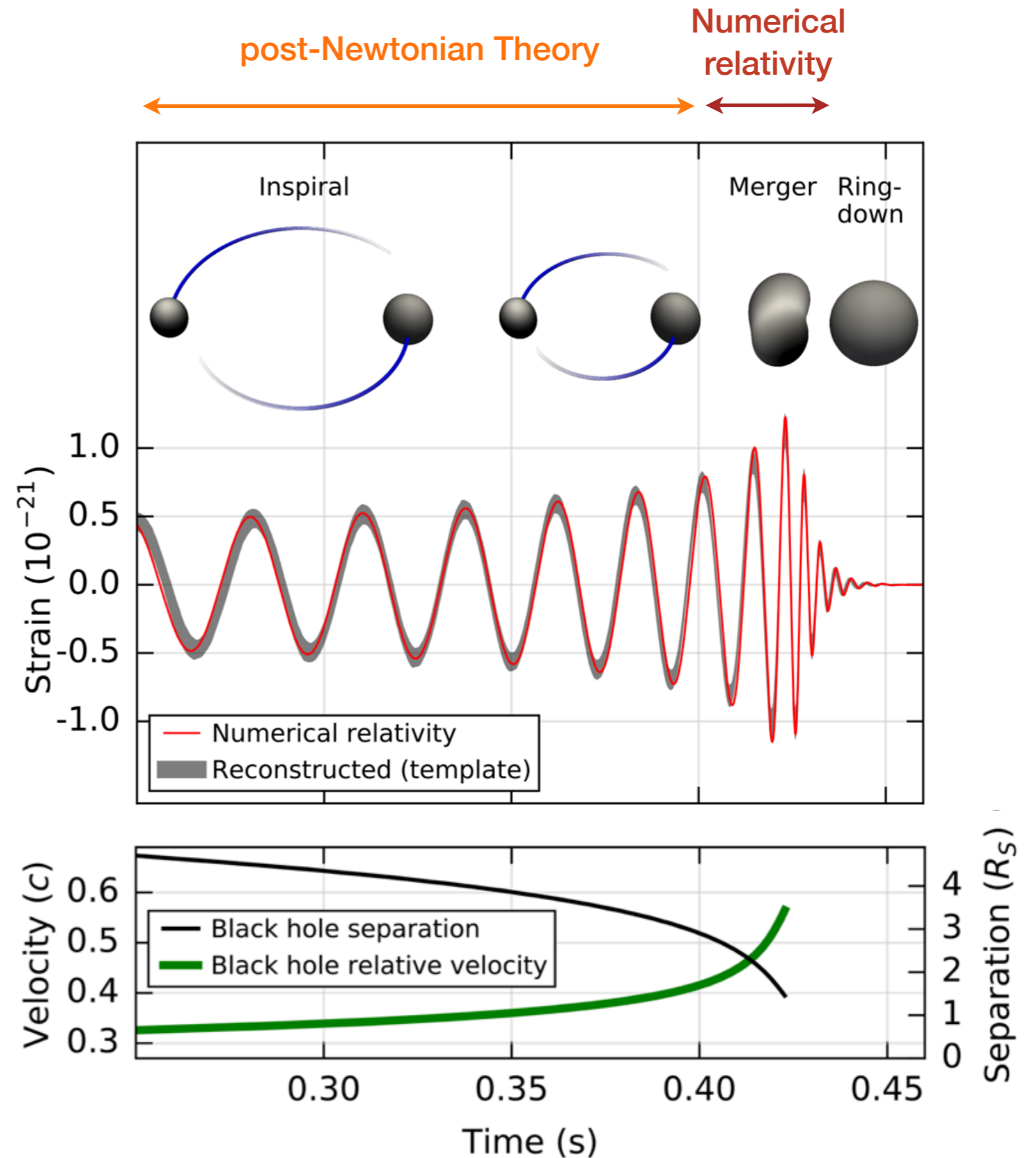
# Motivation: Detection of Gravitational Waves



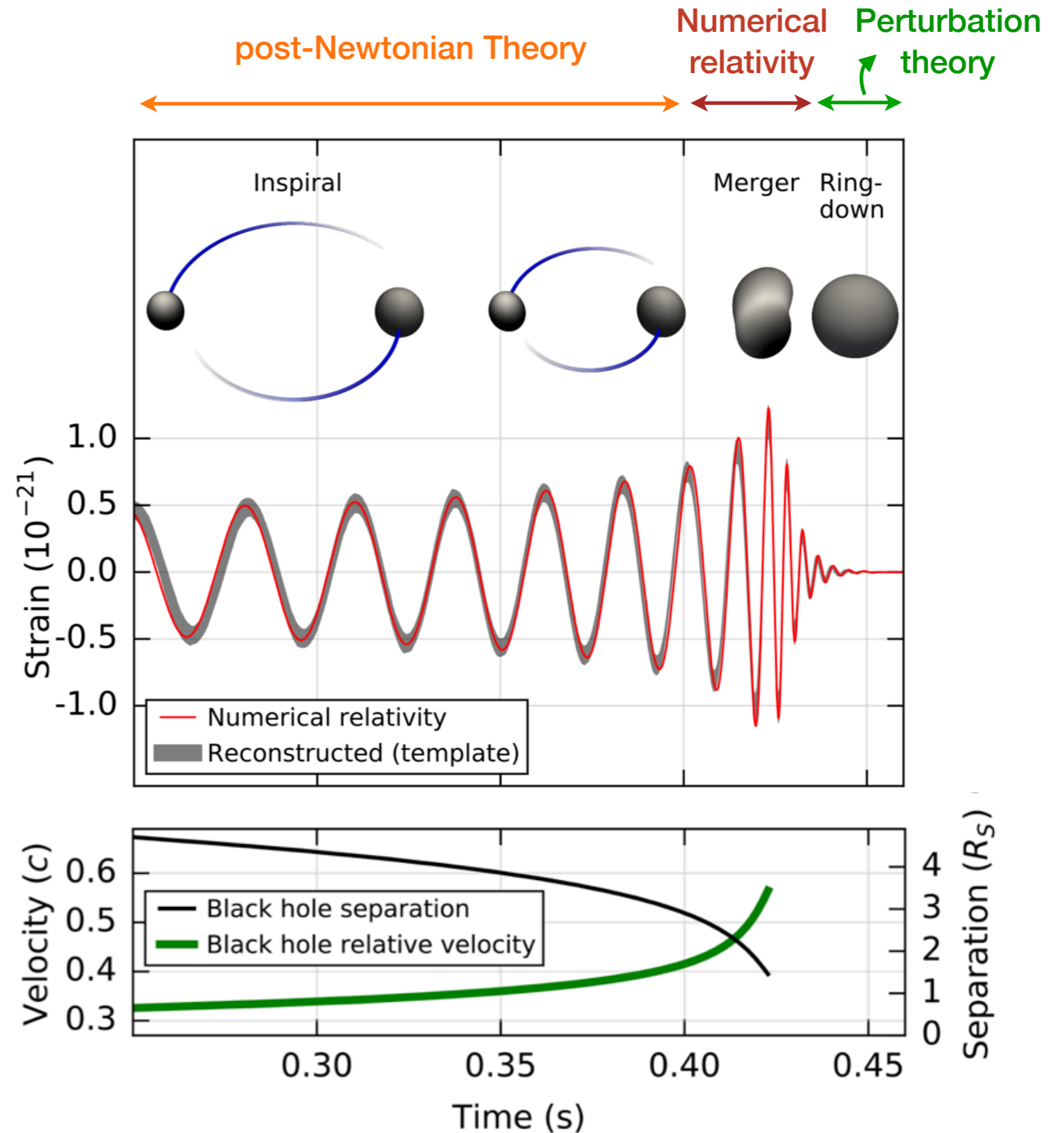
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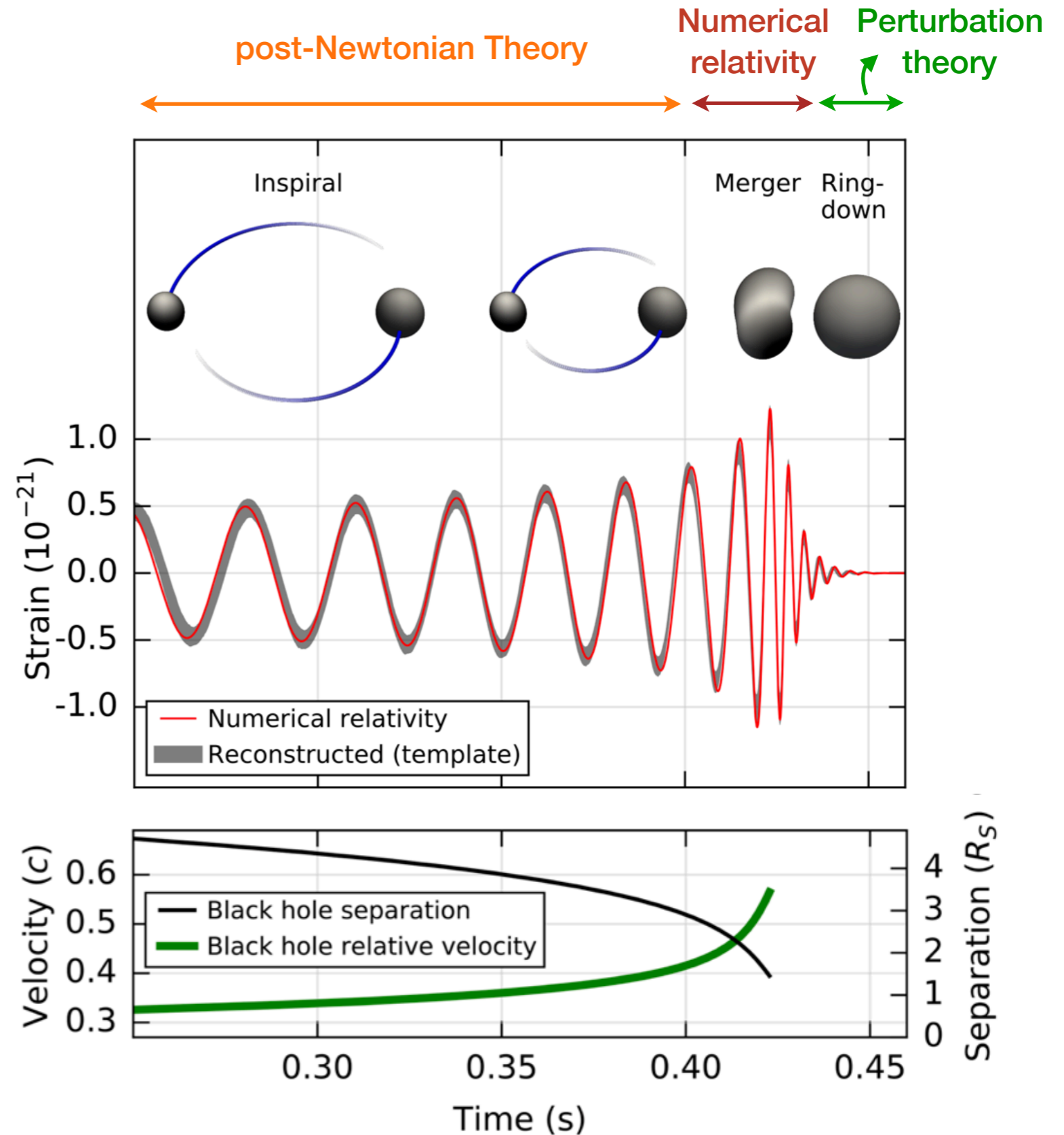


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Basic scalings:

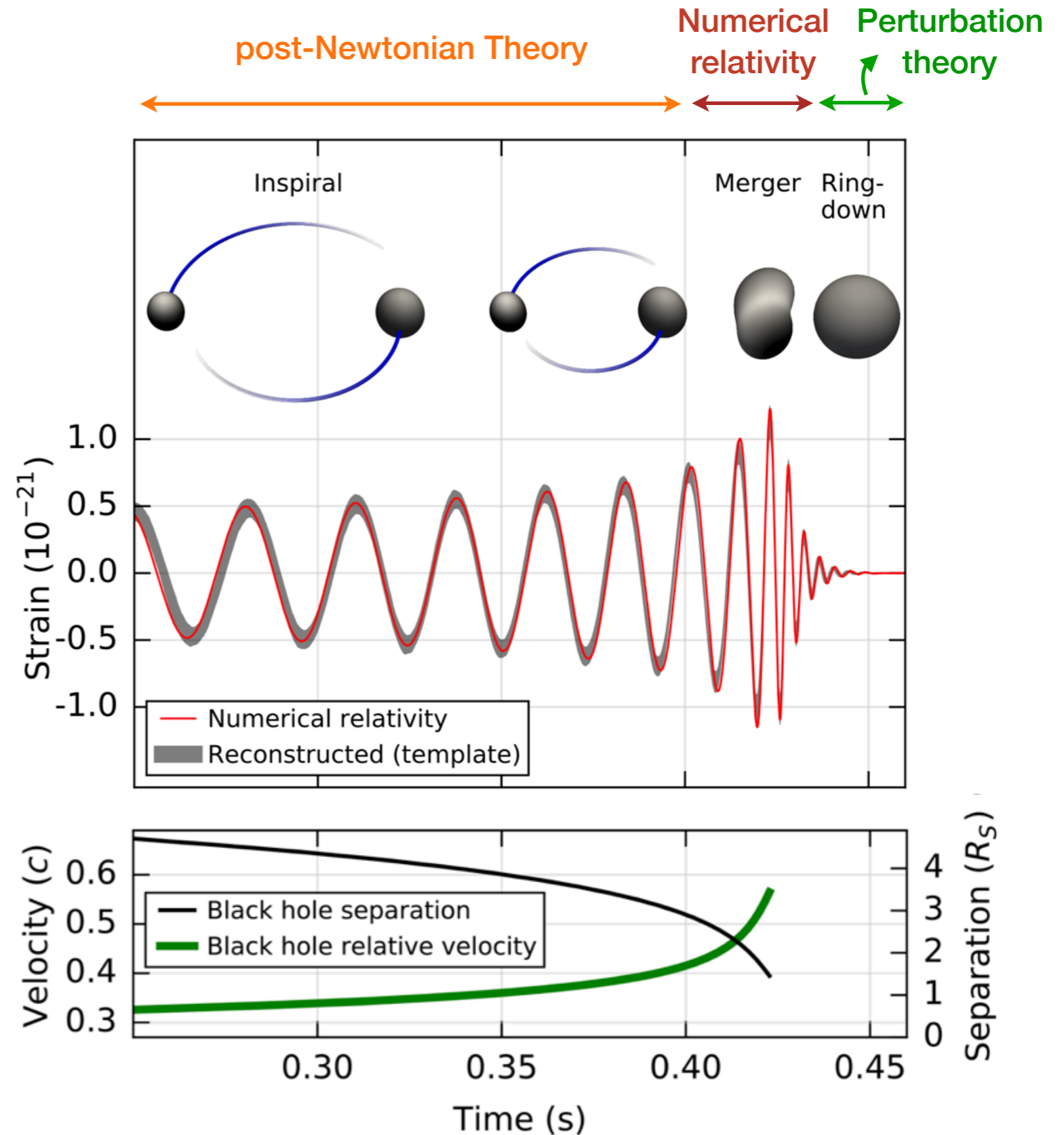


# Motivation: Detection of Gravitational Waves

Basic scalings:

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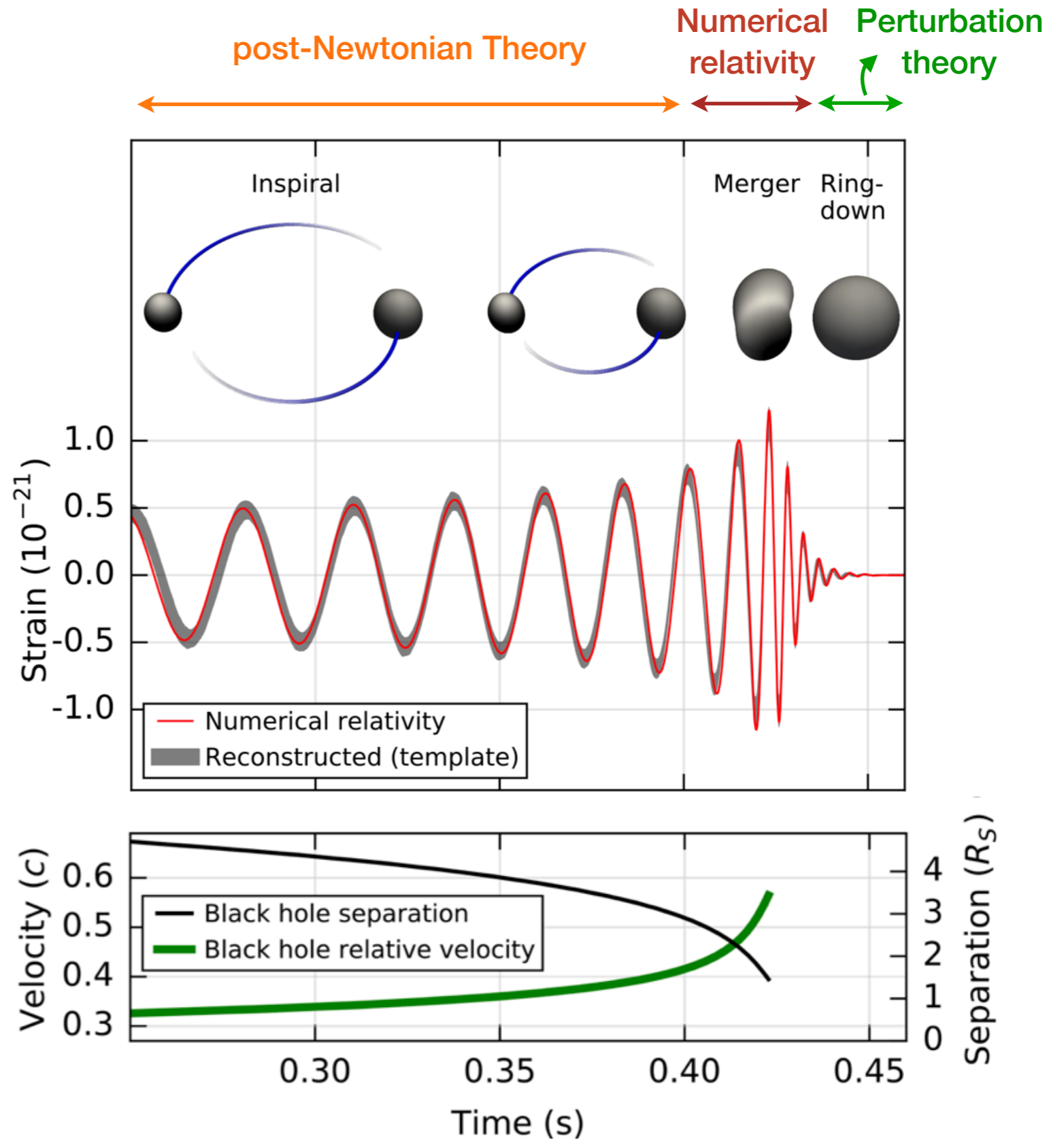
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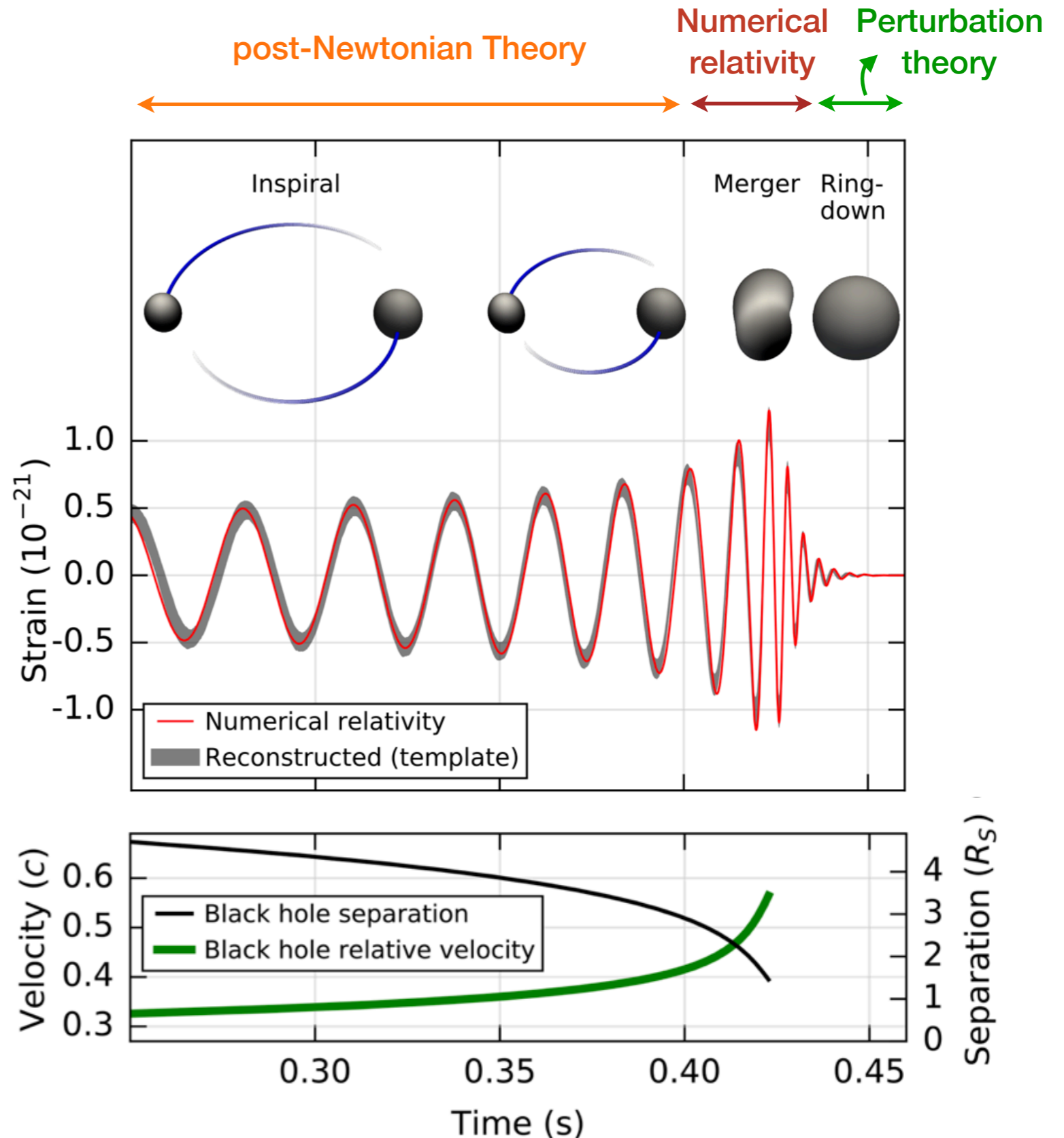
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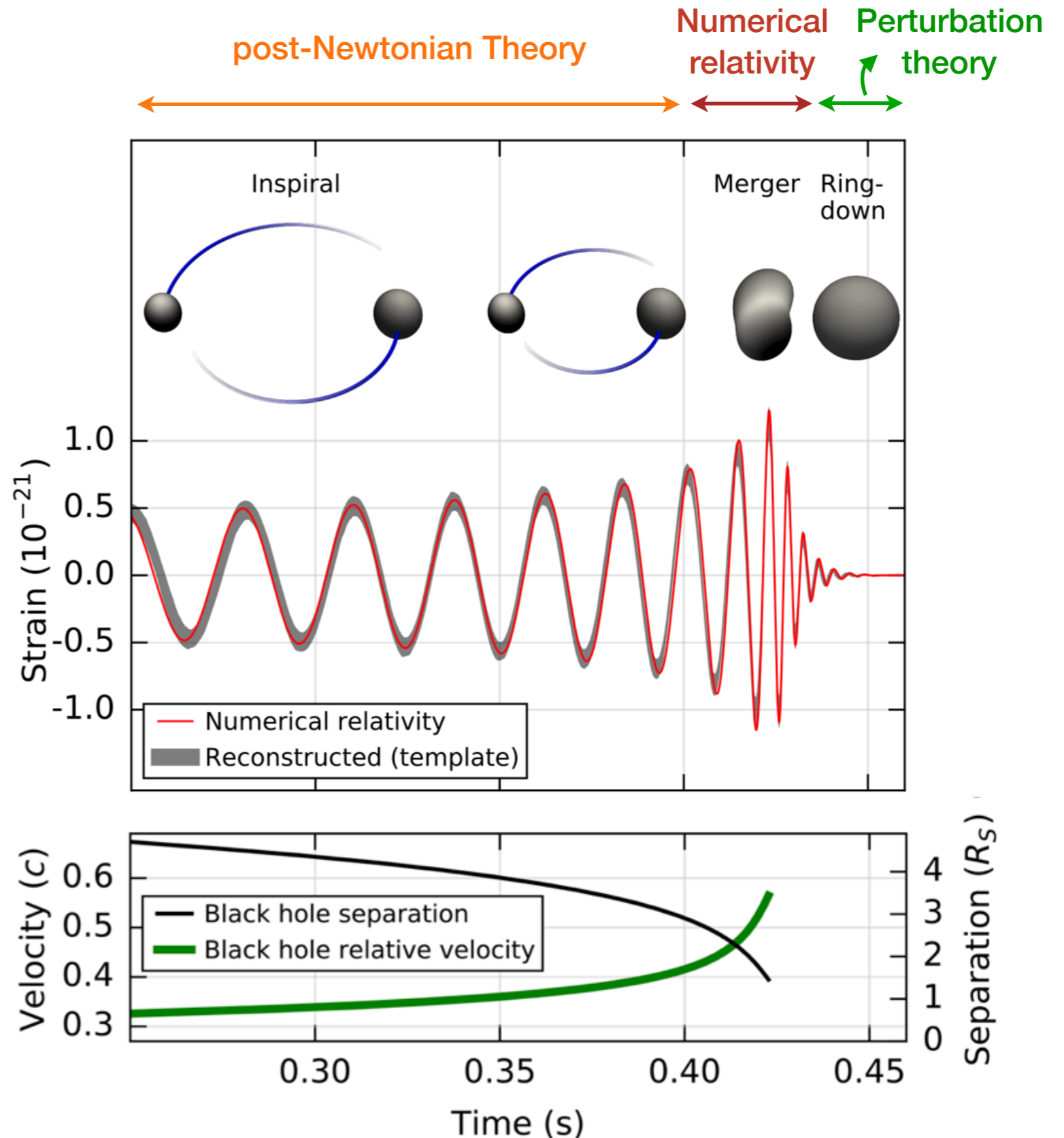
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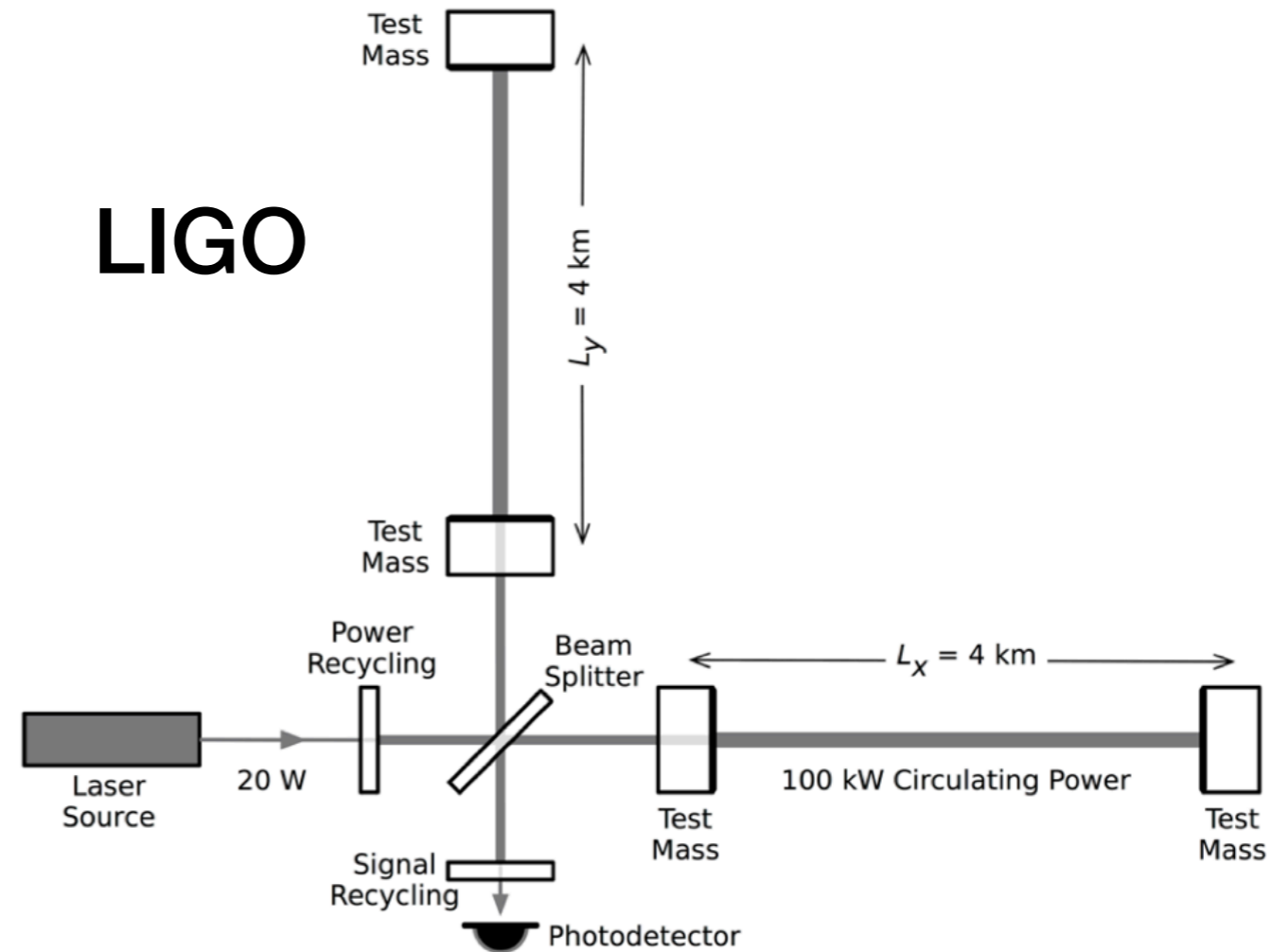
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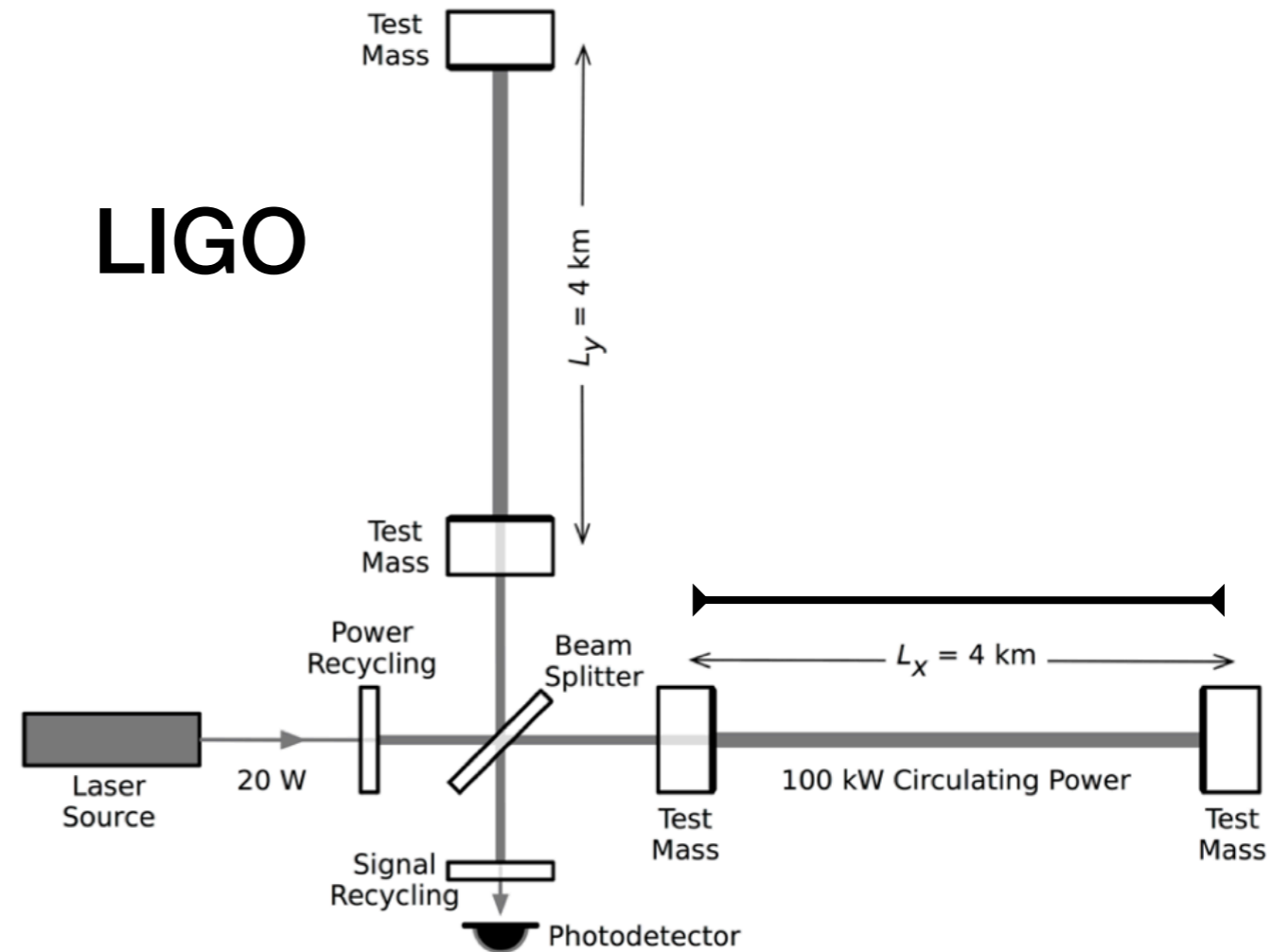
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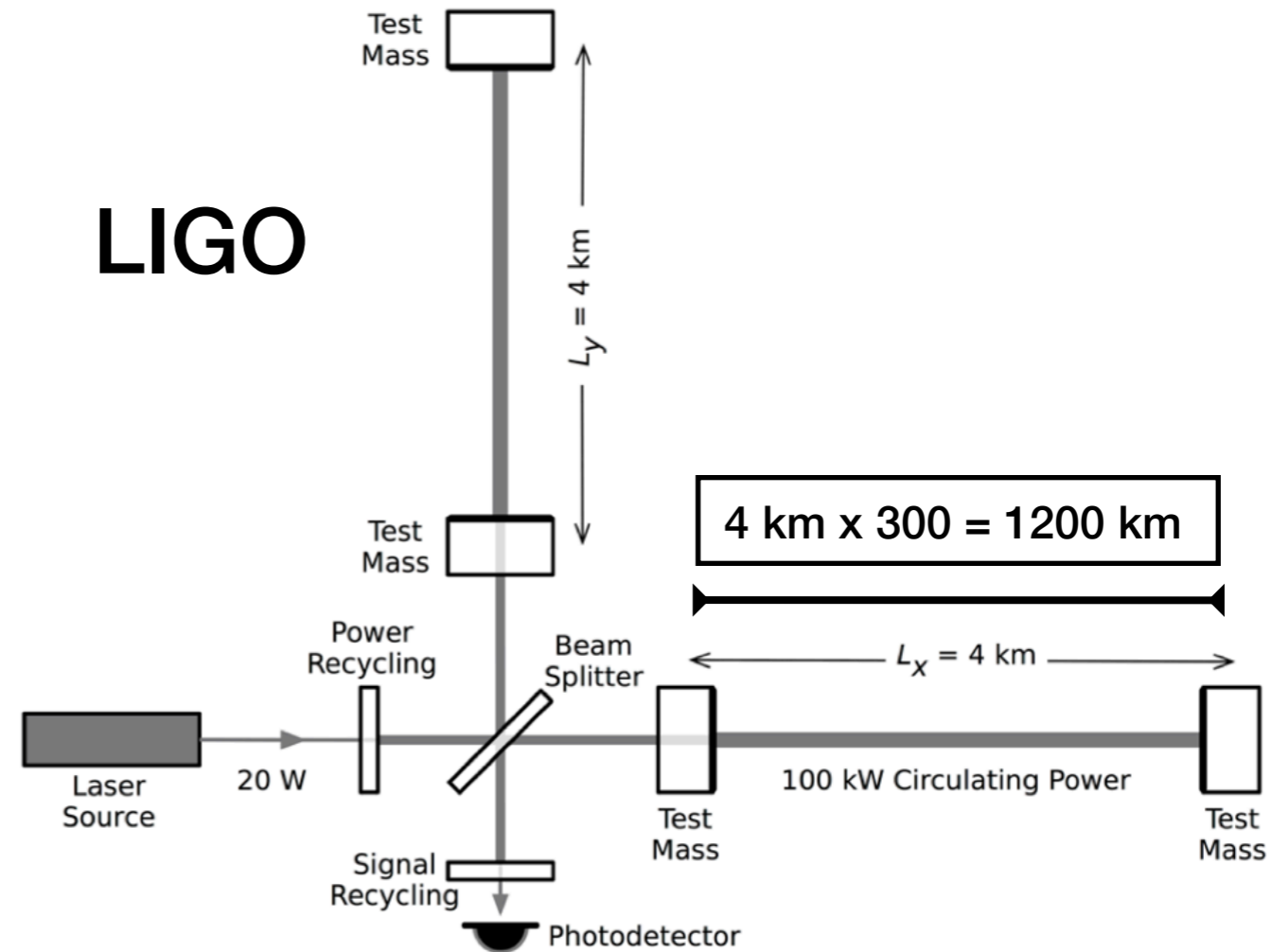
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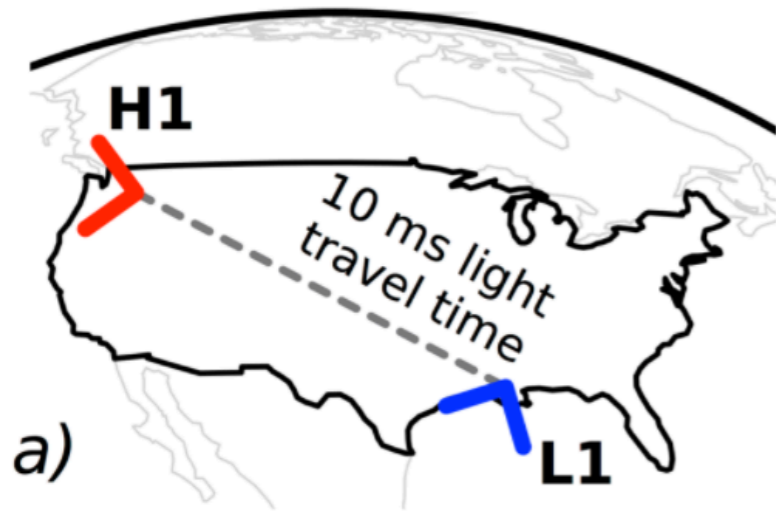
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LIGO is designed to detect mergers of stellar-mass objects.

# Motivation: Detection of Gravitational Waves

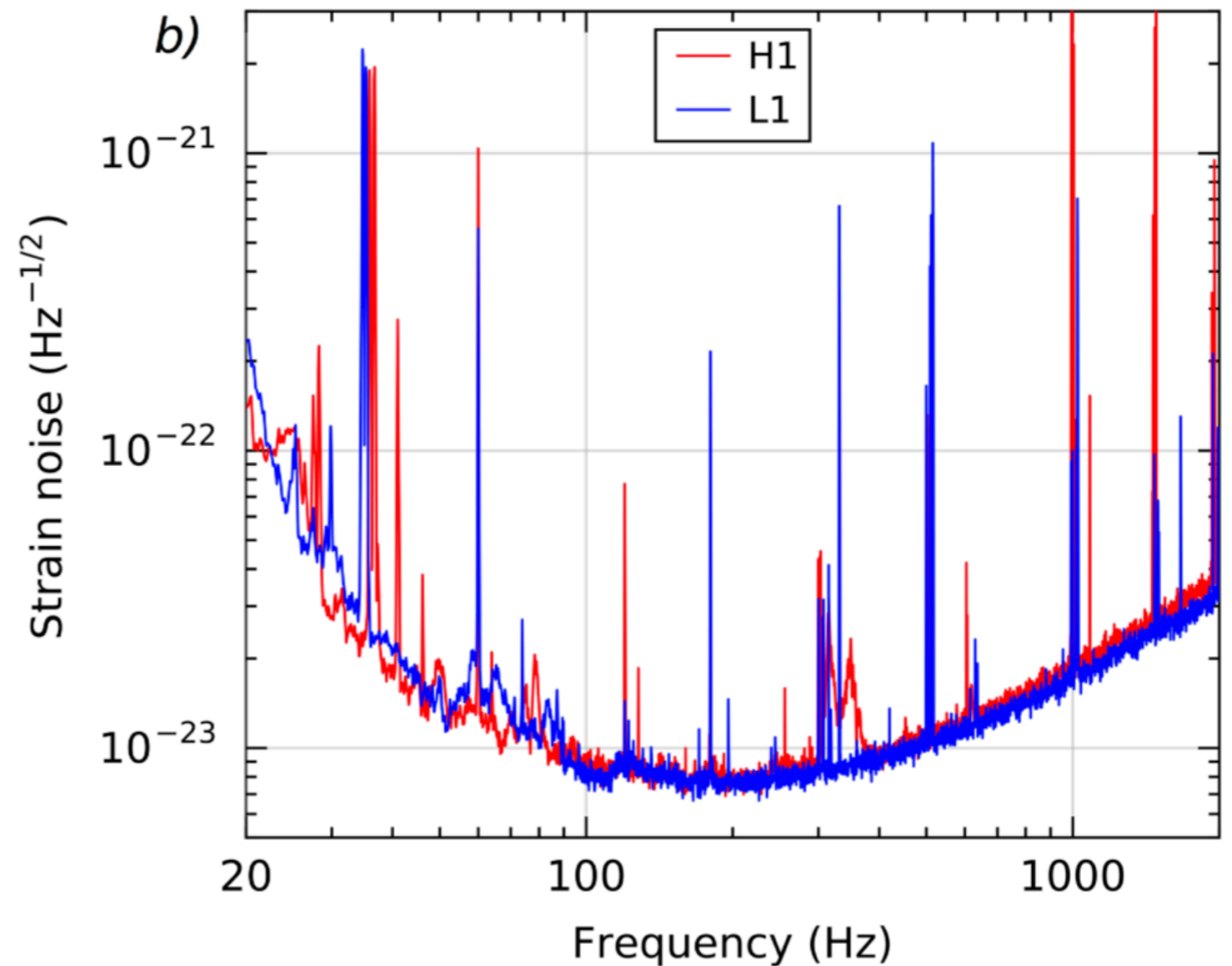
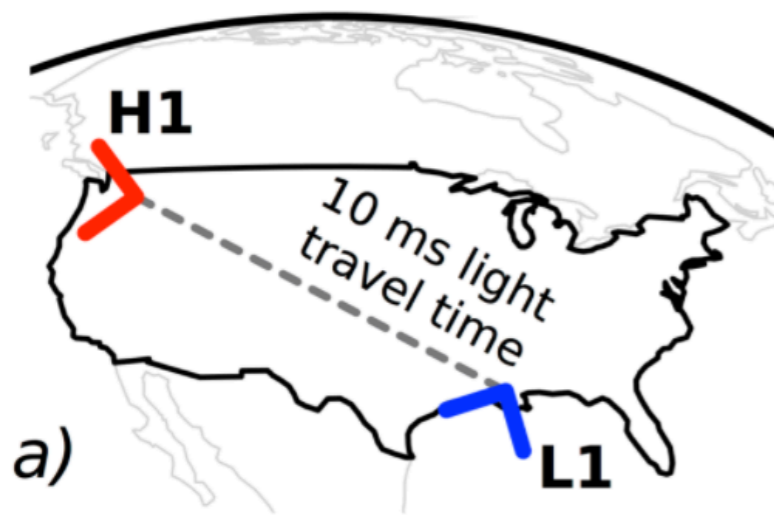
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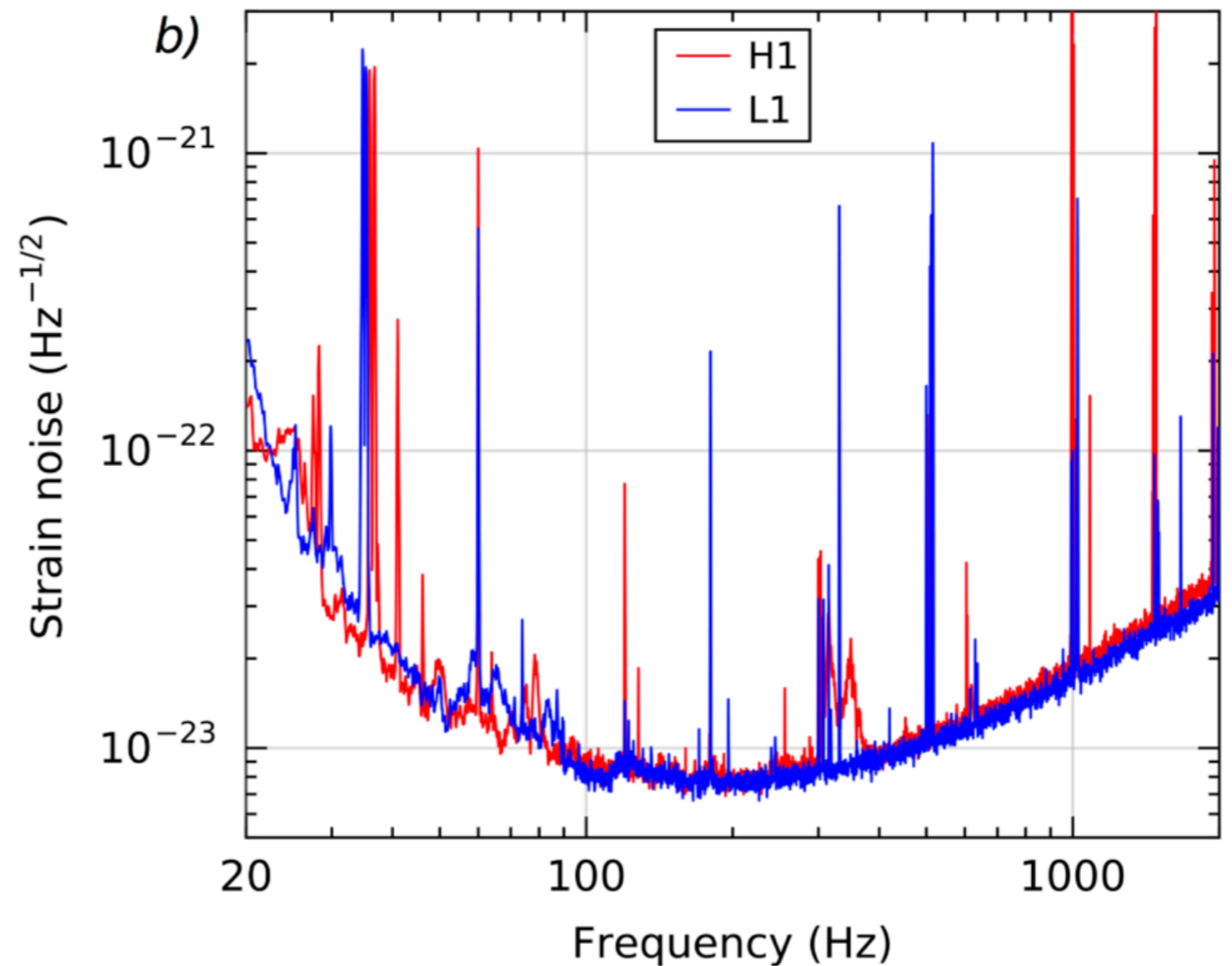
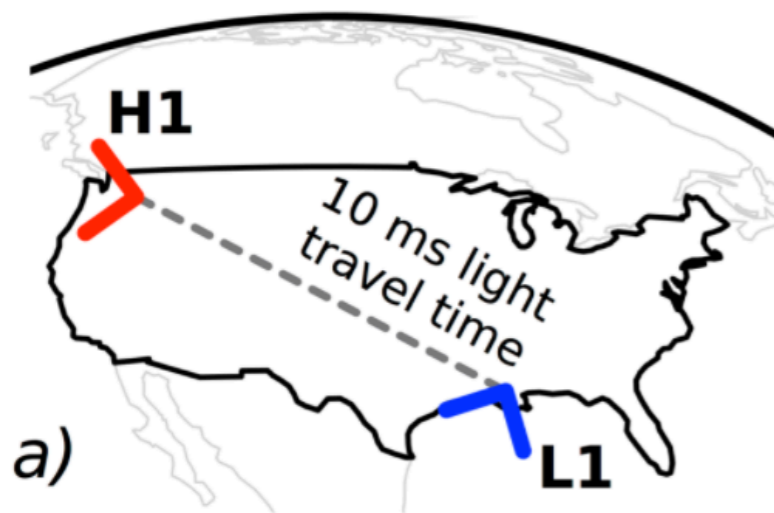
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# Motivation: Detection of Gravitational Waves

LIGO is designed to detect mergers of stellar-mass objects.

Target strain level: 
$$h = 10^{-21} \left( \frac{300 \text{ Mpc}}{d_L} \right) \left( \frac{150 \text{ Hz}}{f_{\text{merger}}} \right)$$



# Motivation: Detection of Gravitational Waves

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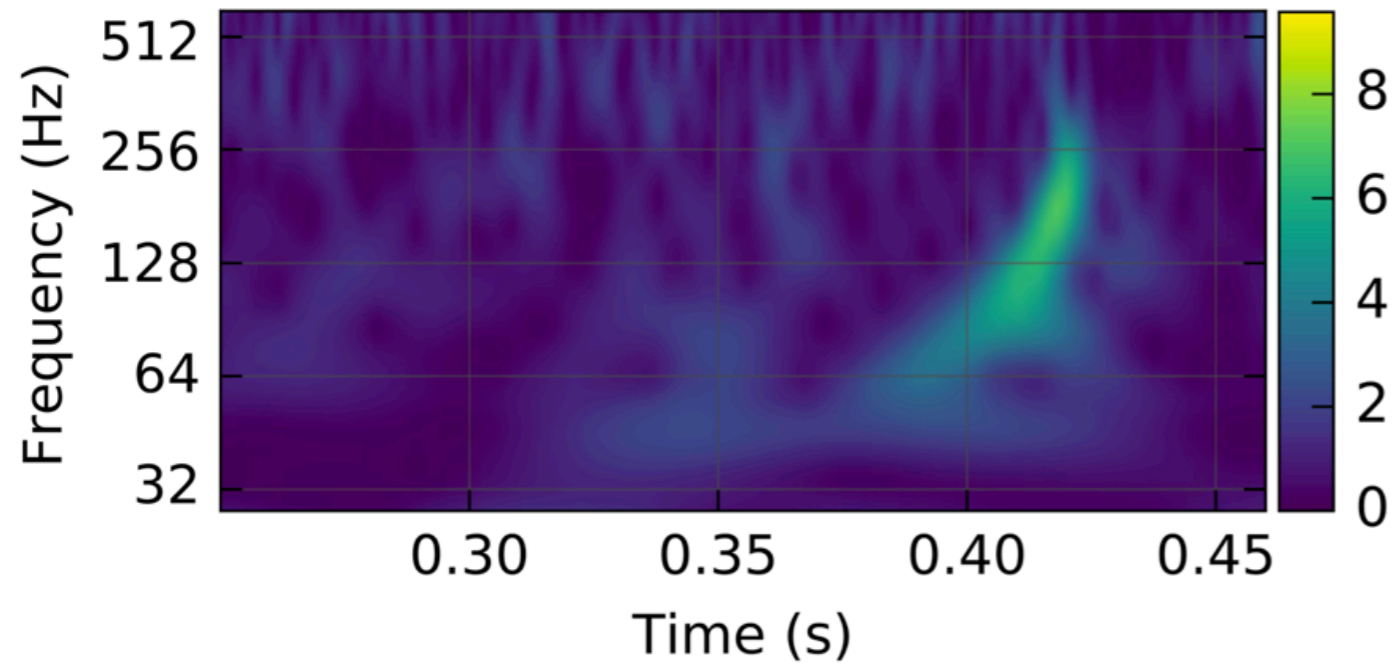
On September 14, 2015 at 09:50:45 GMT:

# Motivation: Detection of Gravitational Waves

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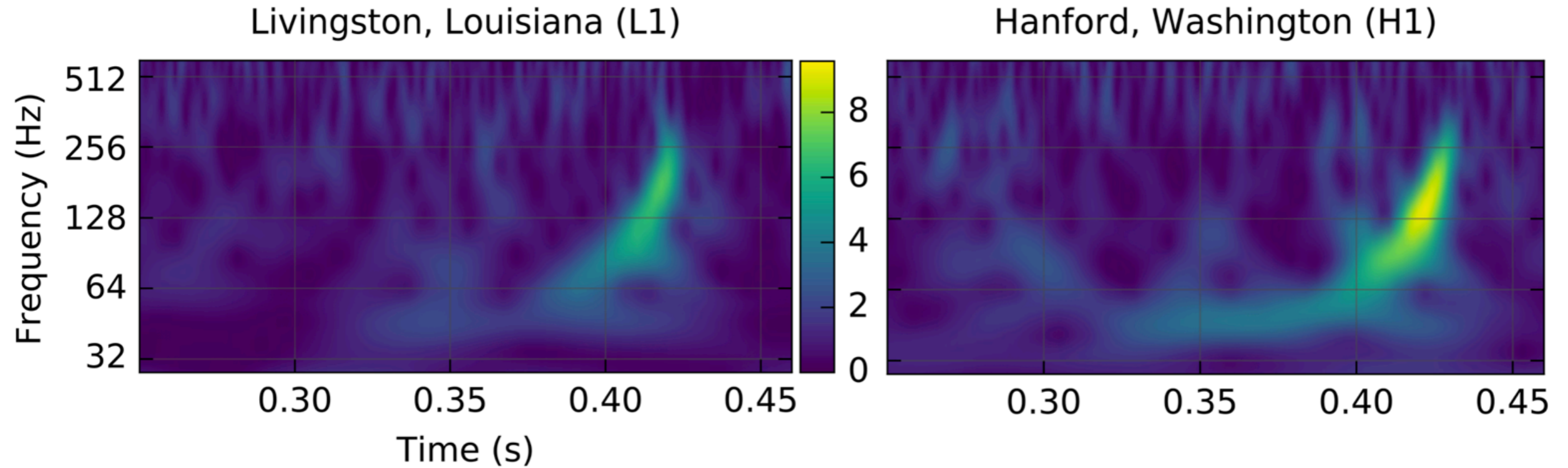
Livingston, Louisiana (L1)



# Motivation: Detection of Gravitational Waves

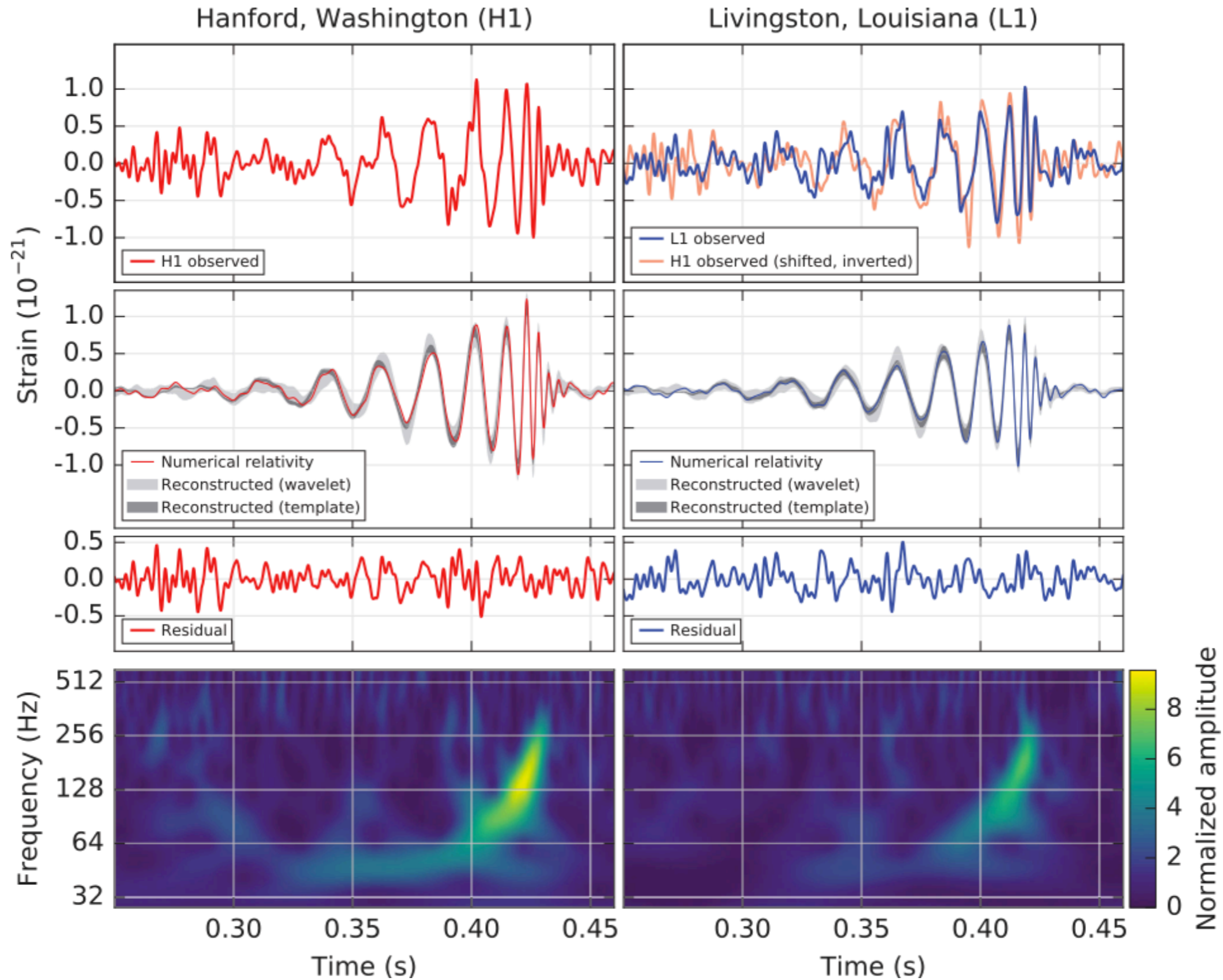
---

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Parameter estimation (basics):

# Motivation: Detection of Gravitational Waves

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Measuring the Chirp Mass:

# Motivation: Detection of Gravitational Waves

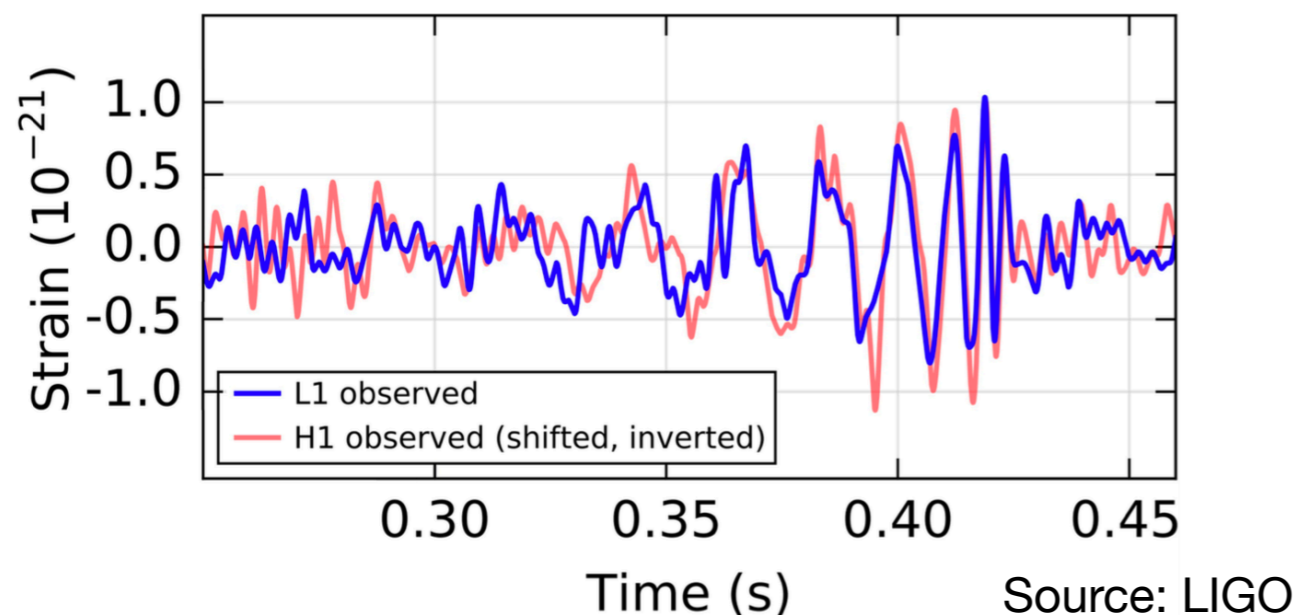
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# Motivation: Detection of Gravitational Waves

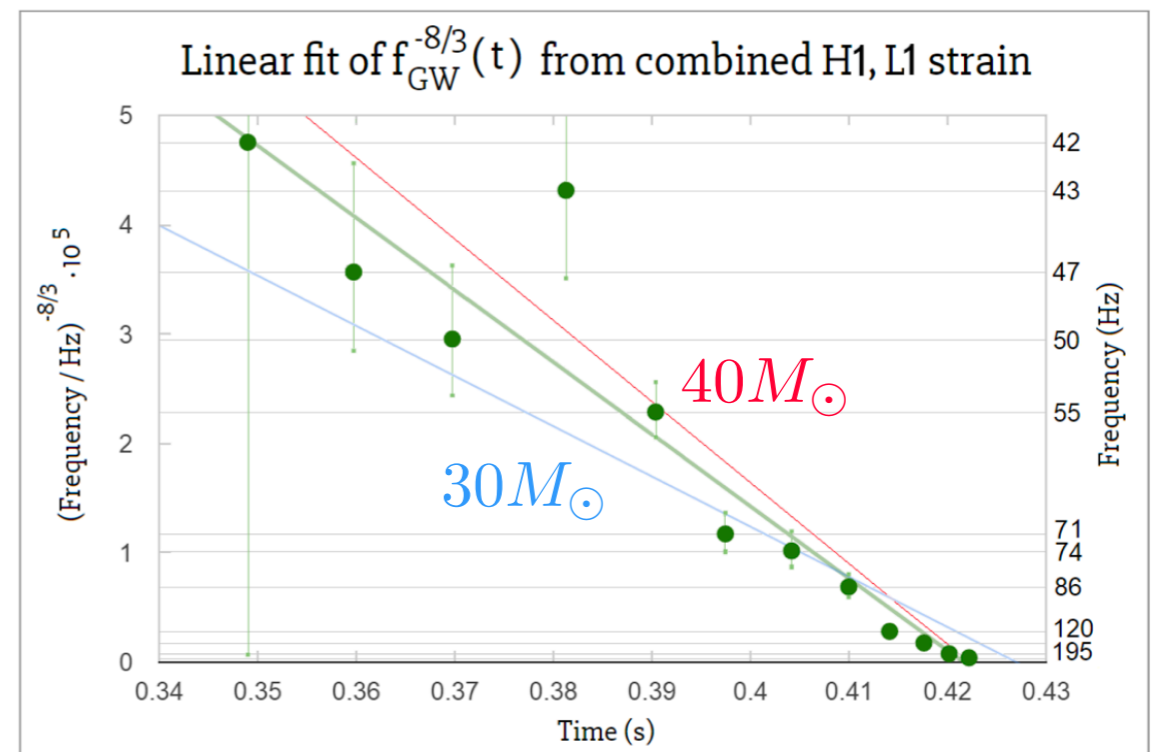
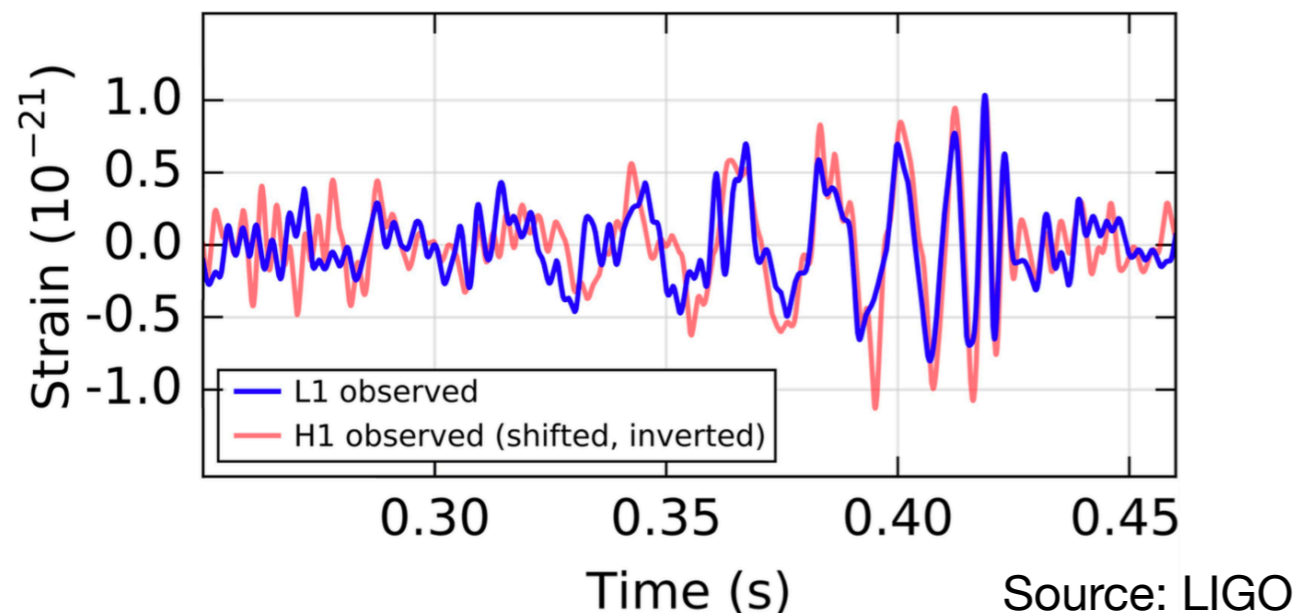
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# Extra Motivation: More Detections!

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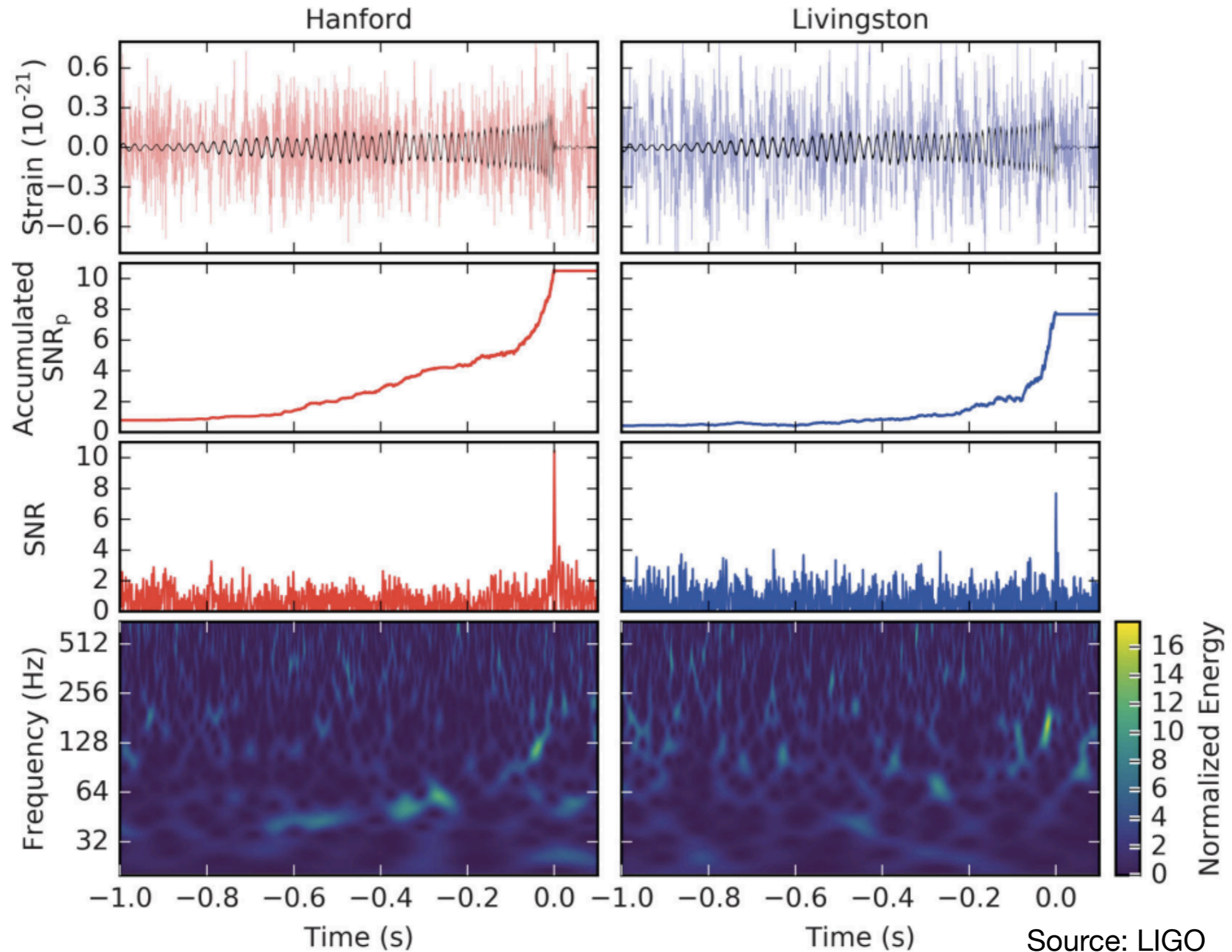
# Extra Motivation: More Detections!

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Boxing-day event GW151226:

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# Motivation: Detection of Gravitational Waves

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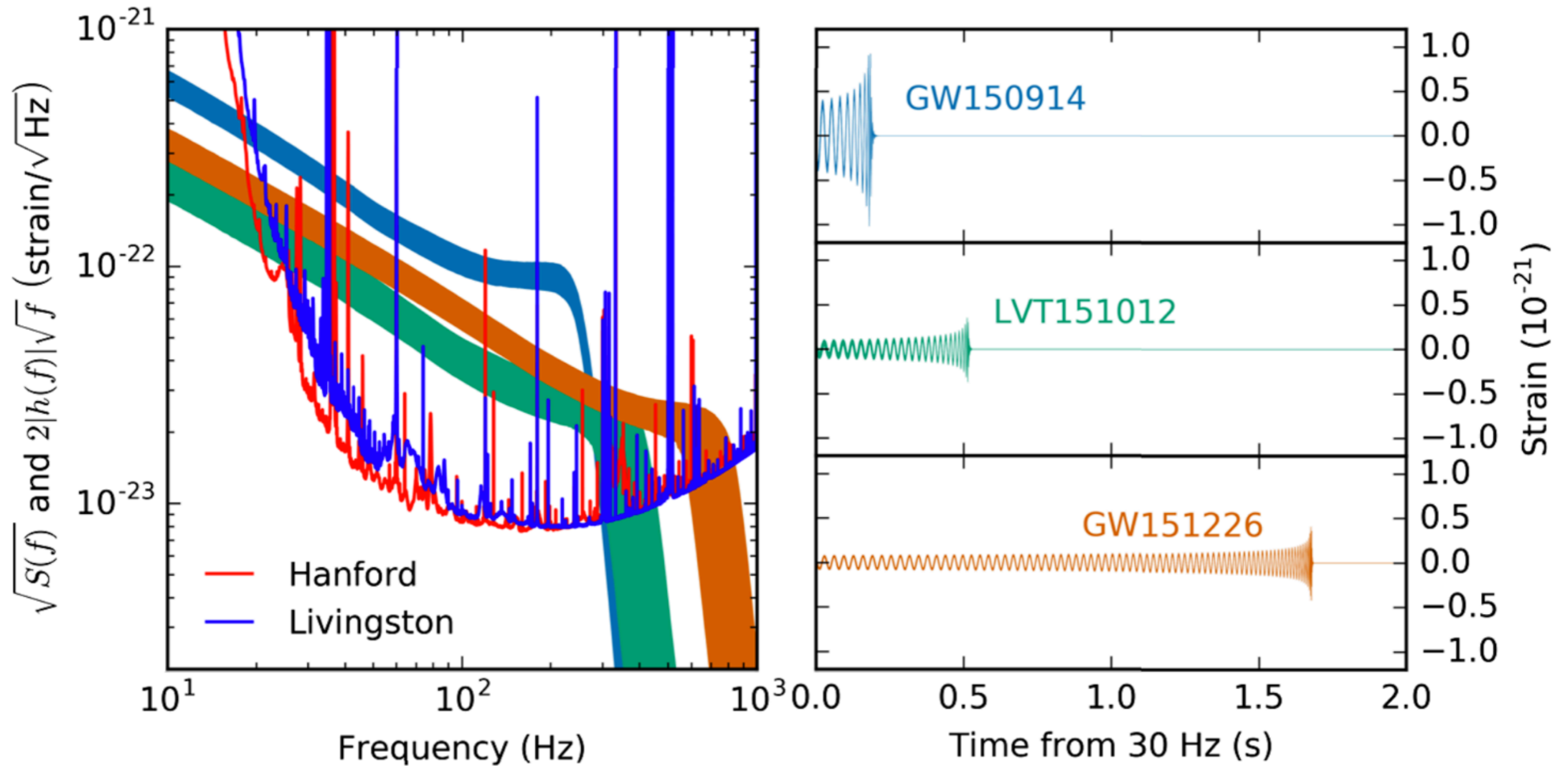
# Motivation: Detection of Gravitational Waves

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Over 51.5 days, three events singled out:

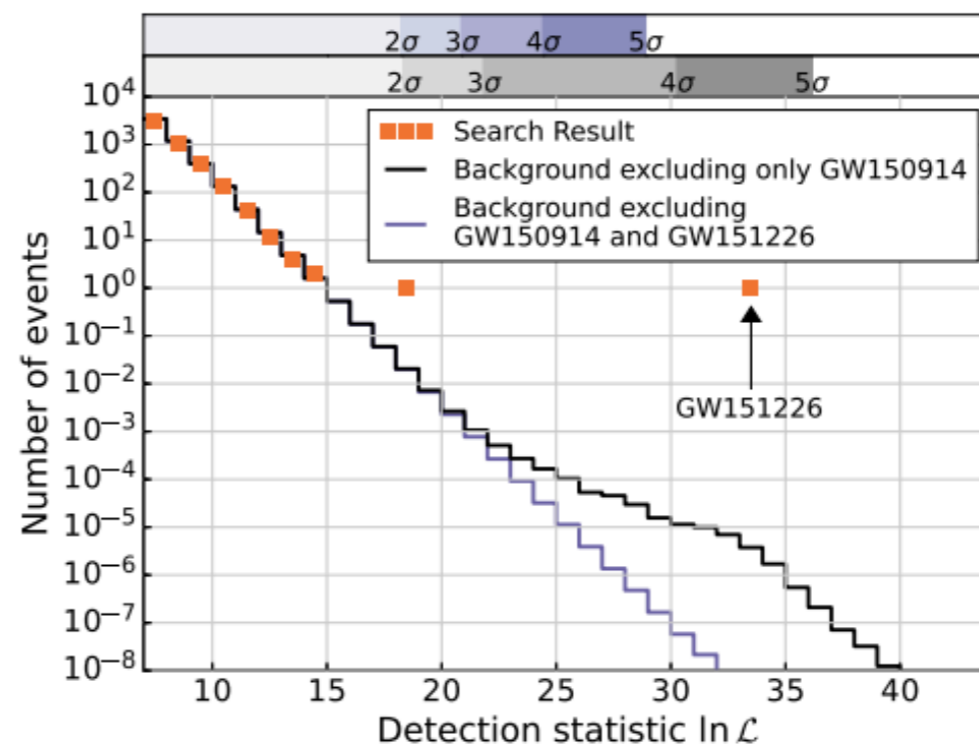
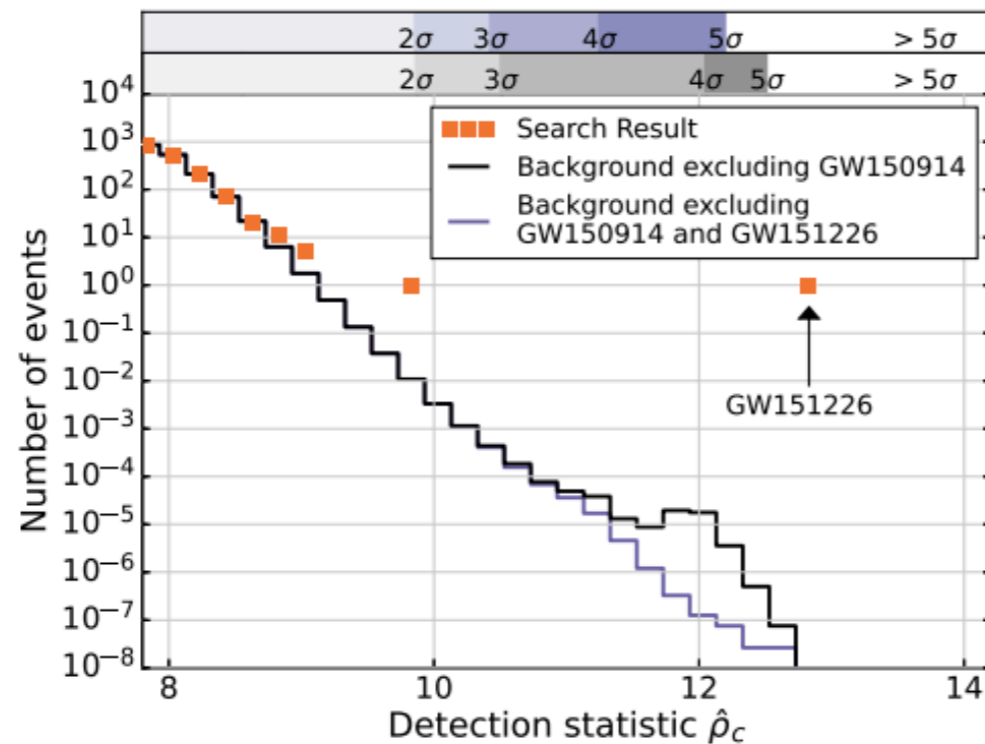
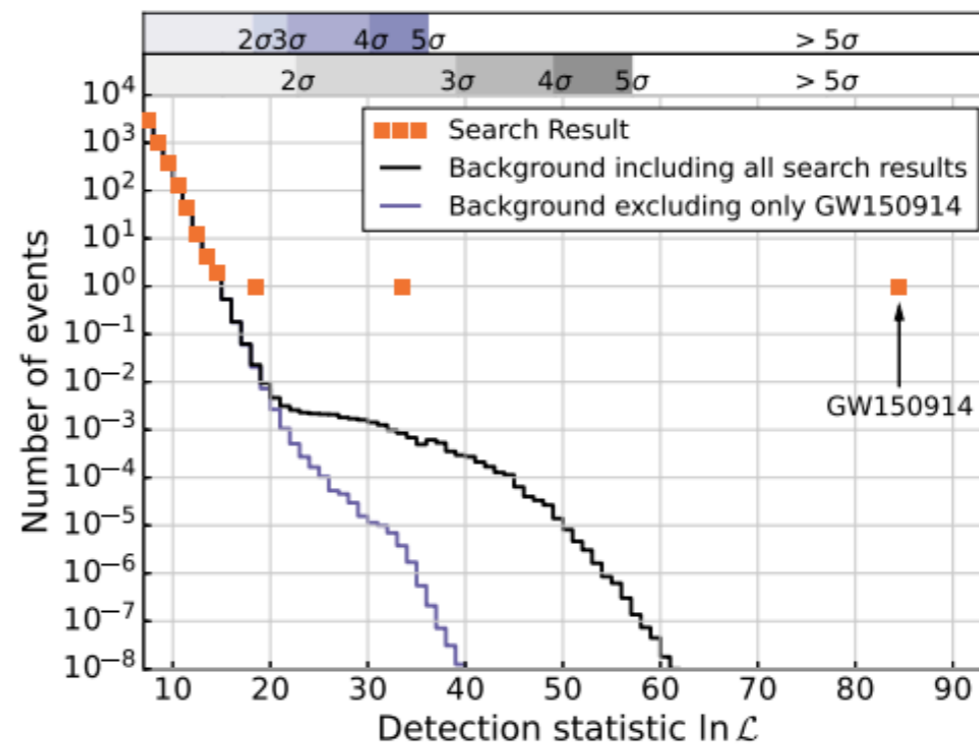
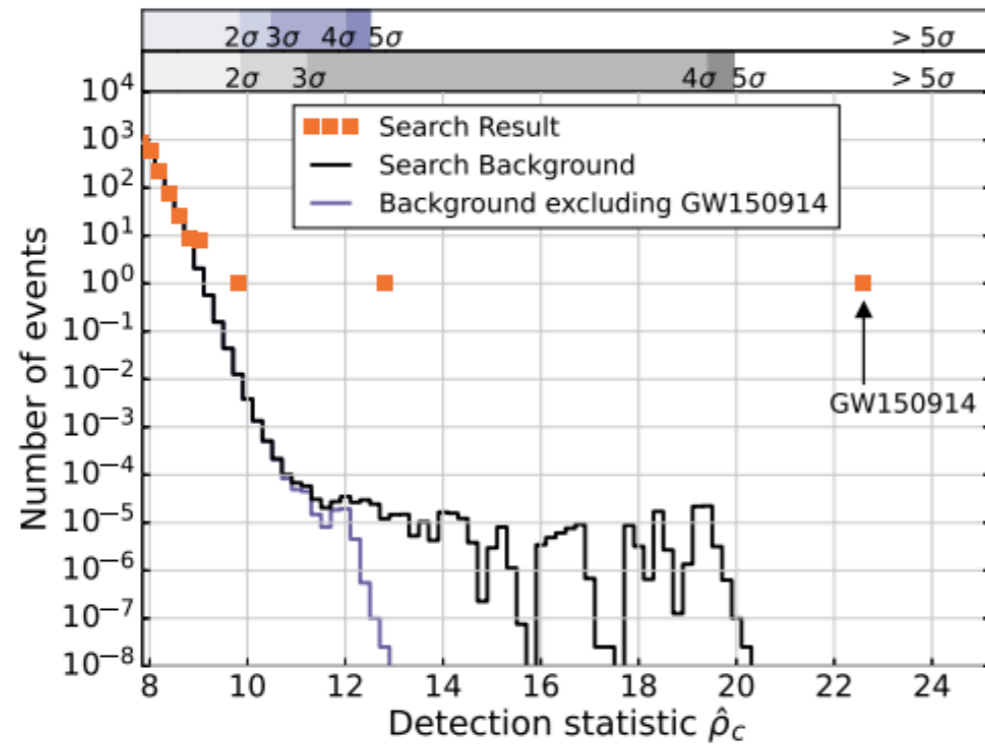
# Motivation: Detection of Gravitational Waves

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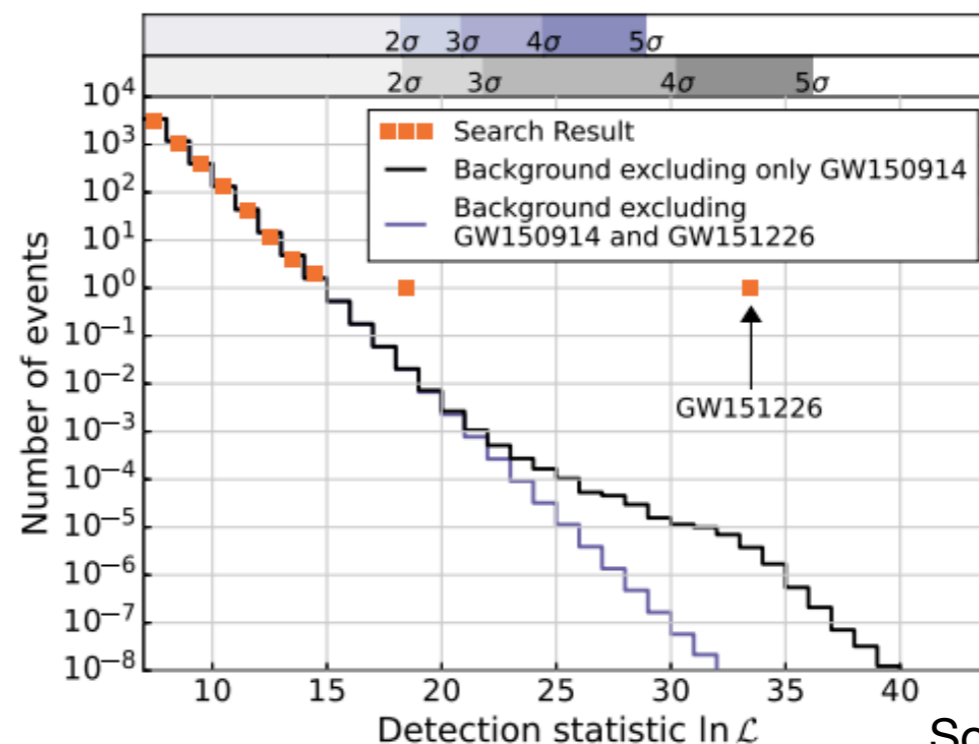
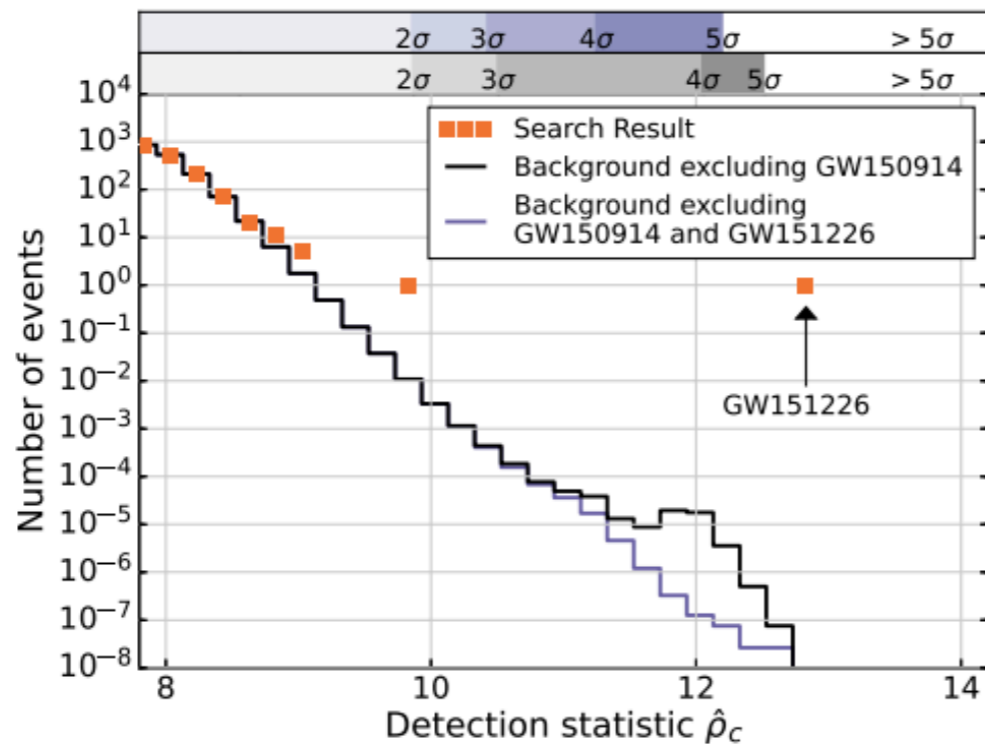
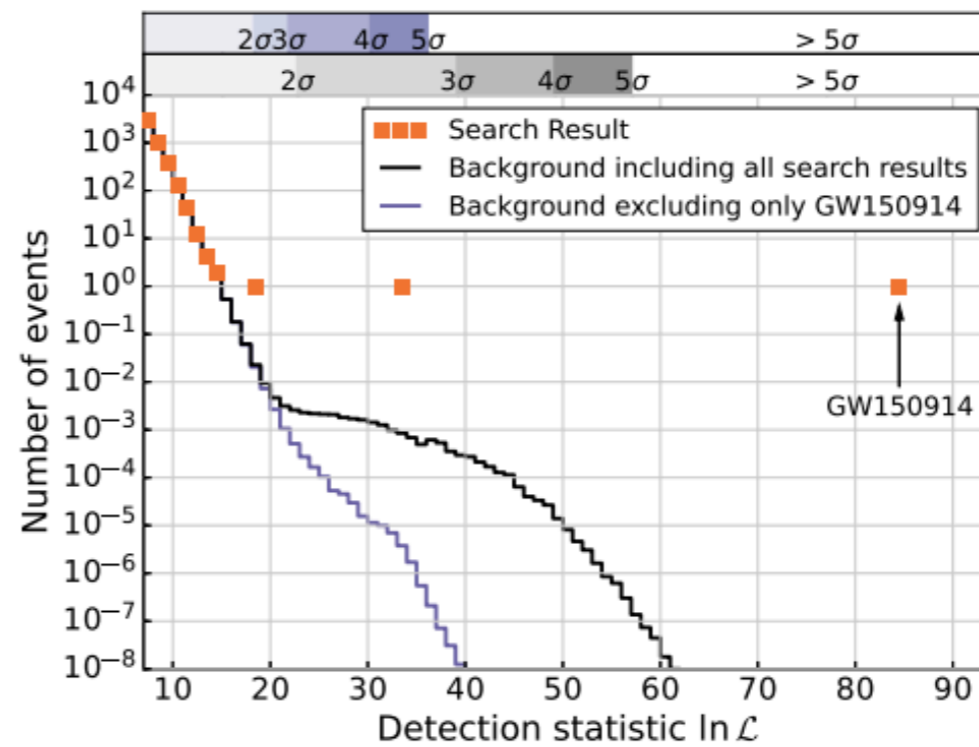
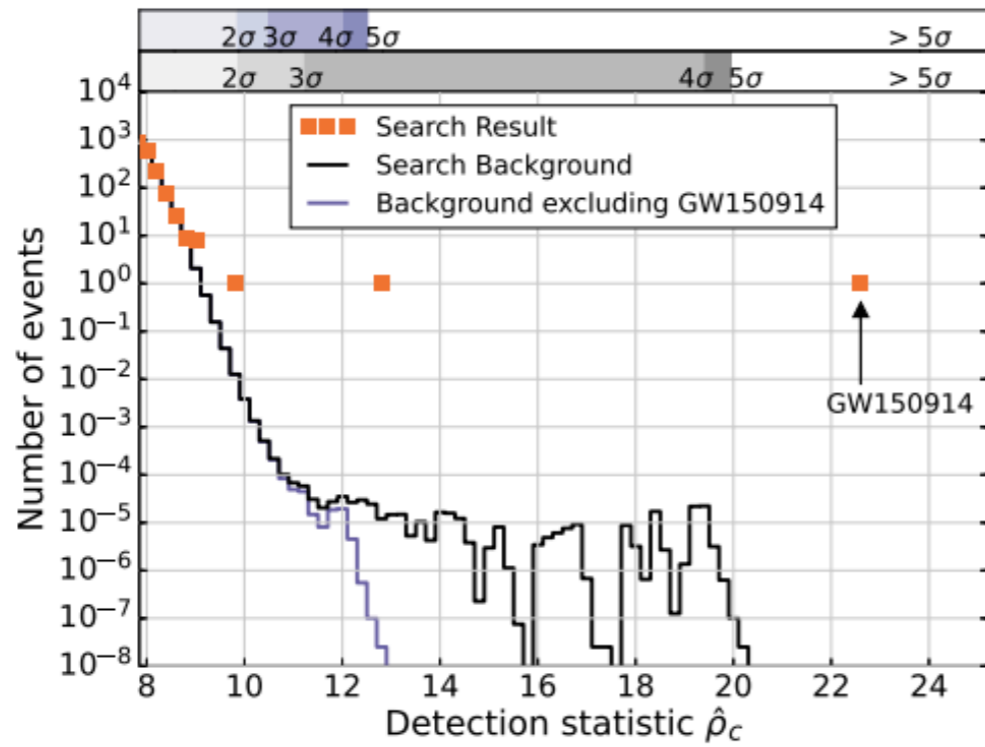
Source: LIGO

# Motivation: Detection of Gravitational Waves



# Motivation: Detection of Gravitational Waves

Significance:



# Motivation: Detection of Gravitational Waves

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Signal-to-noise ratio $\rho$	23.7	13.0	9.7
False alarm rate FAR/yr <sup>-1</sup>	$< 6.0 \times 10^{-7}$	$< 6.0 \times 10^{-7}$	0.37
p-value	$7.5 \times 10^{-8}$	$7.5 \times 10^{-8}$	0.045
Significance	$> 5.3\sigma$	$> 5.3\sigma$	$1.7\sigma$
Primary mass $m_1^{\text{source}}/M_\odot$	$36.2^{+5.2}_{-3.8}$	$14.2^{+8.3}_{-3.7}$	$23^{+18}_{-6}$
Secondary mass $m_2^{\text{source}}/M_\odot$	$29.1^{+3.7}_{-4.4}$	$7.5^{+2.3}_{-2.3}$	$13^{+4}_{-5}$
Chirp mass $\mathcal{M}^{\text{source}}/M_\odot$	$28.1^{+1.8}_{-1.5}$	$8.9^{+0.3}_{-0.3}$	$15.1^{+1.4}_{-1.1}$
Total mass $M^{\text{source}}/M_\odot$	$65.3^{+4.1}_{-3.4}$	$21.8^{+5.9}_{-1.7}$	$37^{+13}_{-4}$
Effective inspiral spin $\chi_{\text{eff}}$	$-0.06^{+0.14}_{-0.14}$	$0.21^{+0.20}_{-0.10}$	$0.0^{+0.3}_{-0.2}$
Final mass $M_f^{\text{source}}/M_\odot$	$62.3^{+3.7}_{-3.1}$	$20.8^{+6.1}_{-1.7}$	$35^{+14}_{-4}$
Final spin $a_f$	$0.68^{+0.05}_{-0.06}$	$0.74^{+0.06}_{-0.06}$	$0.66^{+0.09}_{-0.10}$
Radiated energy $E_{\text{rad}}/(M_\odot c^2)$	$3.0^{+0.5}_{-0.4}$	$1.0^{+0.1}_{-0.2}$	$1.5^{+0.3}_{-0.4}$
Peak luminosity $\ell_{\text{peak}}/(\text{erg s}^{-1})$	$3.6^{+0.5}_{-0.4} \times 10^{56}$	$3.3^{+0.8}_{-1.6} \times 10^{56}$	$3.1^{+0.8}_{-1.8} \times 10^{56}$
Luminosity distance $D_L/\text{Mpc}$	$420^{+150}_{-180}$	$440^{+180}_{-190}$	$1000^{+500}_{-500}$
Source redshift $z$	$0.09^{+0.03}_{-0.04}$	$0.09^{+0.03}_{-0.04}$	$0.20^{+0.09}_{-0.09}$
Sky localization $\Delta\Omega/\text{deg}^2$	230	850	1600

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Chirp mass $\mathcal{M}^{\text{source}}/M_\odot$	$28.1_{-1.5}^{+1.8}$	$8.9_{-0.3}^{+0.3}$	$15.1_{-1.1}^{+1.4}$
Total mass $M^{\text{source}}/M_\odot$	$65.3_{-3.4}^{+4.1}$	$21.8_{-1.7}^{+5.9}$	$37_{-4}^{+13}$
Effective inspiral spin $\chi_{\text{eff}}$	$-0.06_{-0.14}^{+0.14}$	$0.21_{-0.10}^{+0.20}$	$0.0_{-0.2}^{+0.3}$
Final mass $M_f^{\text{source}}/M_\odot$	$62.3_{-3.1}^{+3.7}$	$20.8_{-1.7}^{+6.1}$	$35_{-4}^{+14}$
Final spin $a_f$	$0.68_{-0.06}^{+0.05}$	$0.74_{-0.06}^{+0.06}$	$0.66_{-0.10}^{+0.09}$
Radiated energy $E_{\text{rad}}/(M_\odot c^2)$	$3.0_{-0.4}^{+0.5}$	$1.0_{-0.2}^{+0.1}$	$1.5_{-0.4}^{+0.3}$
Peak luminosity $\ell_{\text{peak}}/(\text{erg s}^{-1})$	$3.6_{-0.4}^{+0.5} \times 10^{56}$	$3.3_{-1.6}^{+0.8} \times 10^{56}$	$3.1_{-1.8}^{+0.8} \times 10^{56}$
Luminosity distance $D_L/\text{Mpc}$	$420_{-180}^{+150}$	$440_{-190}^{+180}$	$1000_{-500}^{+500}$
Source redshift $z$	$0.09_{-0.04}^{+0.03}$	$0.09_{-0.04}^{+0.03}$	$0.20_{-0.09}^{+0.09}$
Sky localization $\Delta\Omega/\text{deg}^2$	230	850	1600

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LIGO estimates after O1 run:

Mass distribution	$R / (\text{Gpc}^{-3} \text{ yr}^{-1})$		
	PyCBC	GstLAL	Combined
	Event based		
GW150914	$3.2^{+8.3}_{-2.7}$	$3.6^{+9.1}_{-3.0}$	$3.4^{+8.8}_{-2.8}$
LVT151012	$9.2^{+30.3}_{-8.5}$	$9.2^{+31.4}_{-8.5}$	$9.1^{+31.0}_{-8.5}$
GW151226	$35^{+92}_{-29}$	$37^{+94}_{-31}$	$36^{+95}_{-30}$
All	$53^{+100}_{-40}$	$56^{+105}_{-42}$	$55^{+103}_{-41}$
	Astrophysical		
Flat in log mass	$31^{+43}_{-21}$	$29^{+43}_{-21}$	$31^{+42}_{-21}$
Power law (-2.35)	$100^{+136}_{-69}$	$94^{+137}_{-66}$	$97^{+135}_{-67}$

# Outline

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- The Mass Function of Merging Black Holes

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- The Mass Function of Merging Black Holes
- The History of Black Hole Mergers
- Summary and observational outlook

# Outline

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# The Mass Distribution of Stellar Objects

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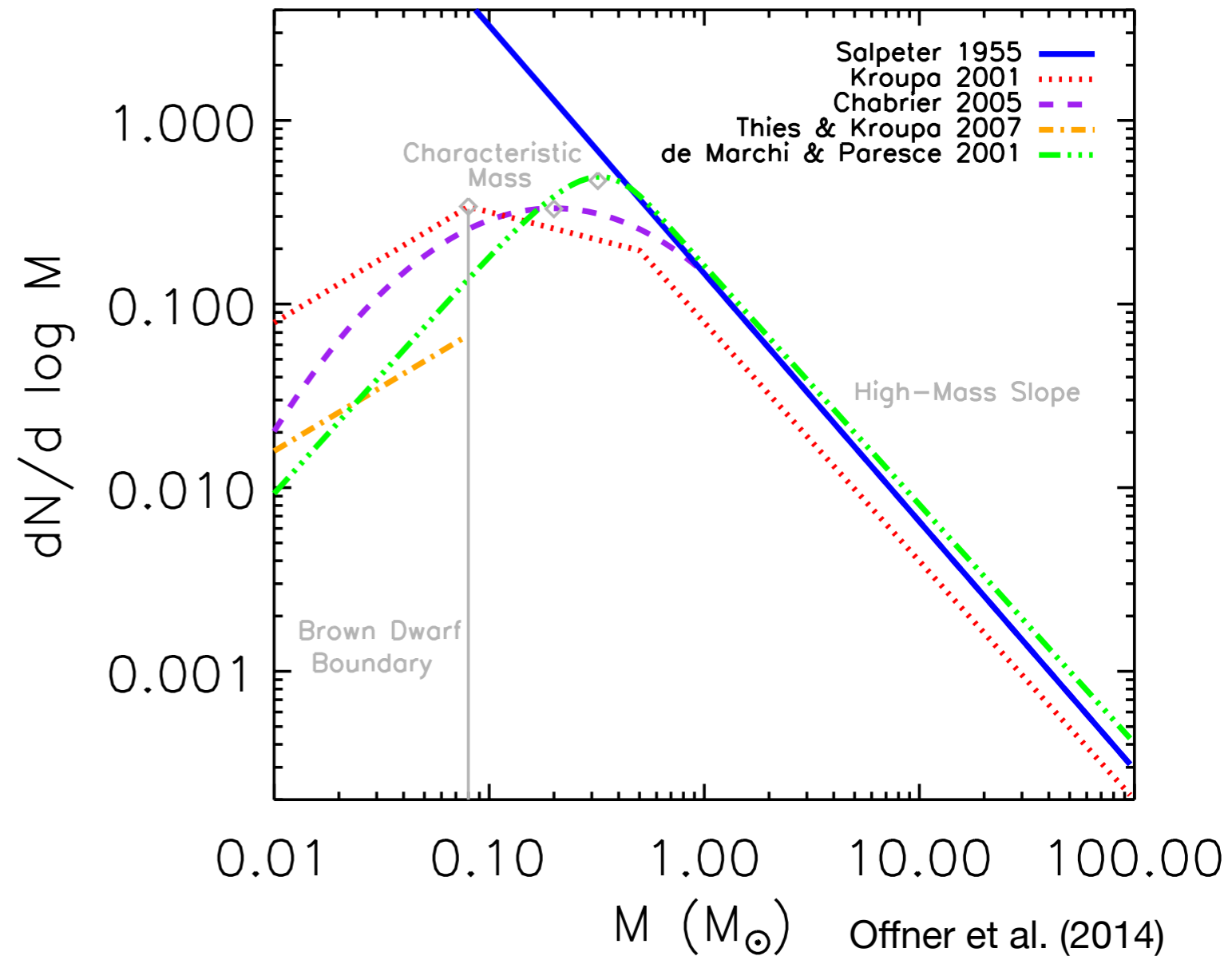
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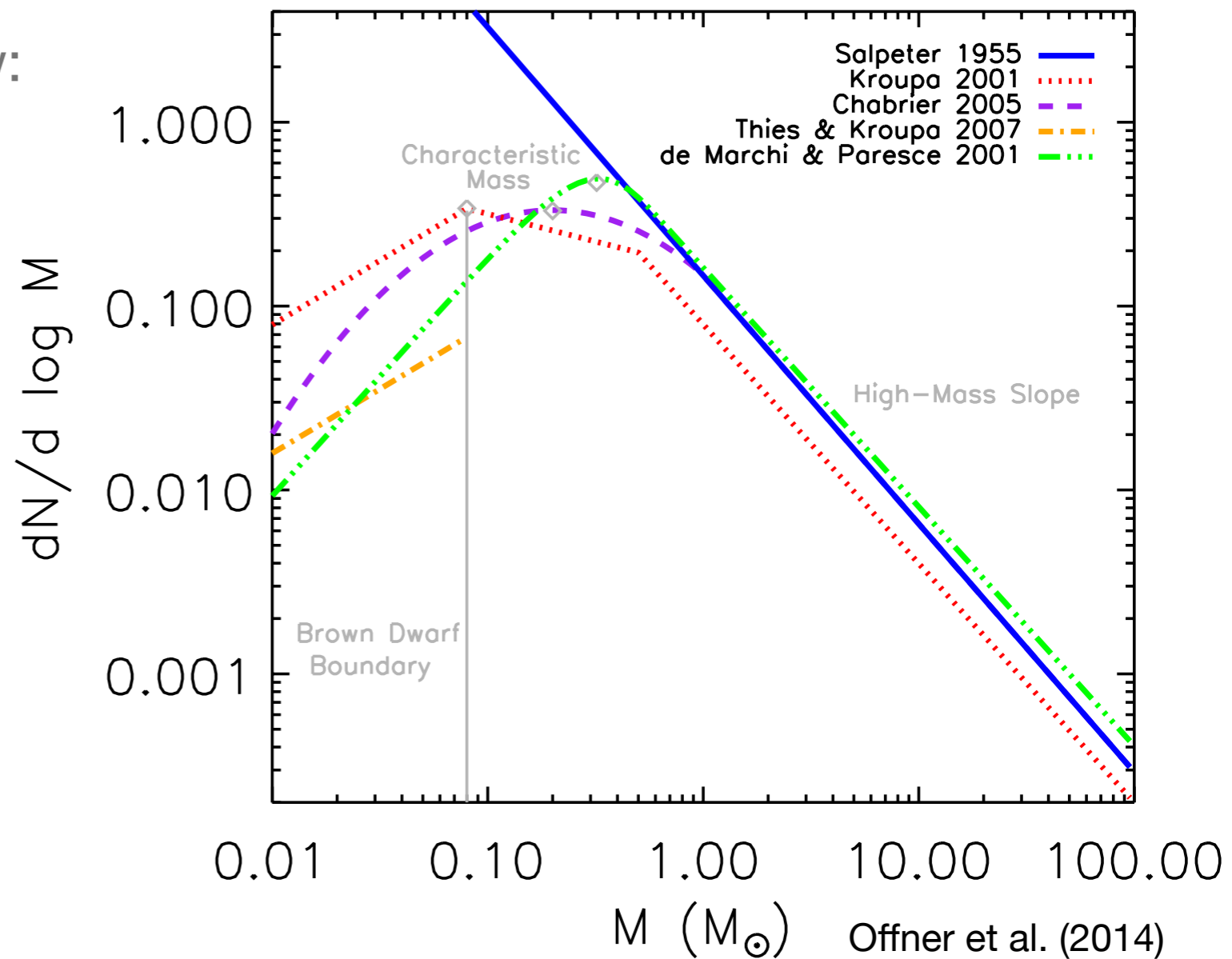
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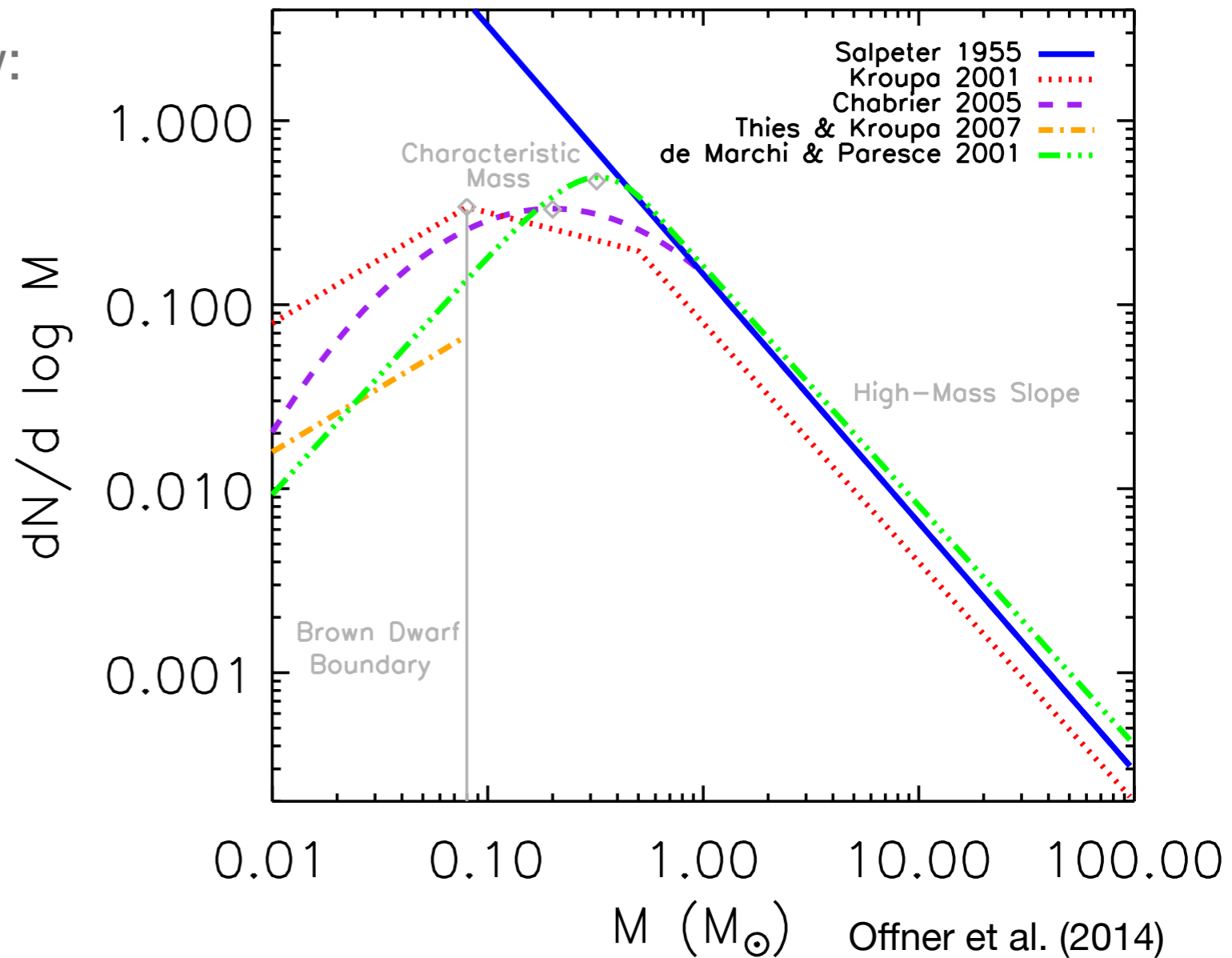


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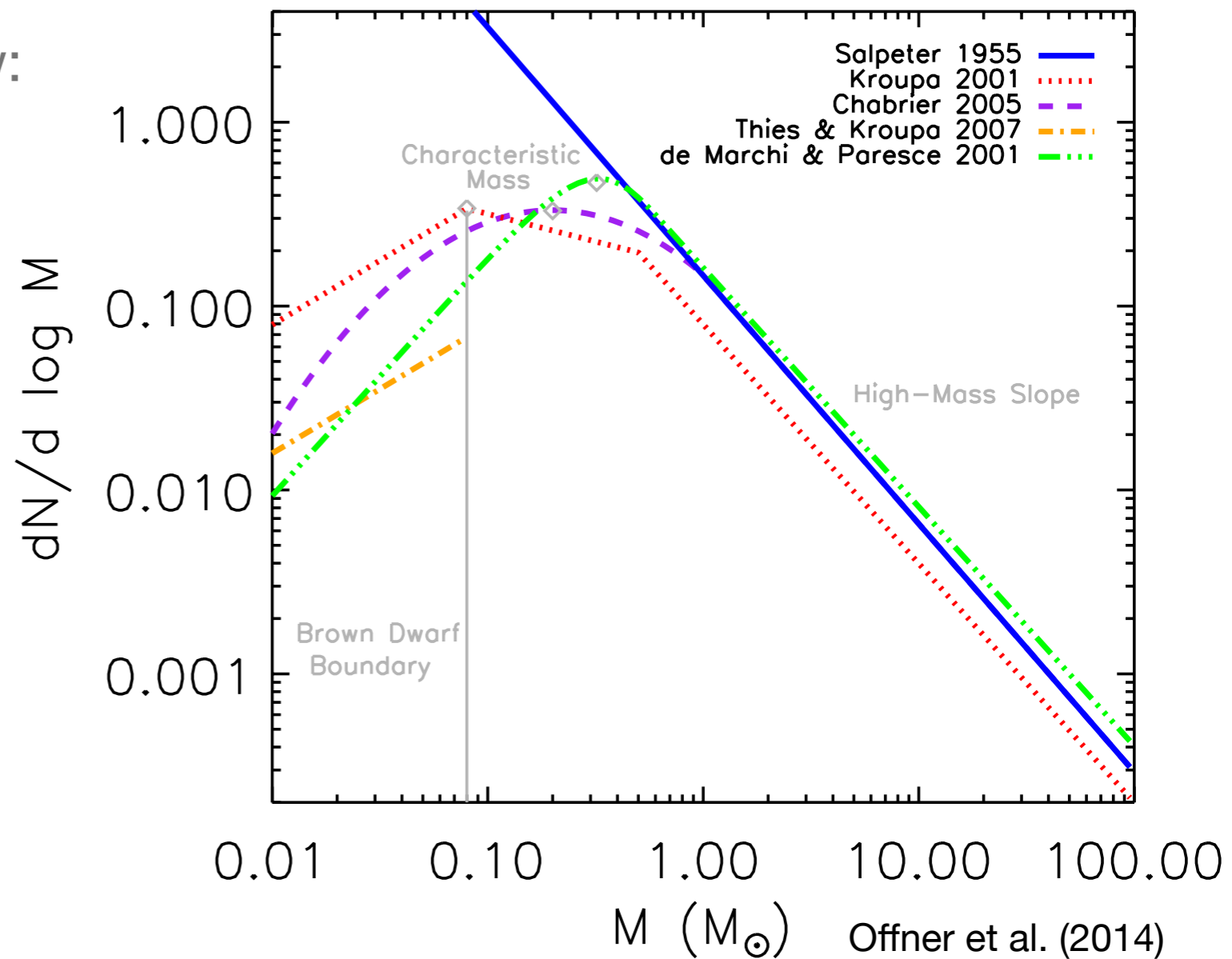
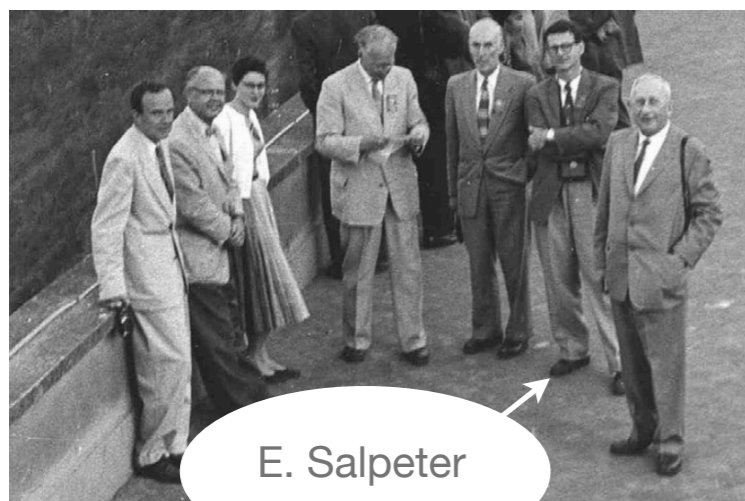
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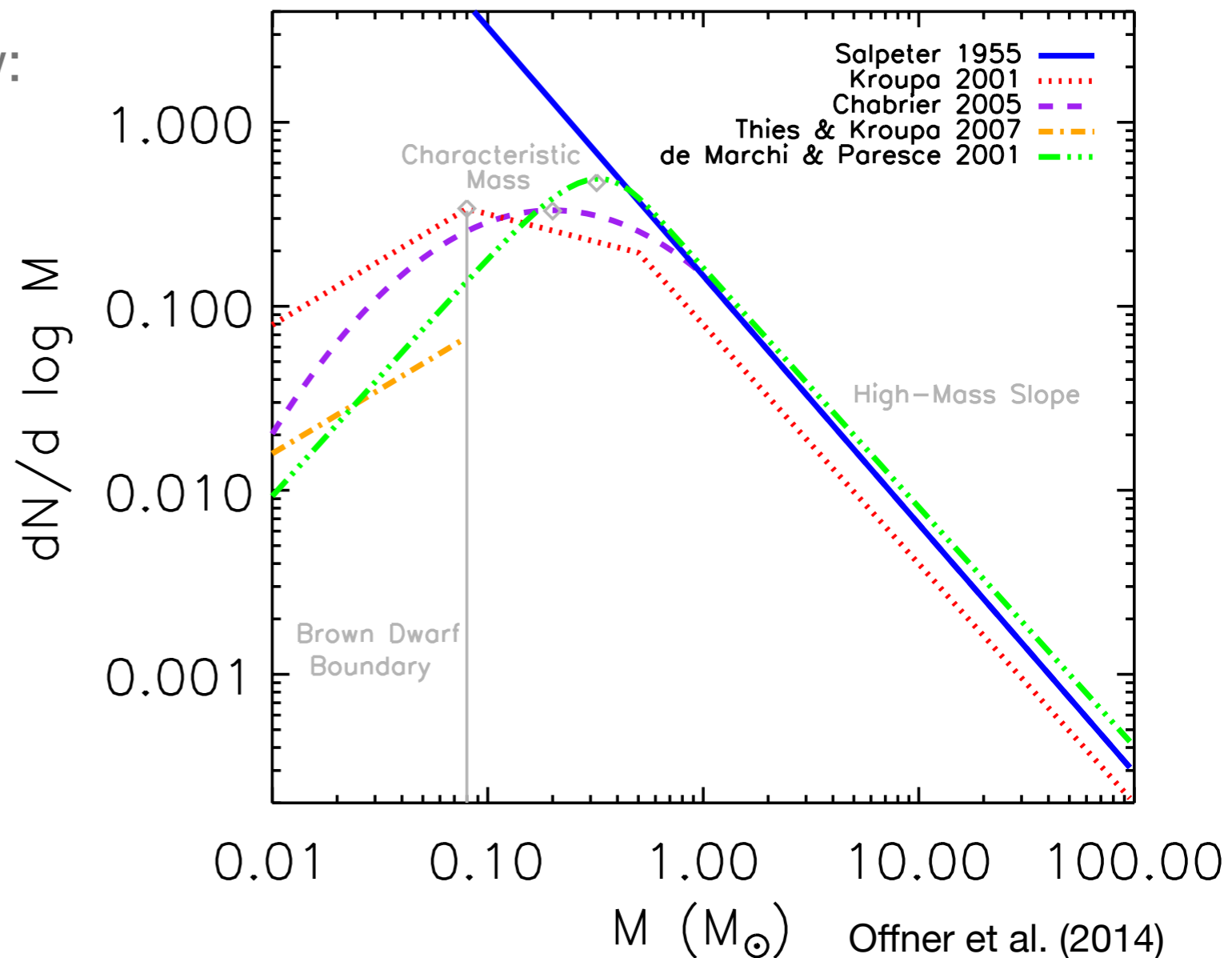
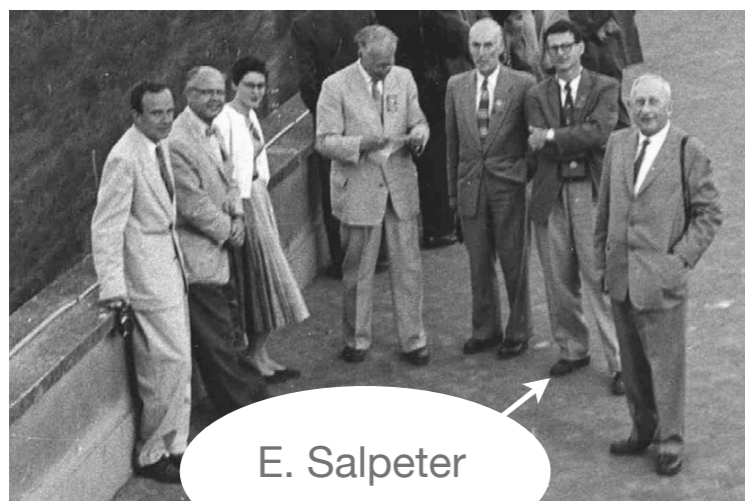
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Value remains a good fit after ~60 yrs. Does it extend to Black Hole remnants?

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Function of the opacity, strongly metallicity dependent.

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Maximum mass:  $M_{\max} \lesssim 1.4M_{\odot}$  (Chandrasekhar mass)

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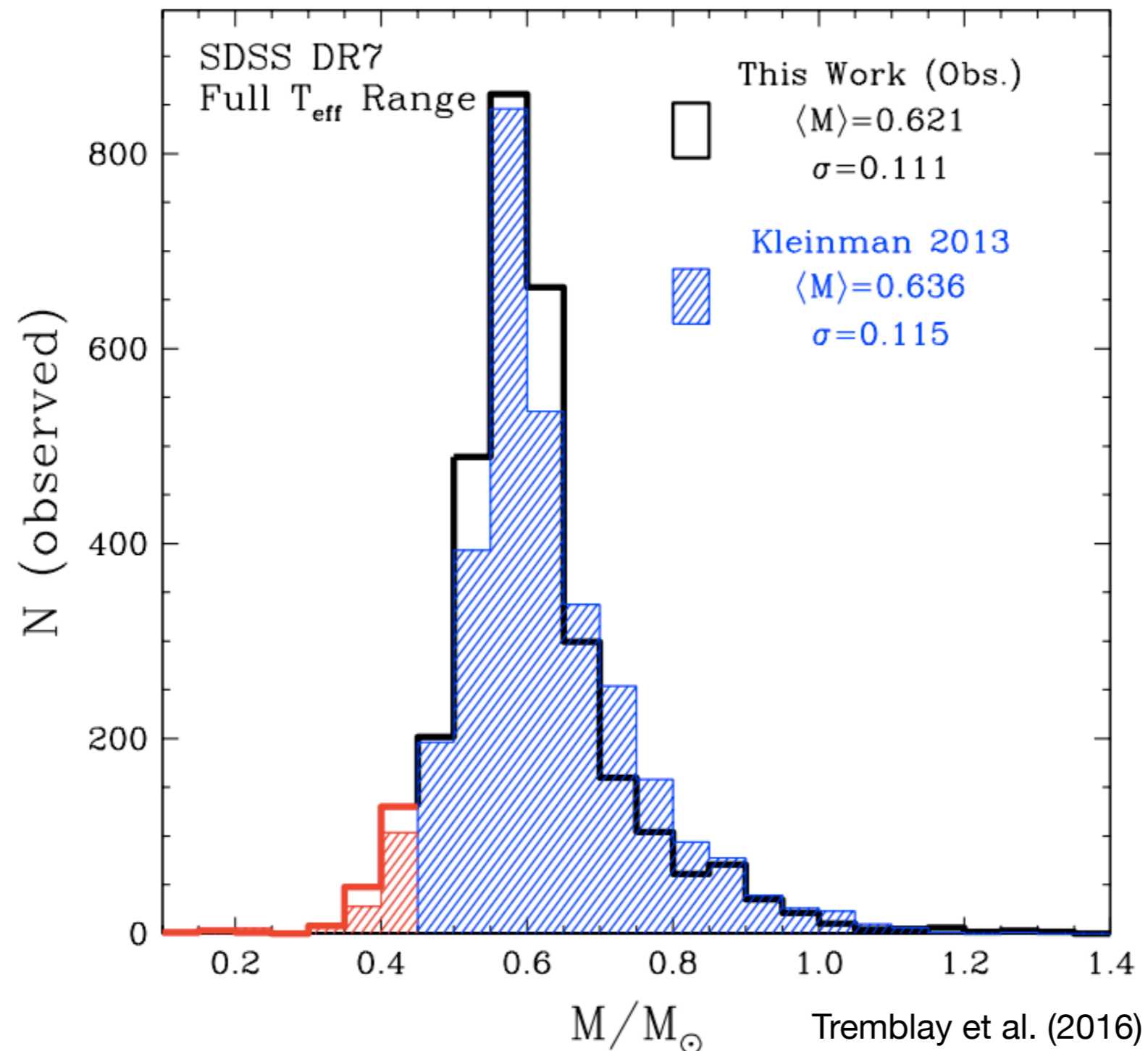
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White Dwarfs: mass distribution overlaps with stars

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# Mass Bounds of Stellar Remnants: Neutron Stars

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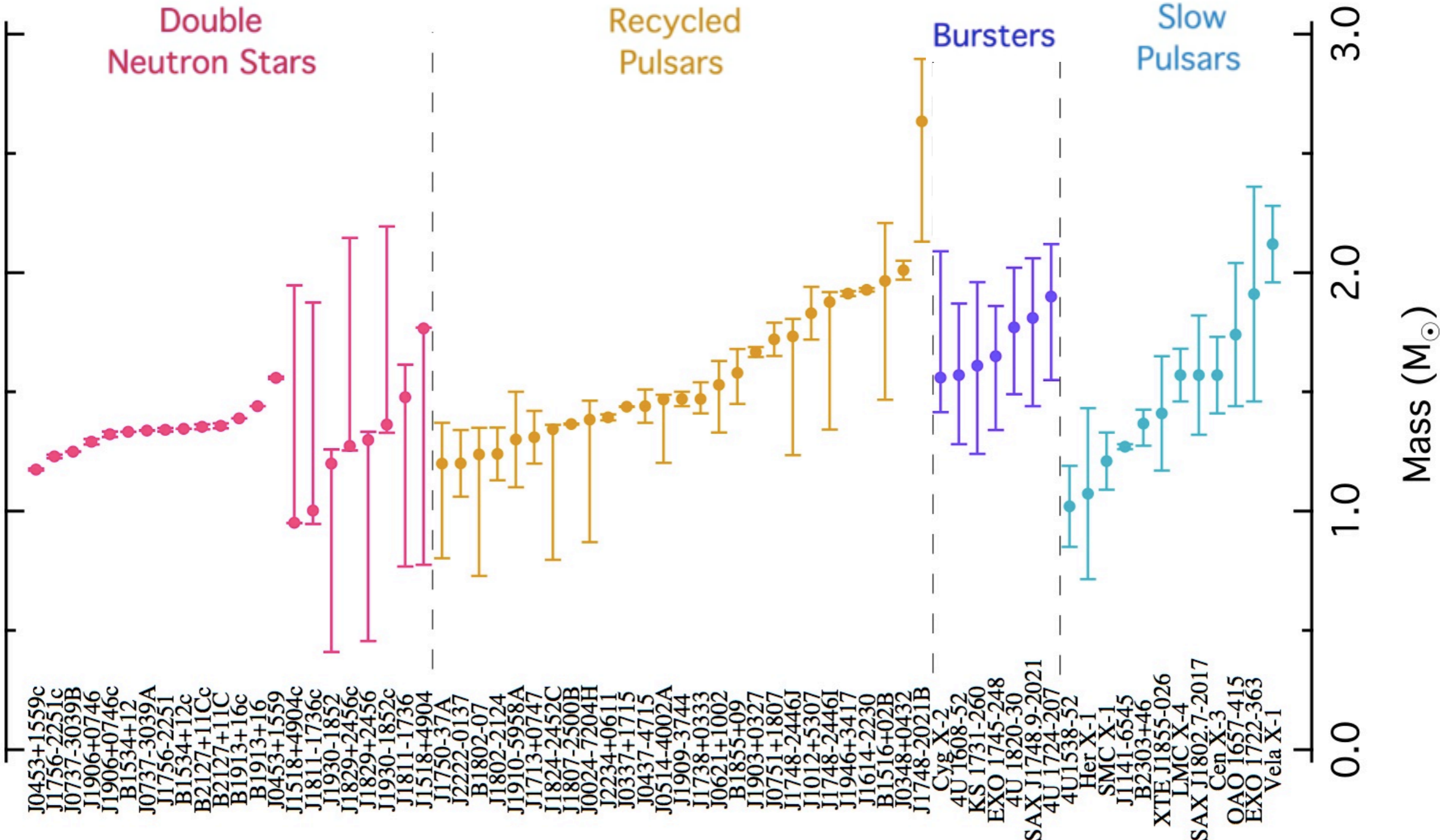
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# Mass Bounds of Stellar Remnants: Black Holes

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Black hole minimum mass: evidence for a gap at low masses?

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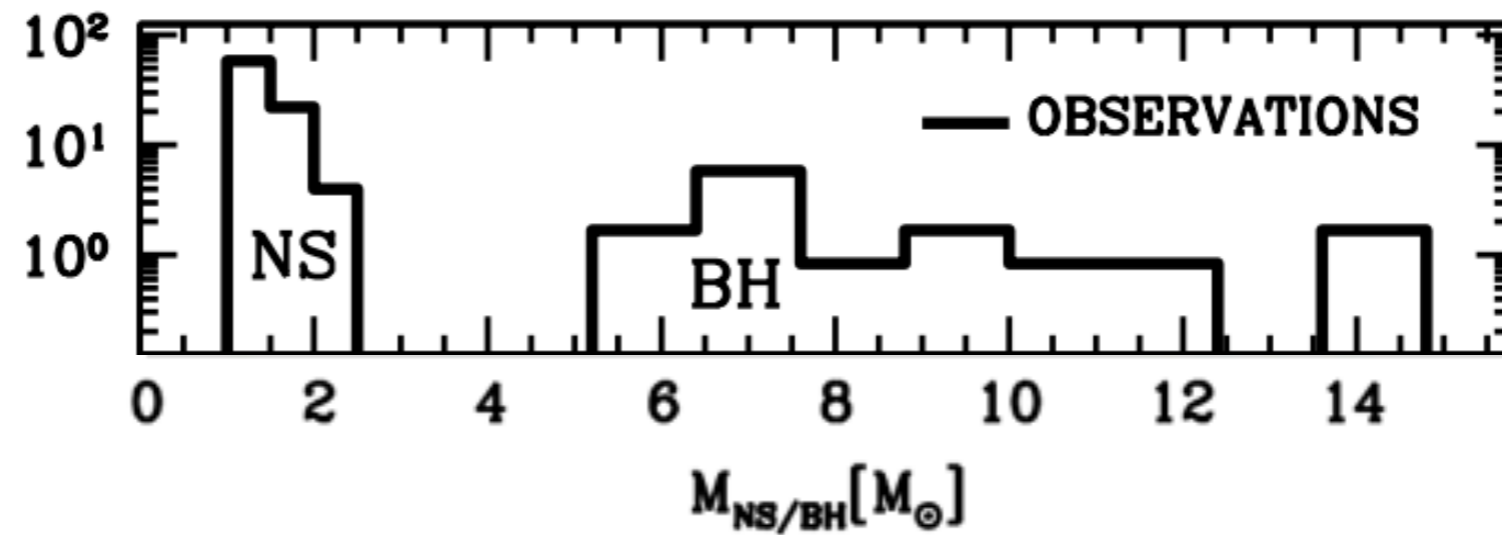
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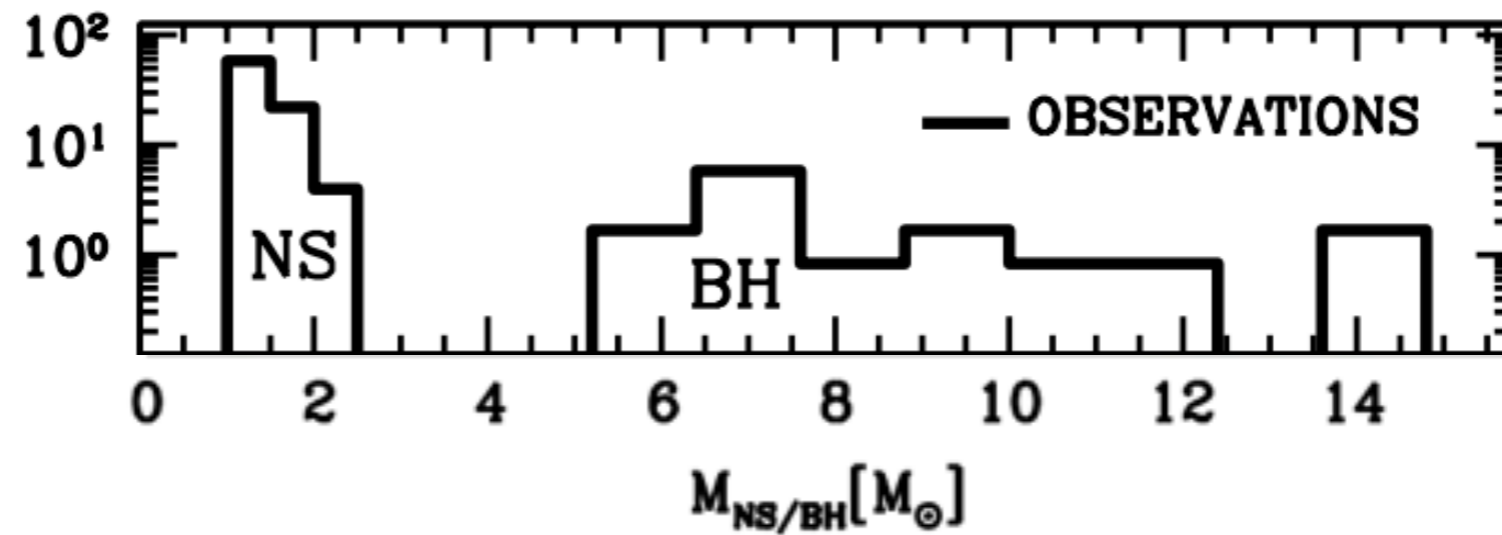


Belczynski et al.  
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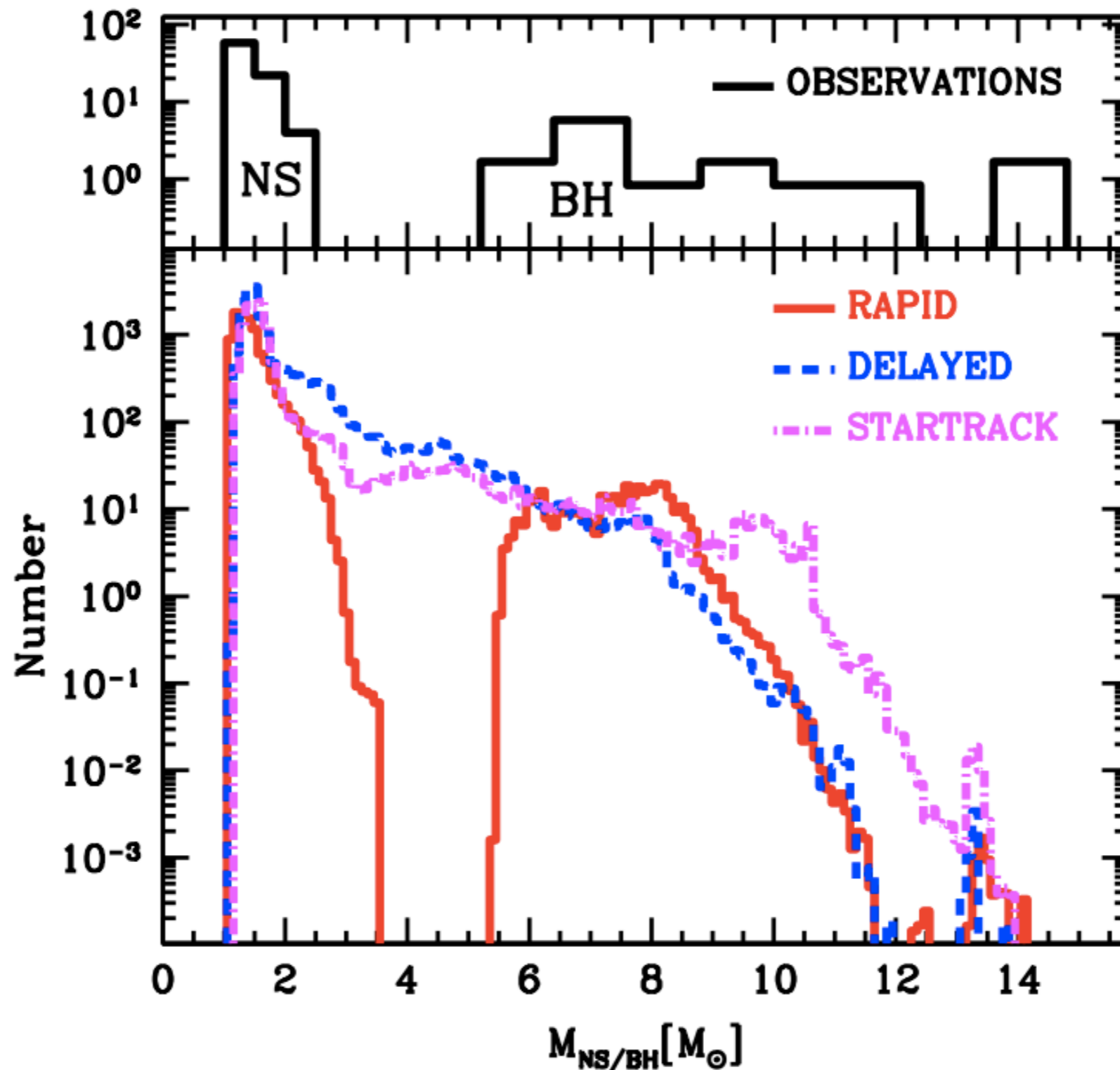
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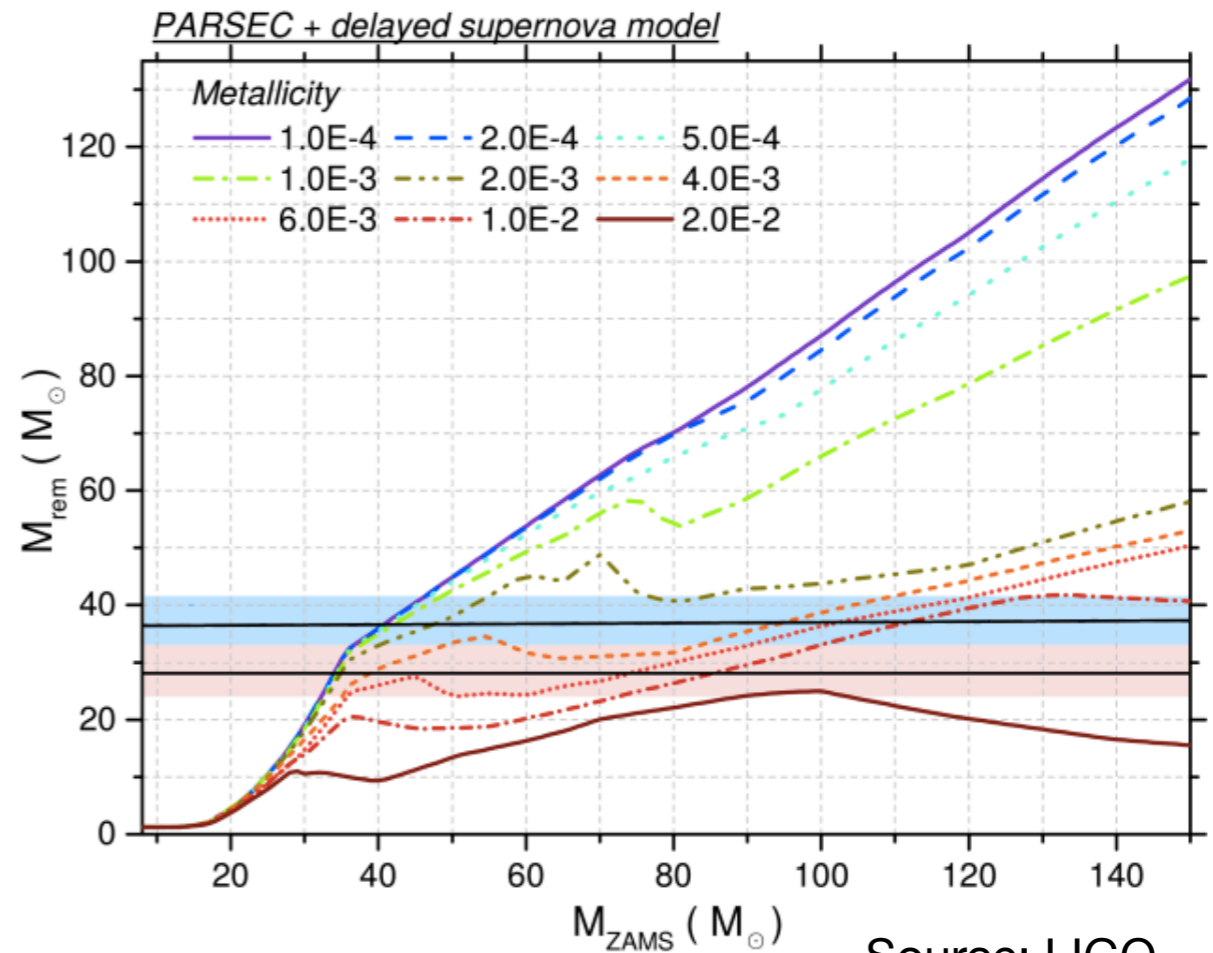
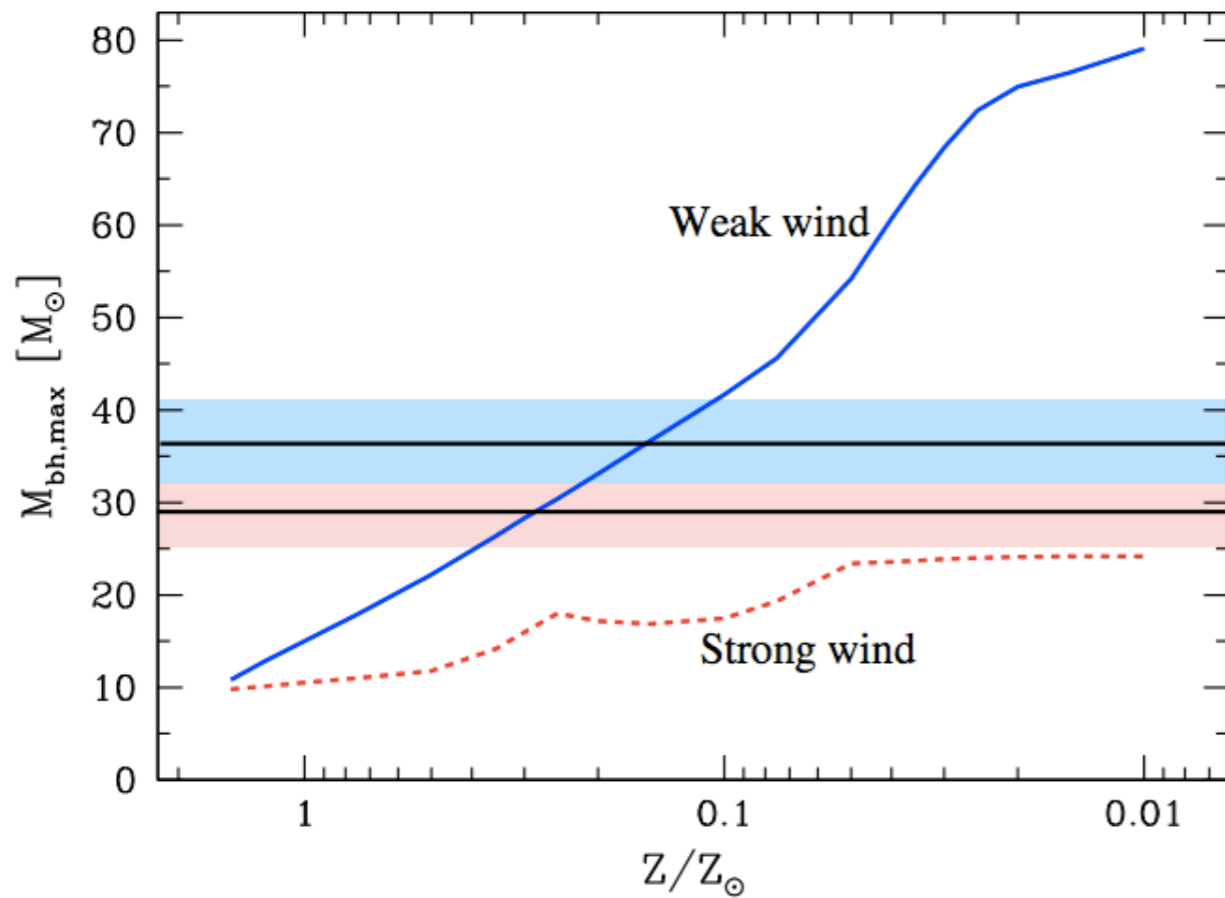
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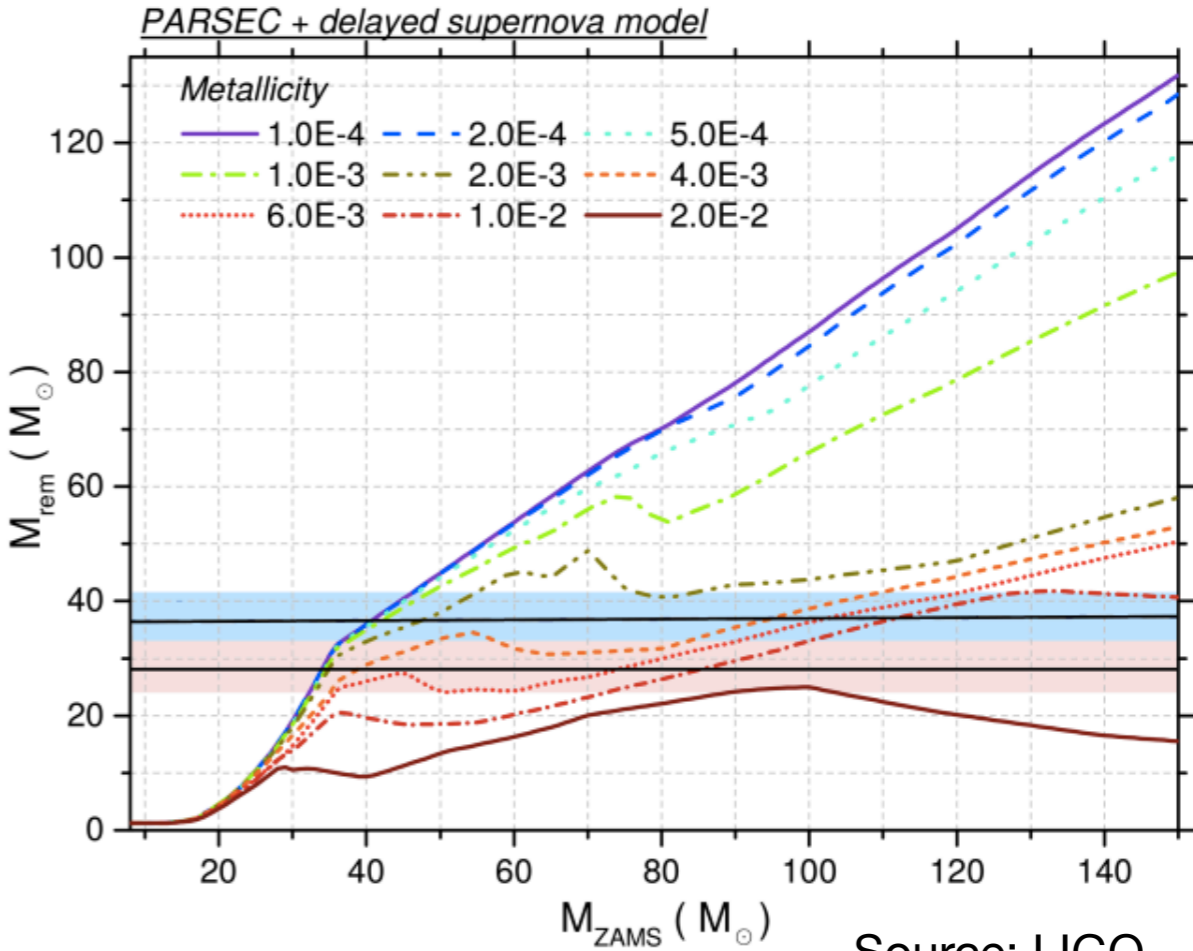
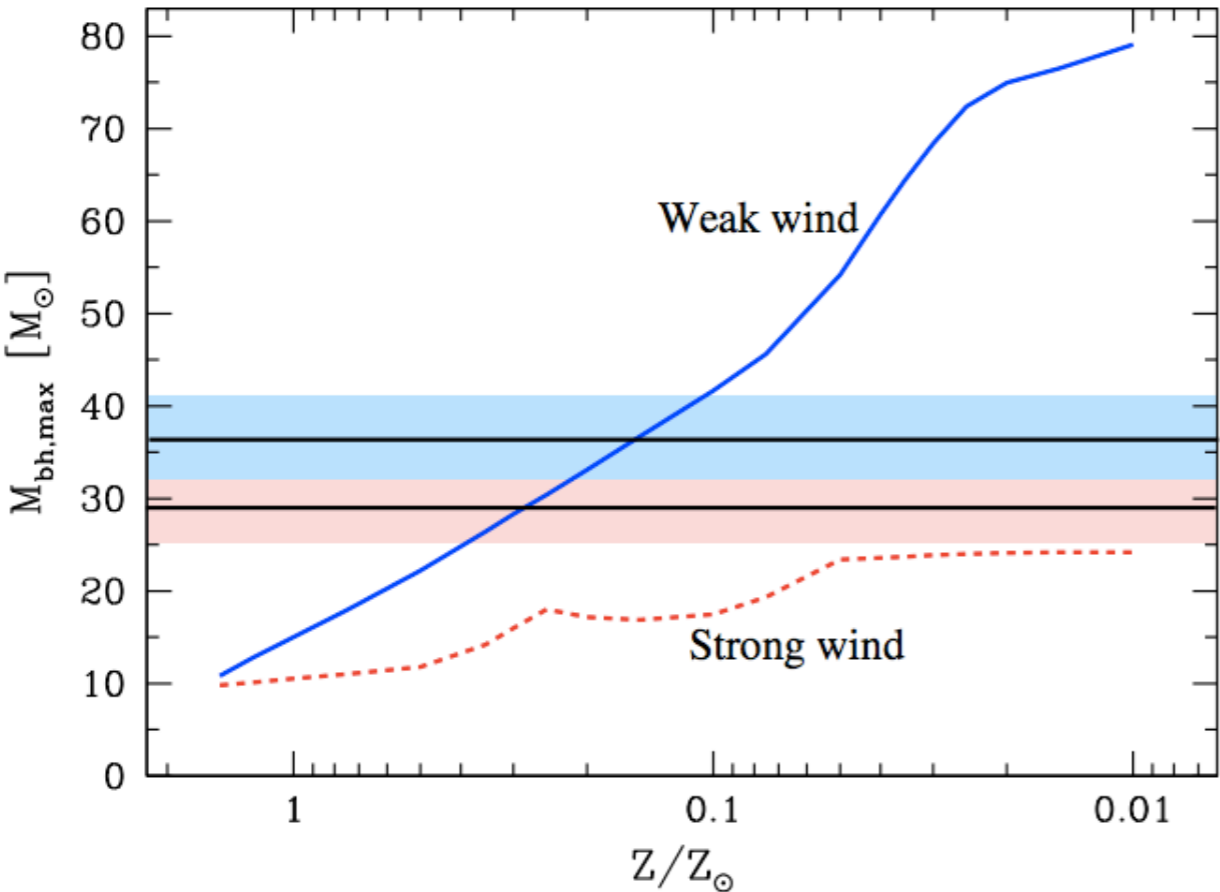


Source: LIGO

# Mass Bounds of Stellar Remnants: Black Holes

Black hole Maximum mass:

Set by wind-driven mass loss  $\longrightarrow$  model dependent

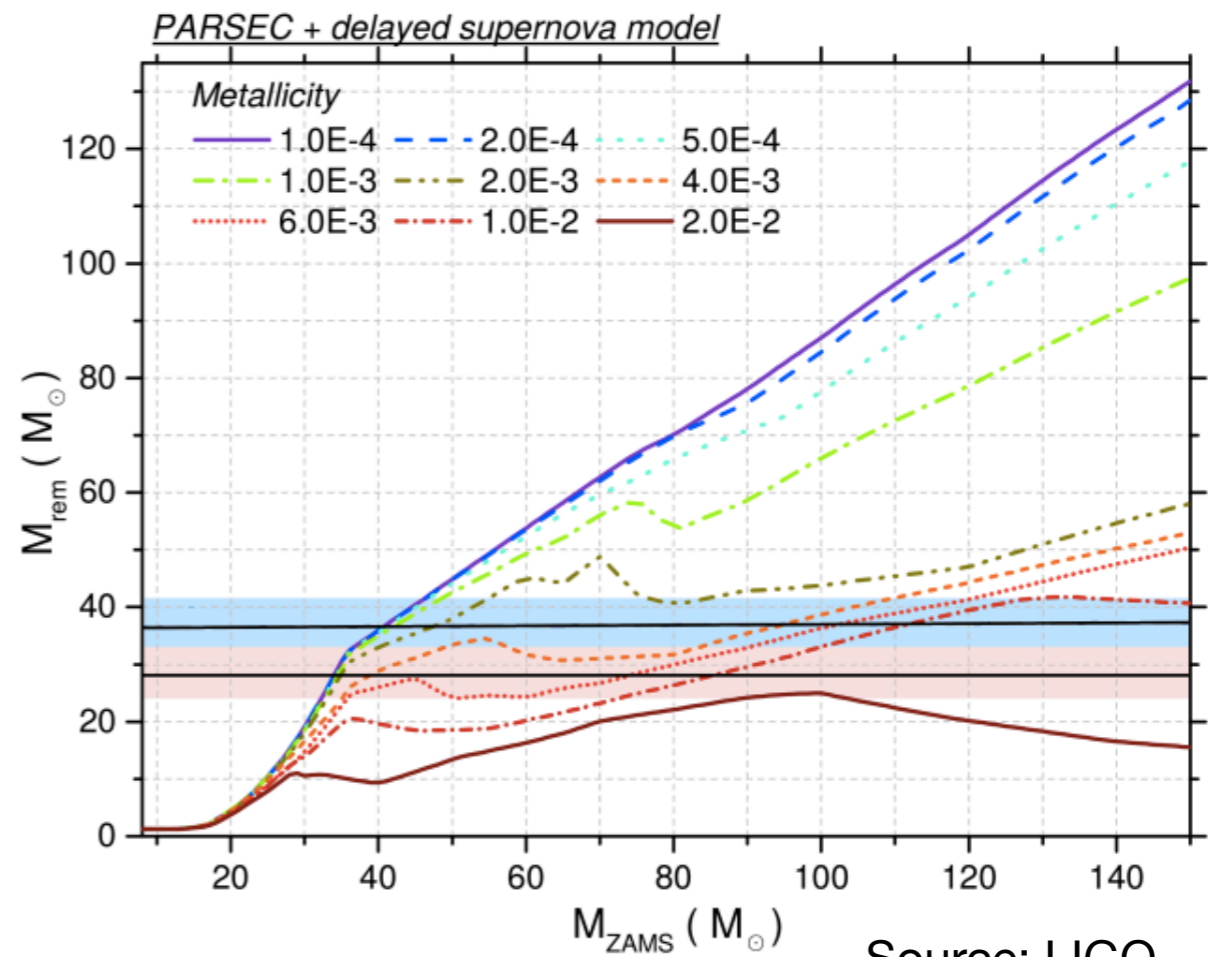
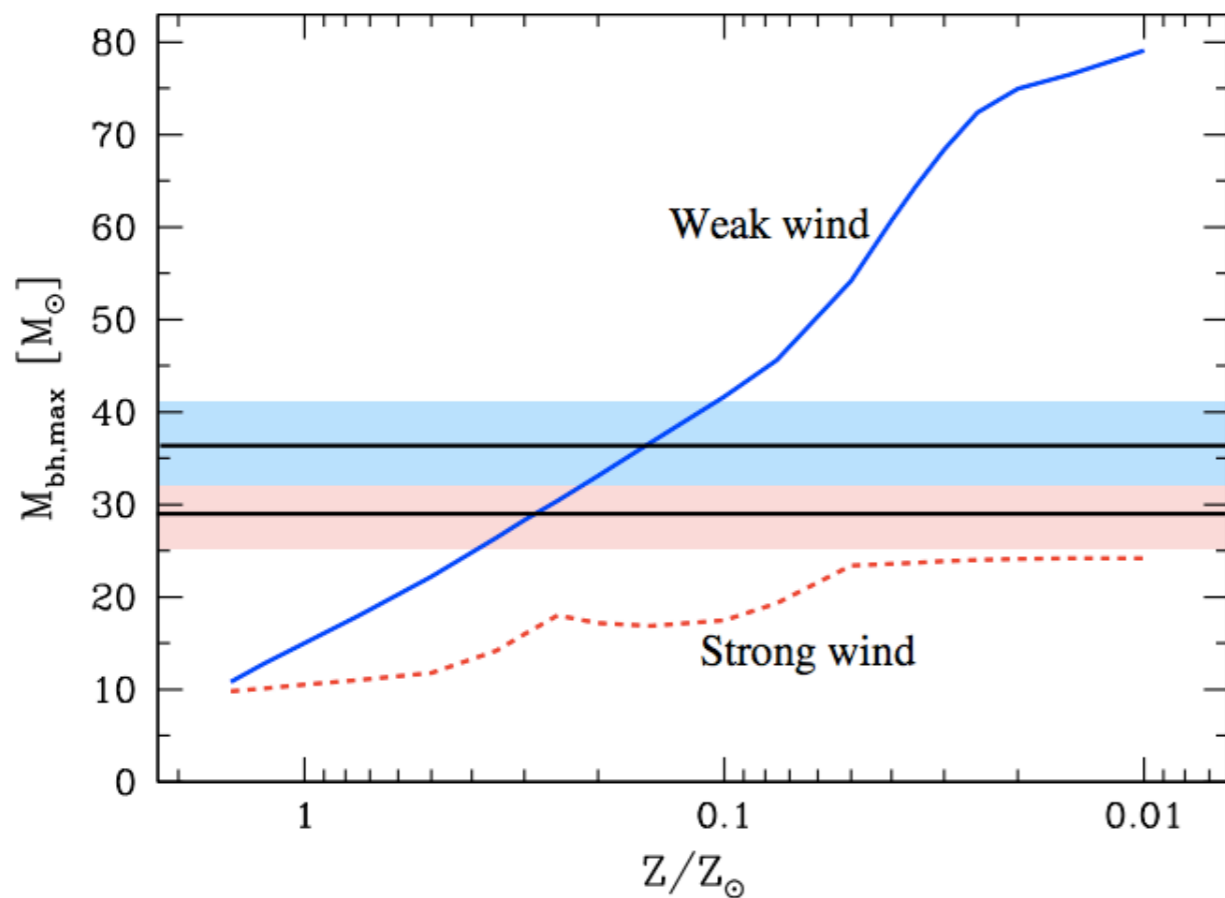


Source: LIGO

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(strongly) metallicity dependent

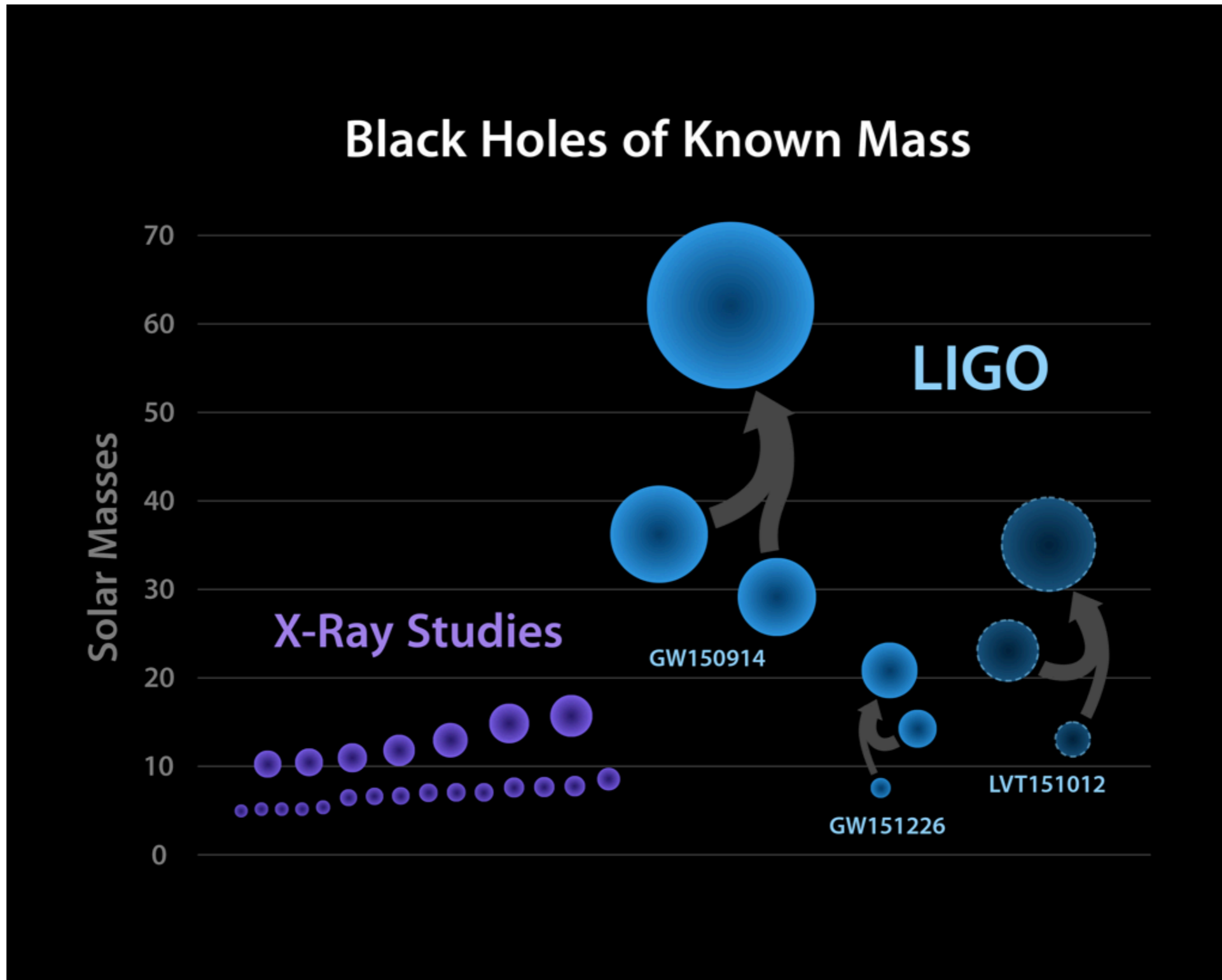


Source: LIGO

# The Black Hole Mass Function Today

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Source: LIGO

# The Black Hole Mass Function from GWs

(EDK et al., arXiv:1611.01157)

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Stellar black hole mass function:

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Ansatz:

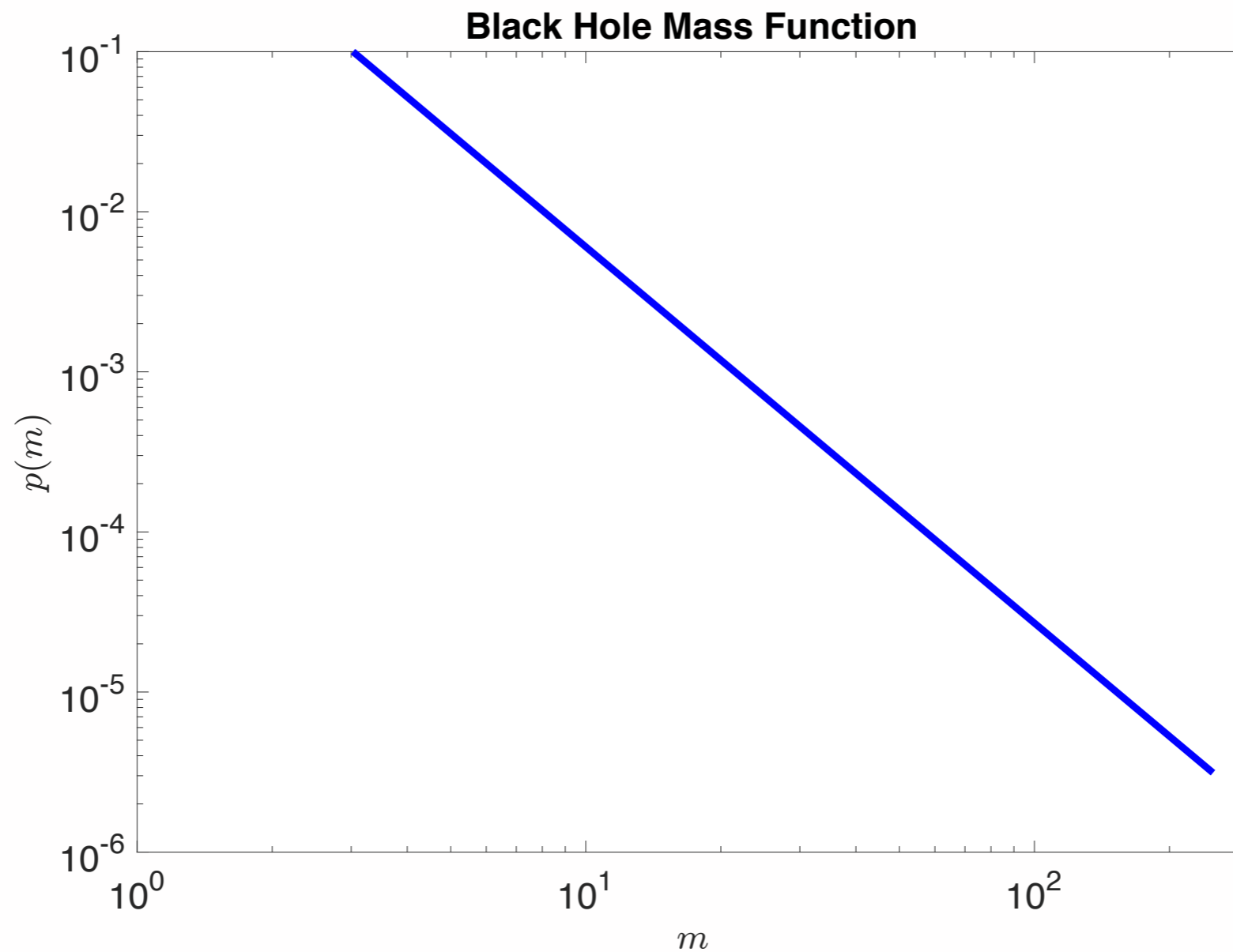
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Stellar black hole mass function:

Ansatz:  $p(m) \propto m^{-\alpha}$



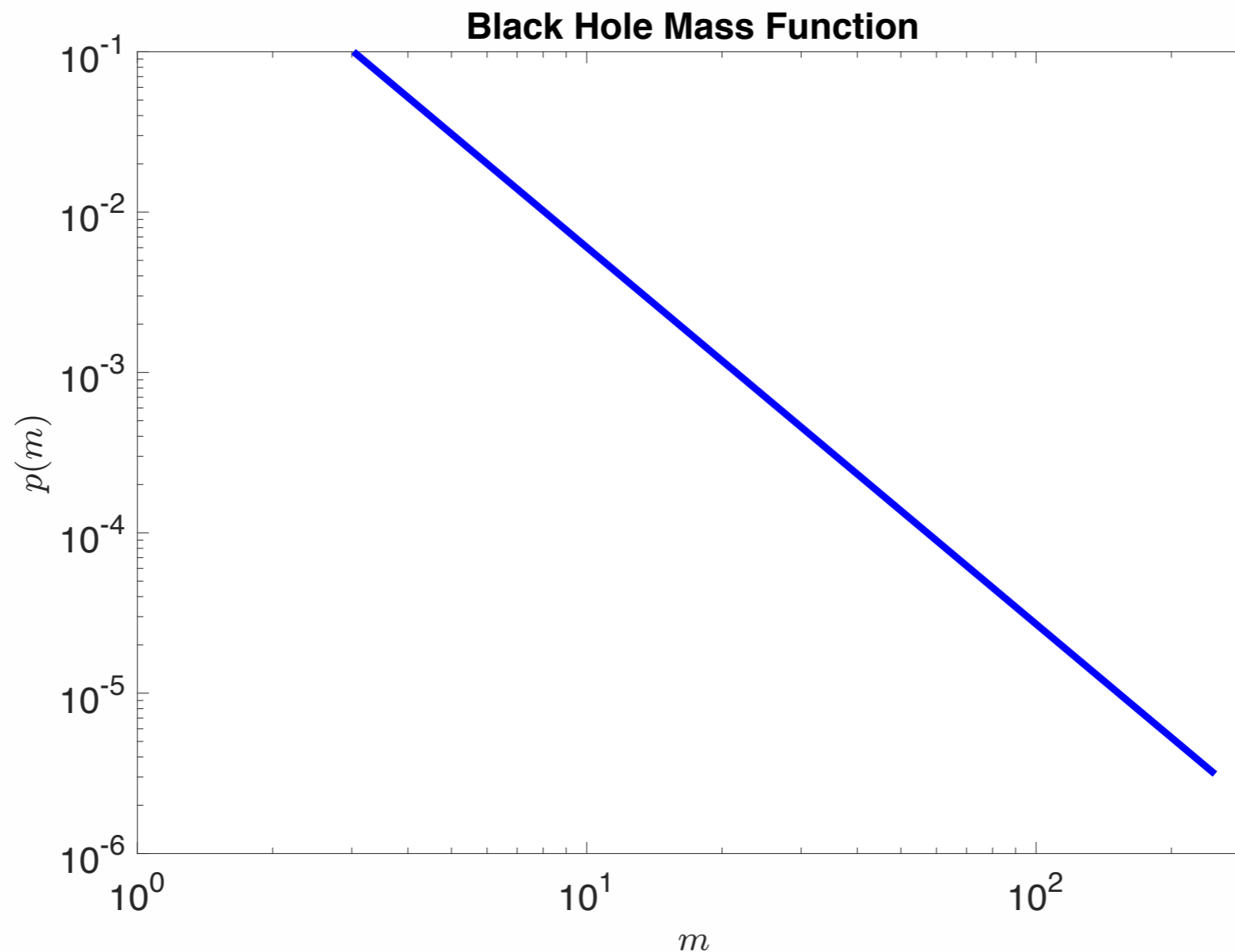
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 $\alpha = 2.35$



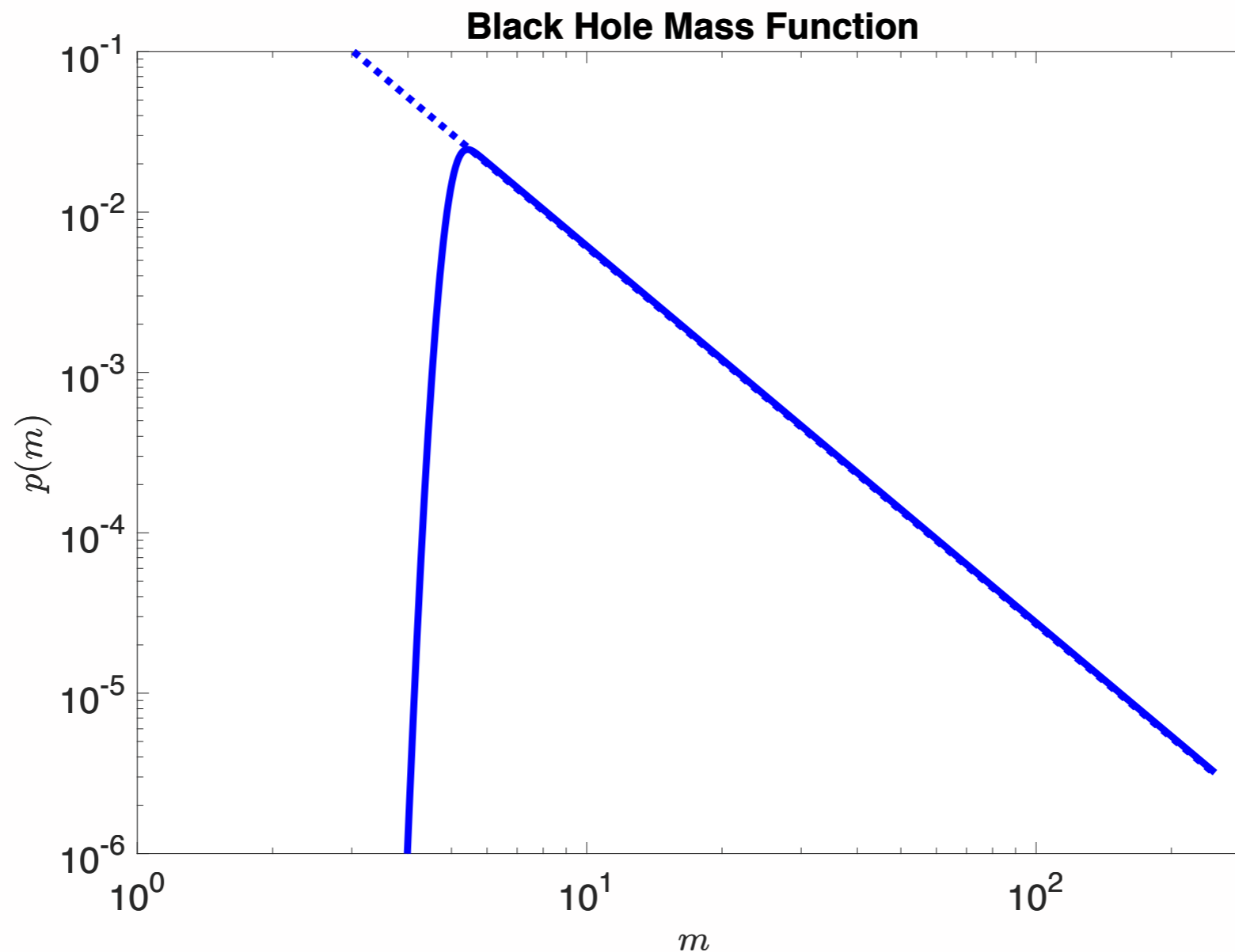
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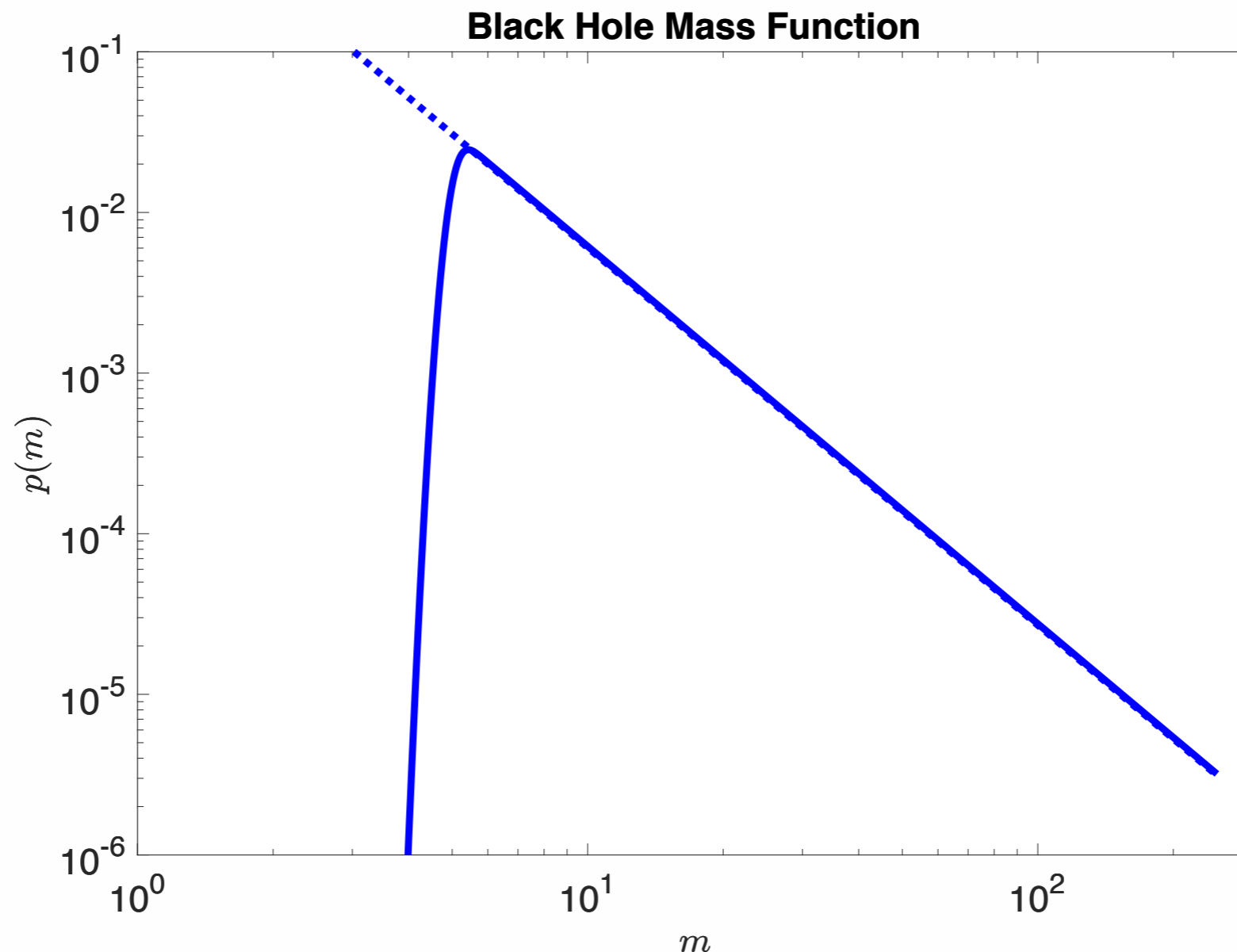
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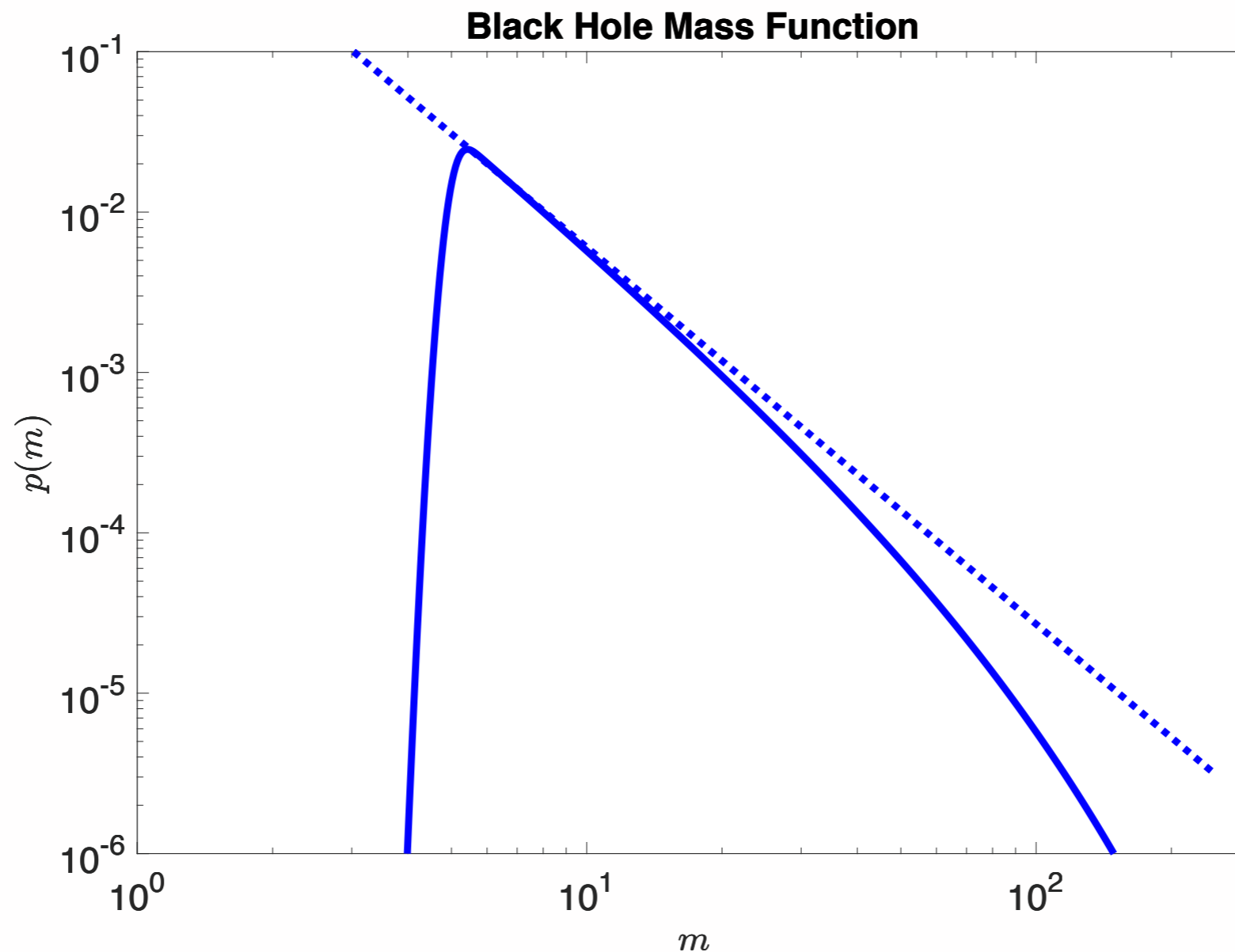
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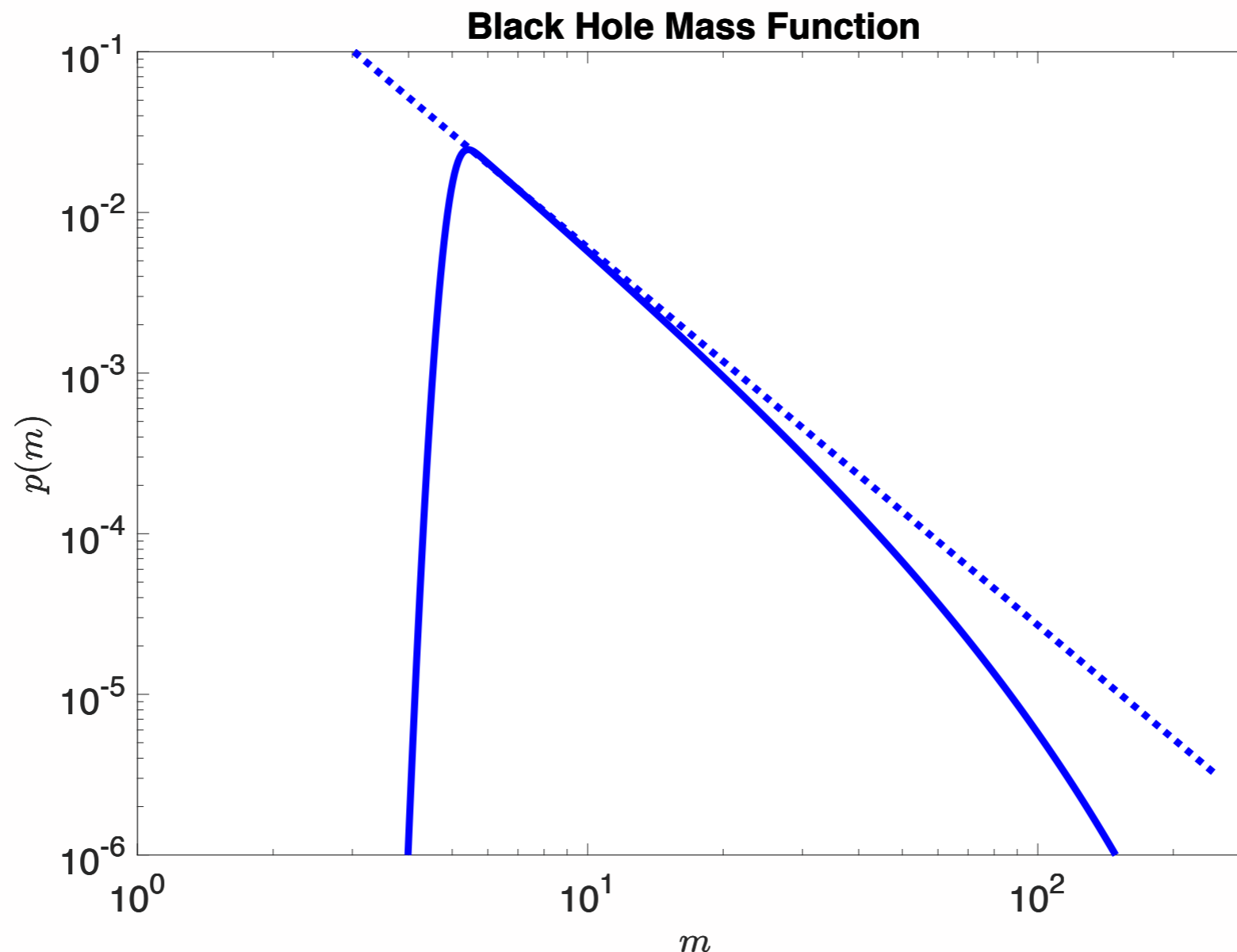
Stellar black hole mass function:

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NS vs. BH  
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Mass Cutoff  
 $M_{\text{Cap}} \sim 60?$

Ansatz:  $p(m) \propto m^{-\alpha} \mathcal{H}(m - m_{\text{Gap}}) e^{-m/m_{\text{Cap}}}$



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Mass measurement error in *observed* BHMF:

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$$\text{Ansatz: } m_{\text{obs}} = xm_{\text{th}} \text{ where: } x \sim \mathcal{N}(1, \sigma^2) \text{ and: } P_G = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-1)^2/2\sigma^2}$$

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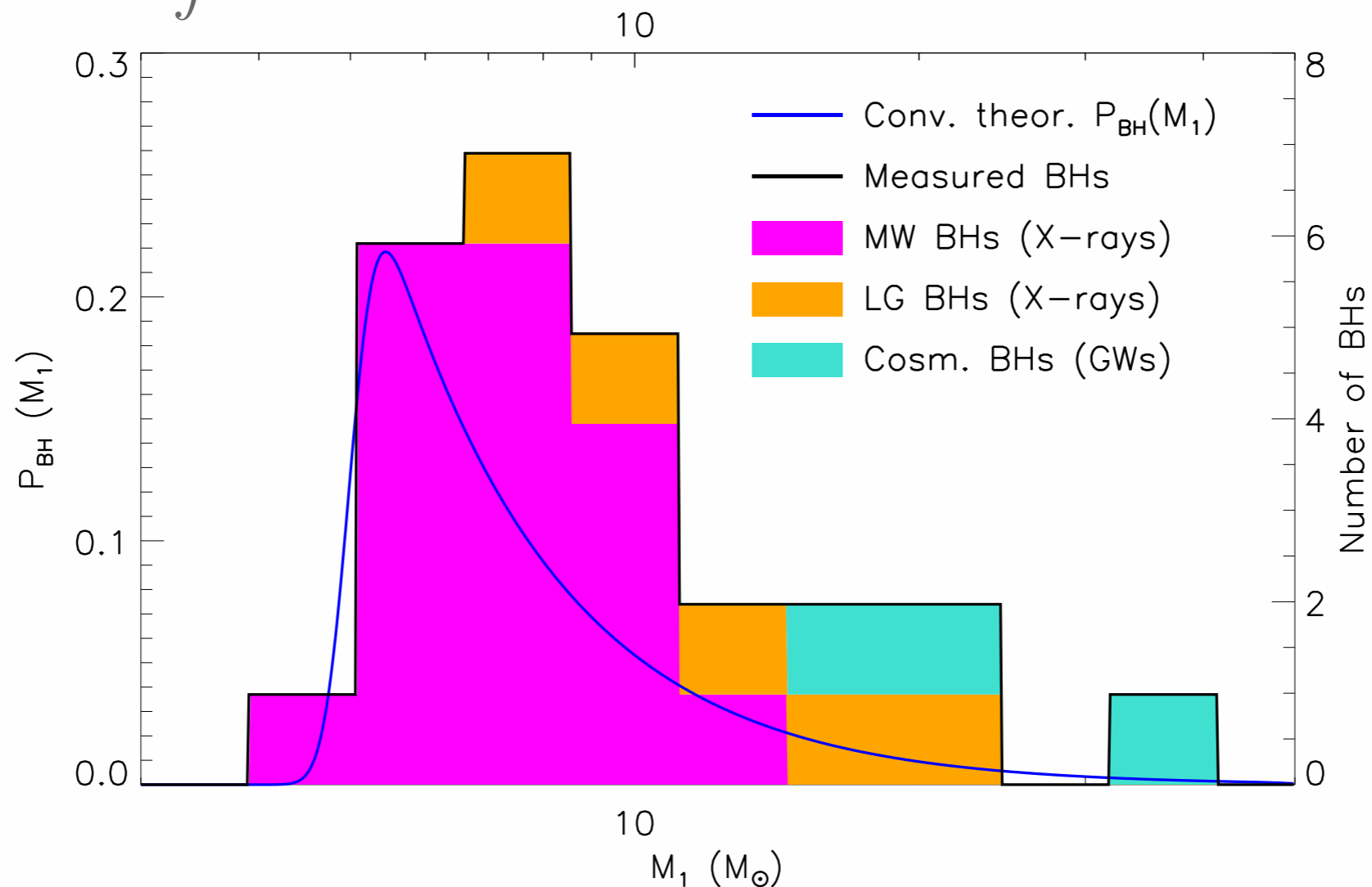
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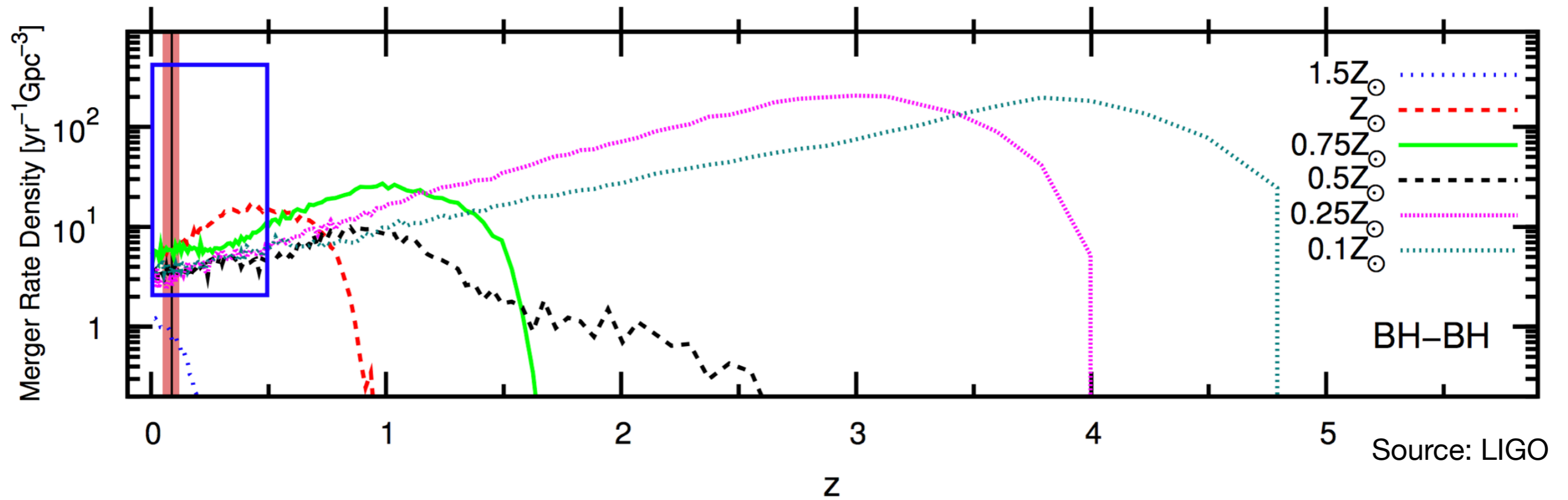
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The merger rate:

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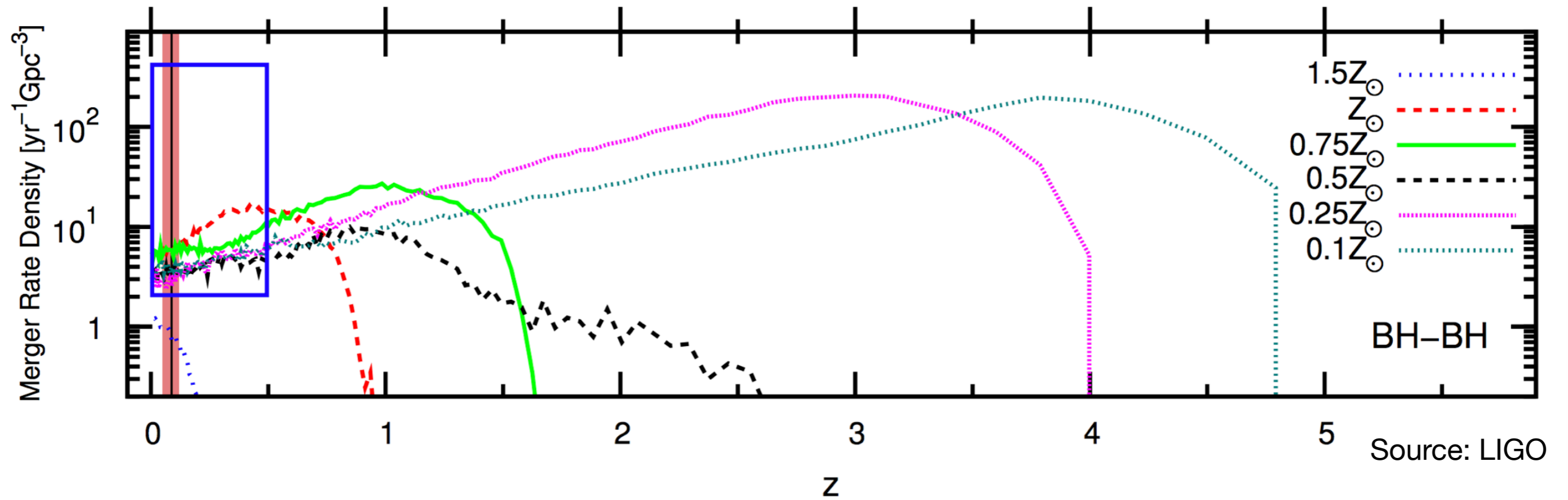
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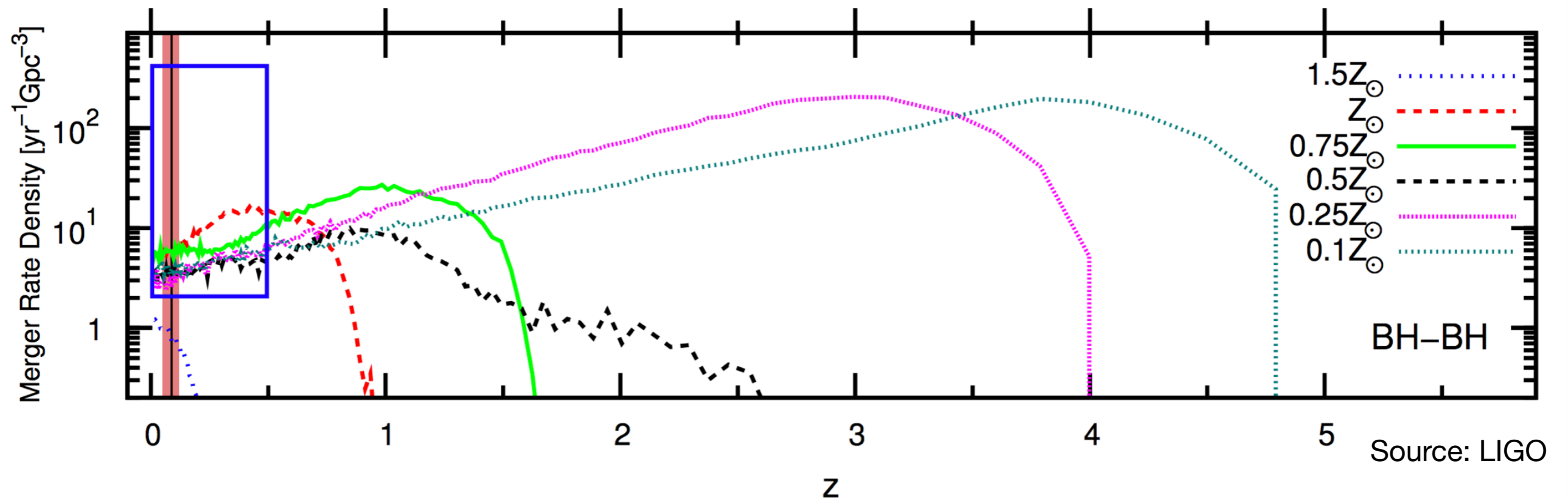


This depends mainly on:

# The Black Hole Mass Function from GWs

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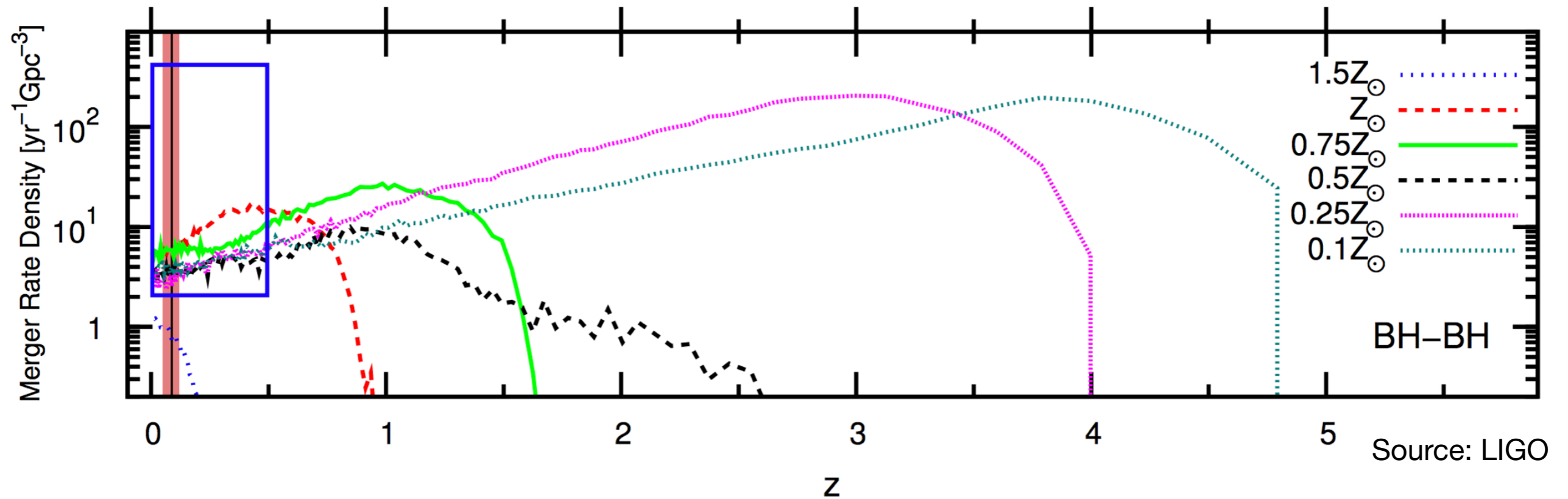
This depends mainly on:

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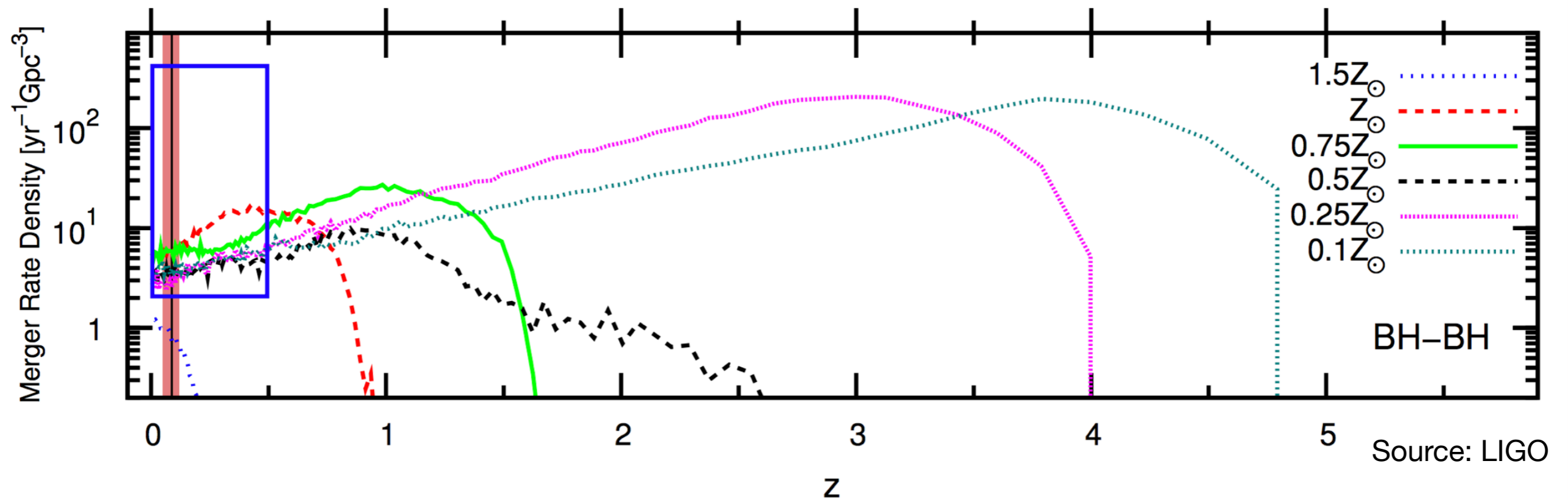
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Ansatz:  $R(z) = R_a (1 + z)^{R_b}$

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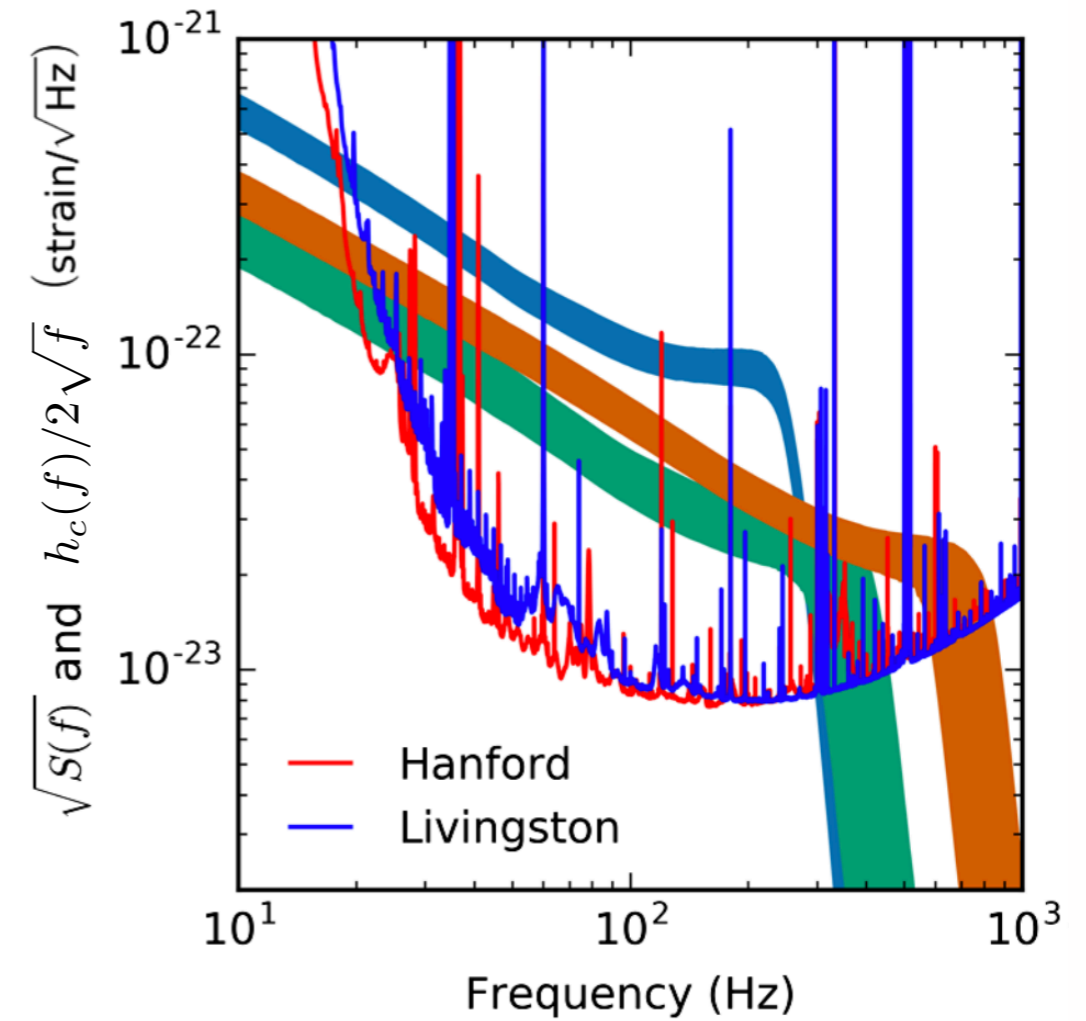
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Signal-to-noise ratio:

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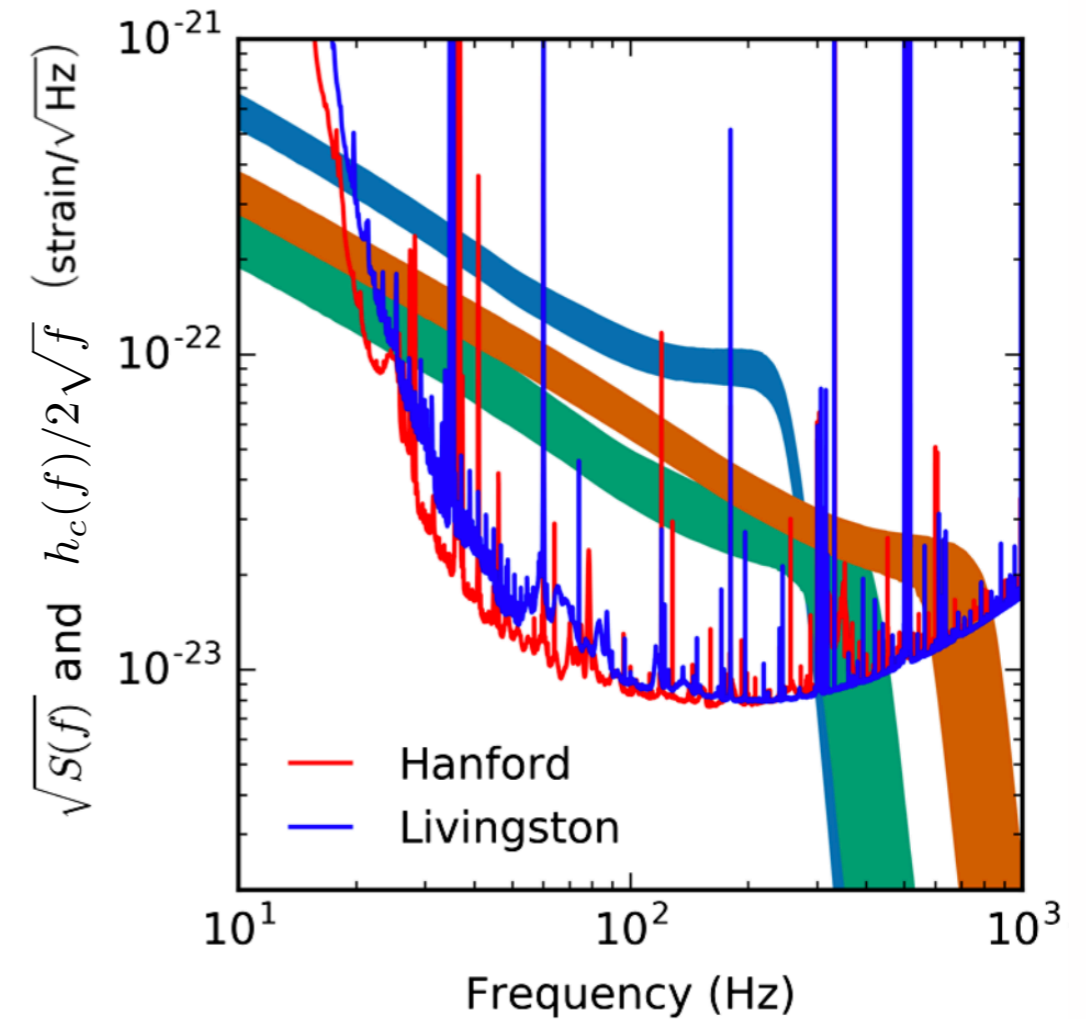


# The Black Hole Mass Function from GWs

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Signal-to-noise ratio:

$$(S/N)^2 = \frac{4}{5} \int_{f_{\min}}^{f_{\max}} \frac{h_c^2(f)}{4f S_n(f)} d \ln f$$



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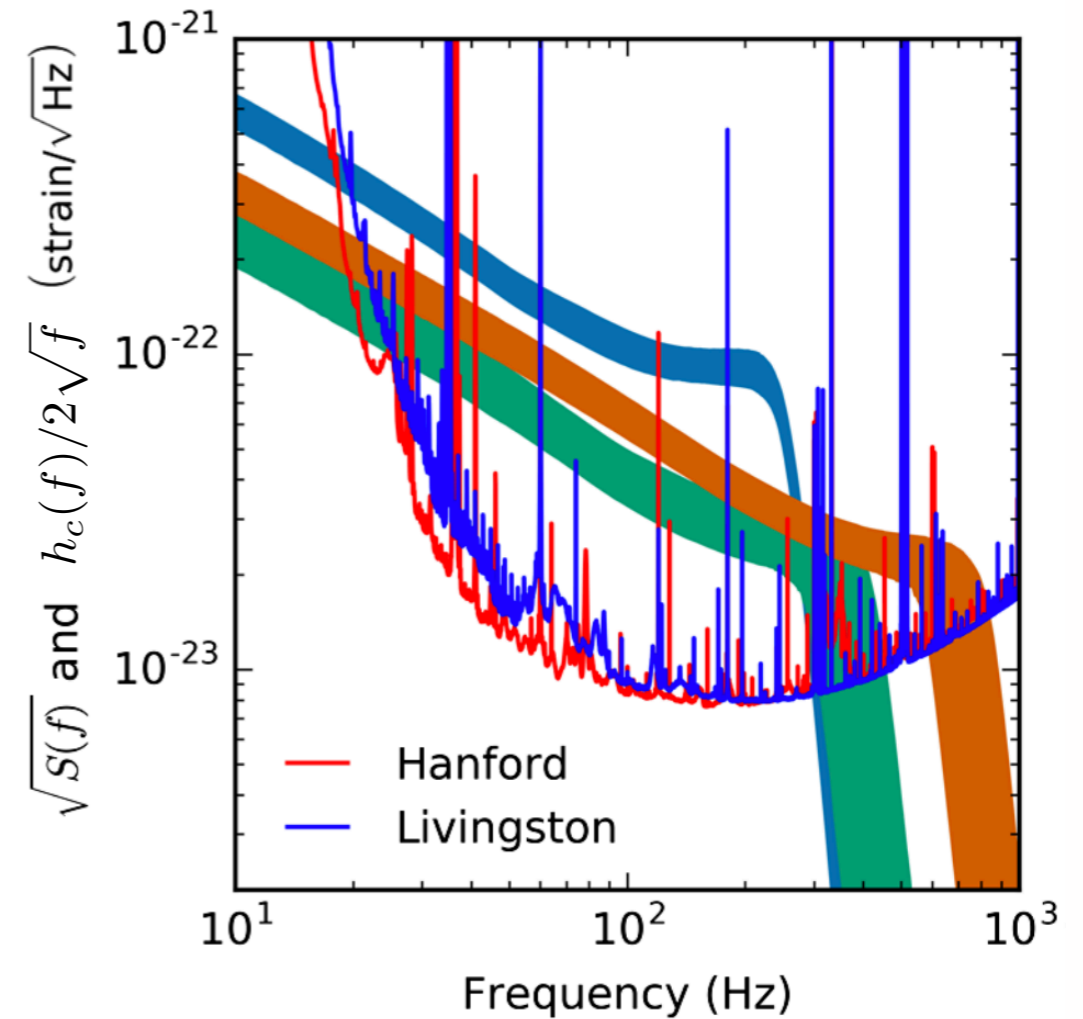
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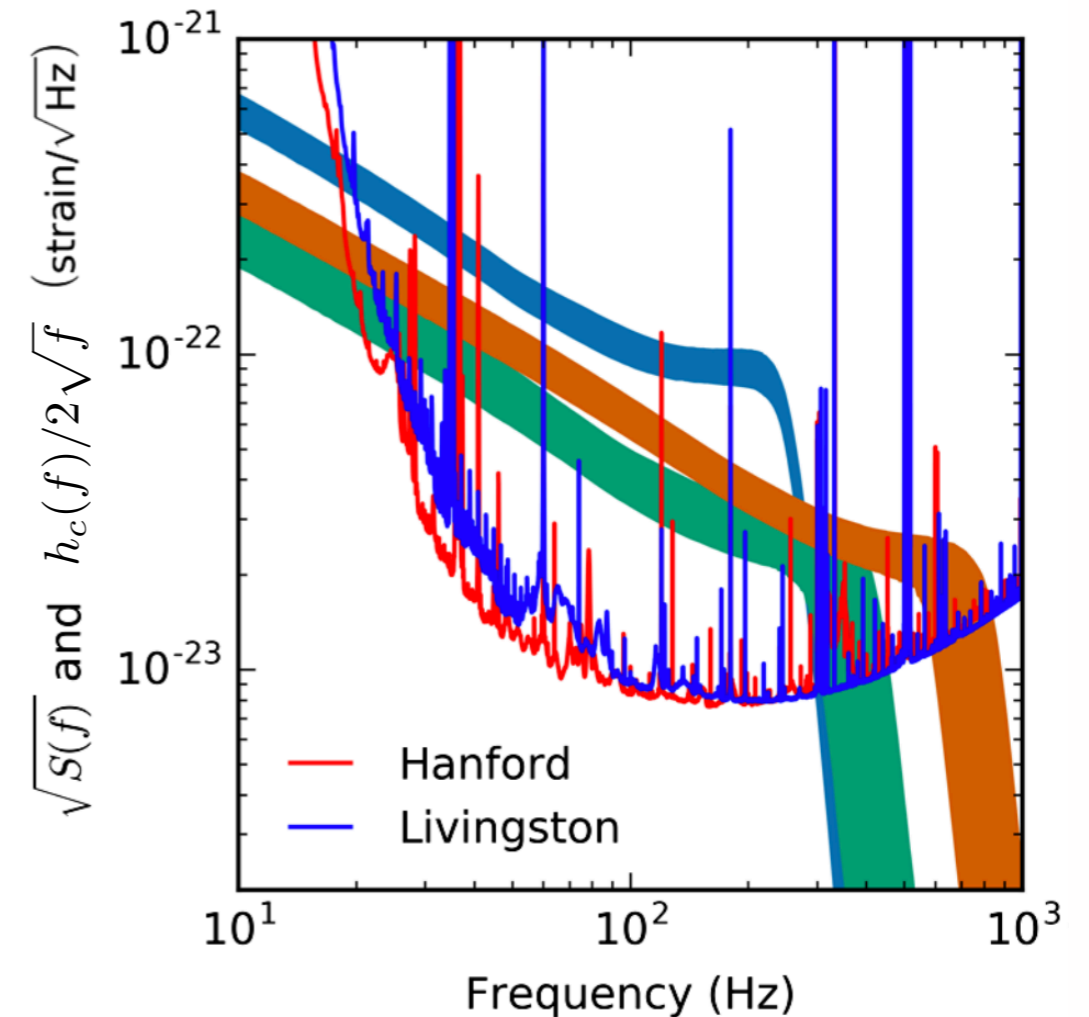
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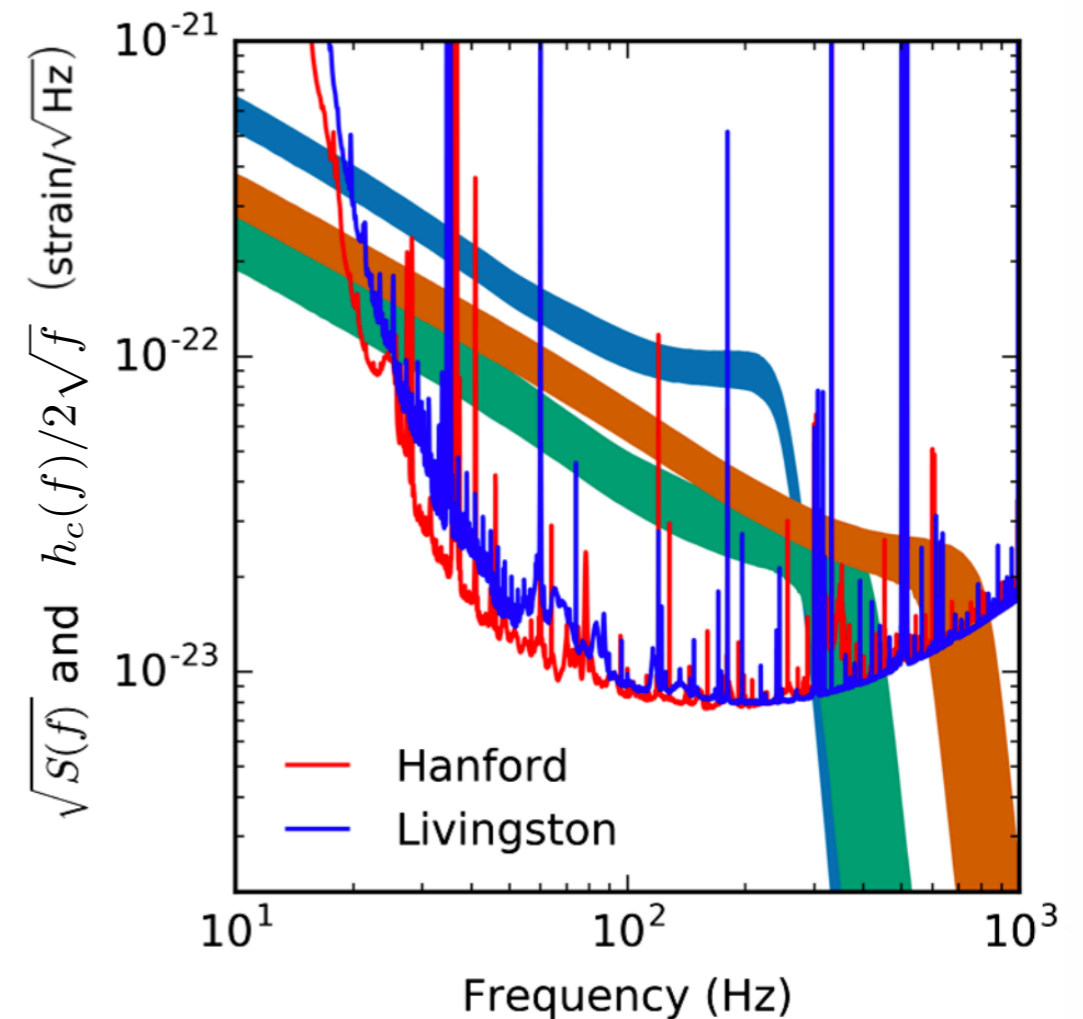
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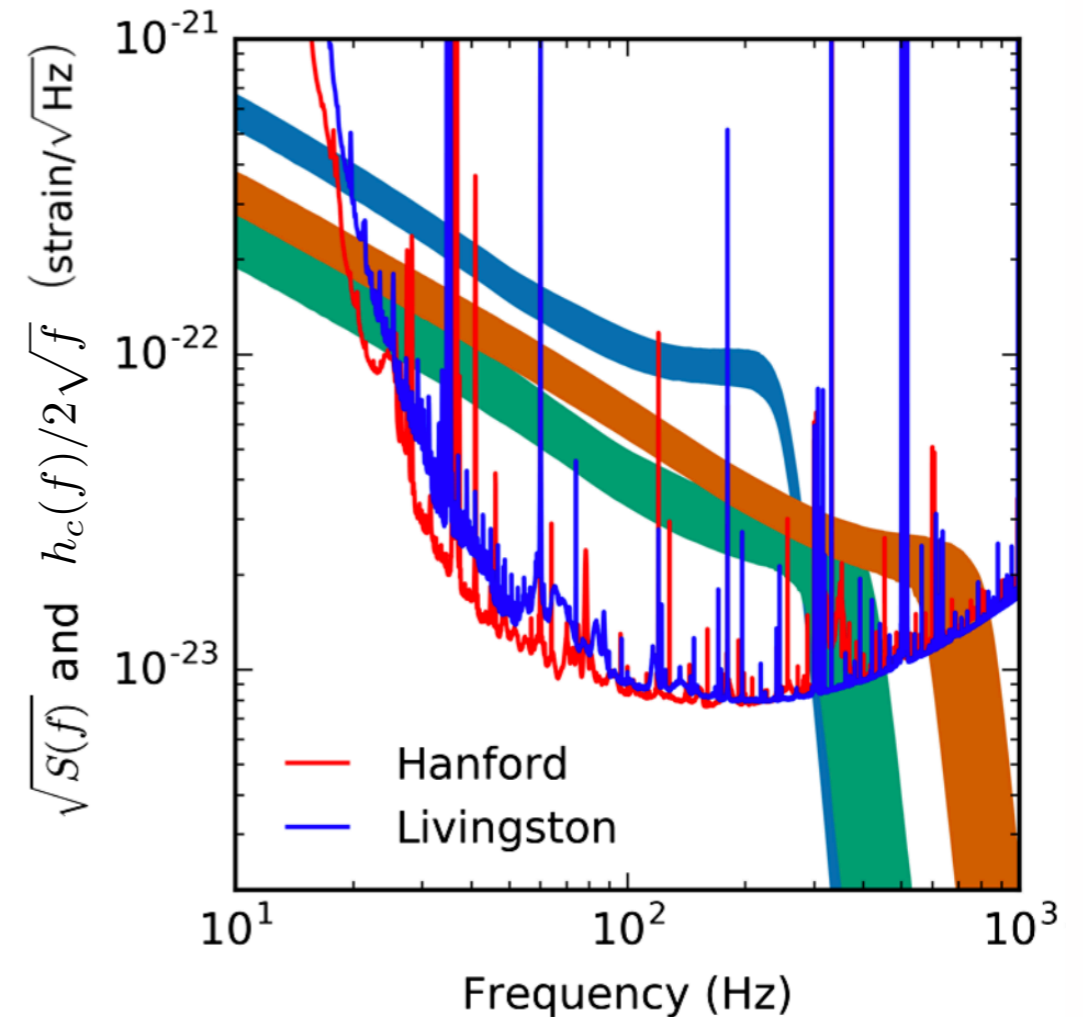
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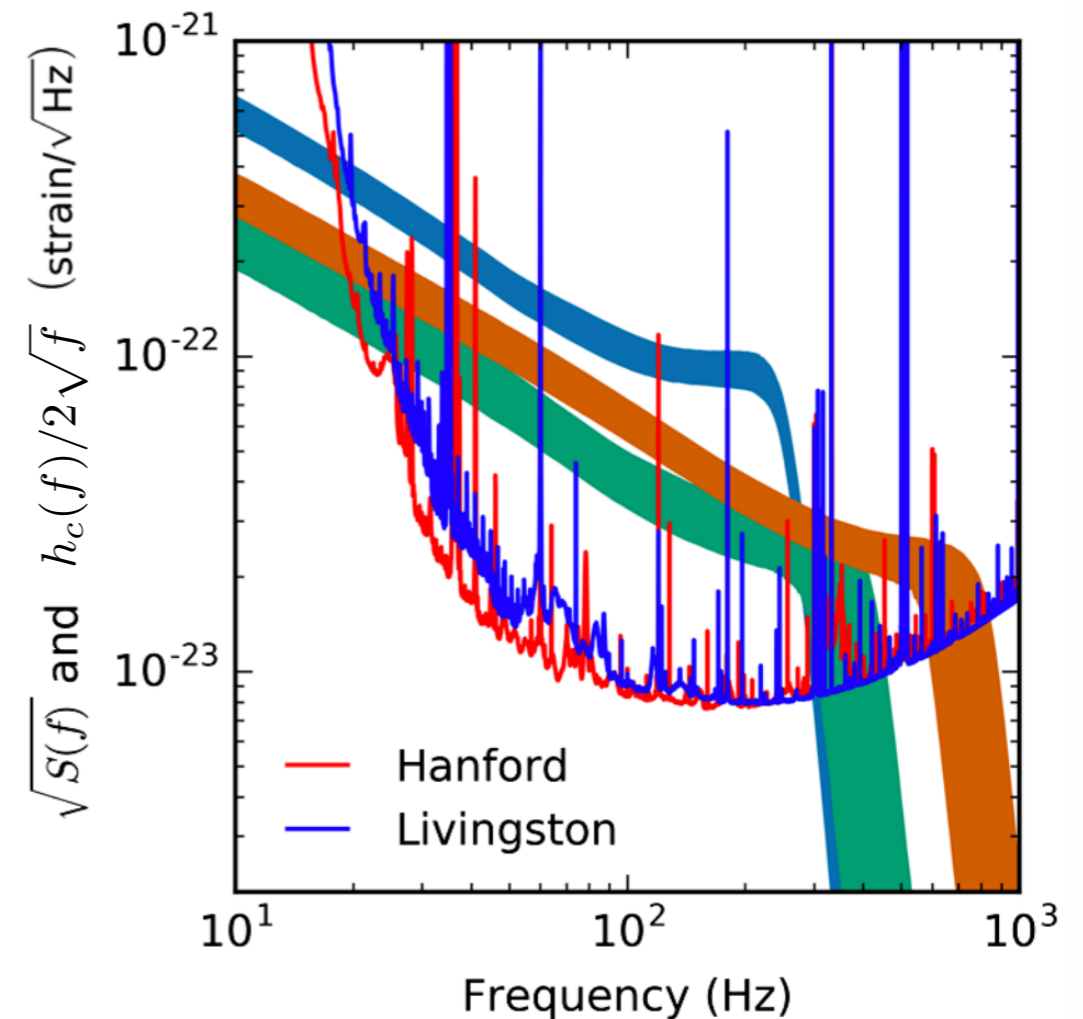
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For GW150914, the maximum distance with aLIGO is:  $z_{\max}^{\text{GW150914}} \sim 0.7$



# The Black Hole Mass Function from GWs

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Binary black hole mass function:

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Heavier mass:  $p(m_1) \propto m_1^{-\alpha} \mathcal{H}(m_1 - m_{\text{Gap}}) e^{-m_1/m_{\text{Cap}}}$

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The number of black holes observed:

$$\frac{dN(m_1)}{dm_1} = 4\pi A_{m_1} p(m_1) \int_{m_{\text{gap}}}^{m_1} A_{m_2} p(m_2) dm_2 \int_0^{z_{\text{max}}(m_1, m_2)} \frac{c\chi(z)^2 R(z)}{(1+z)H(z)} dz$$

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$$N_i = \int_{m_{\text{min},i}}^{m_{\text{max},i}} \frac{dN(m_1)}{dm_1} dm_1$$

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Assume Poisson noise in each bin.

# The Black Hole Mass Function from GWs

(EDK et al., arXiv:1611.01157)

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Observed mass spectrum with 6 years of advanced LIGO data:

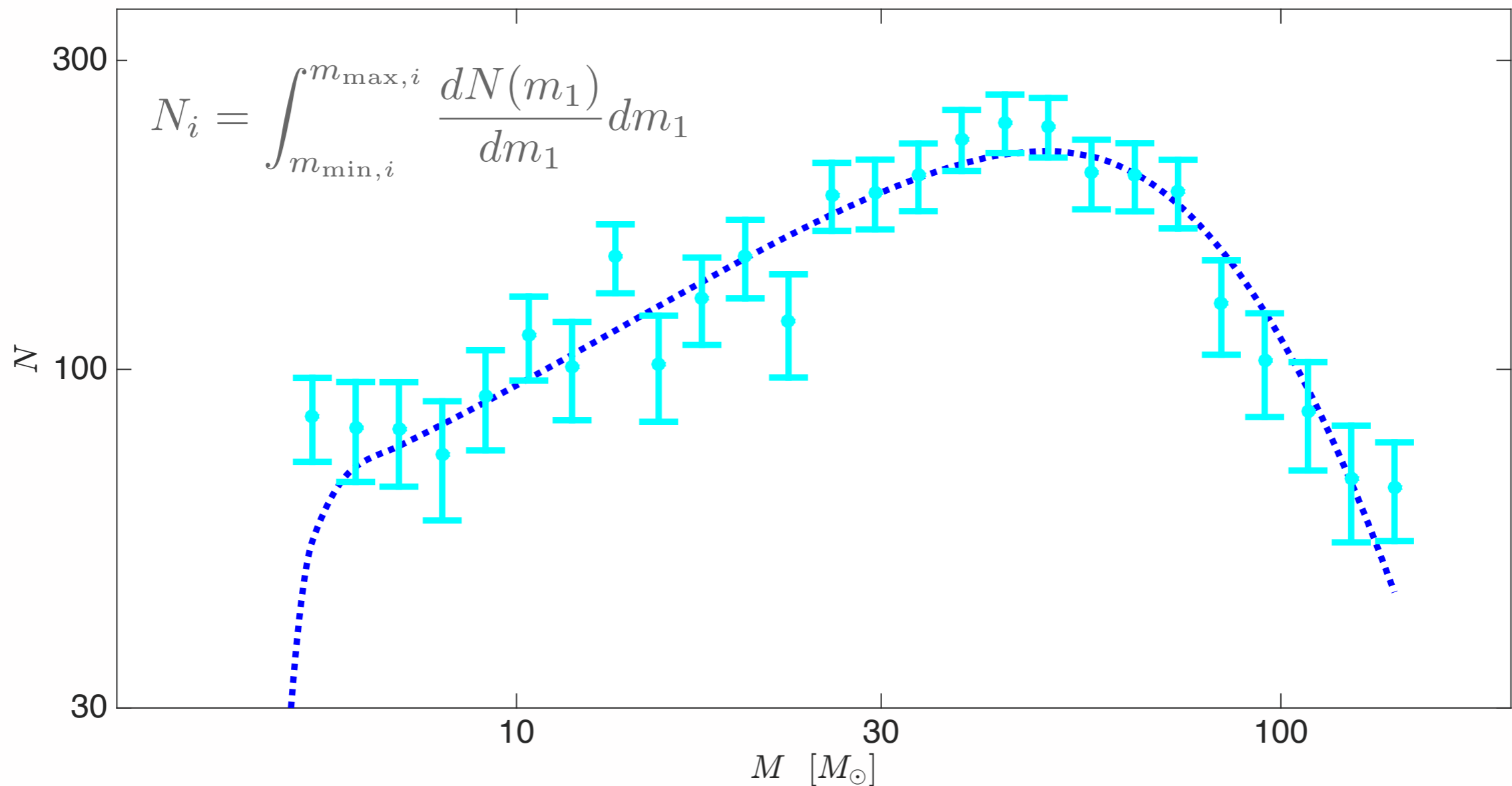
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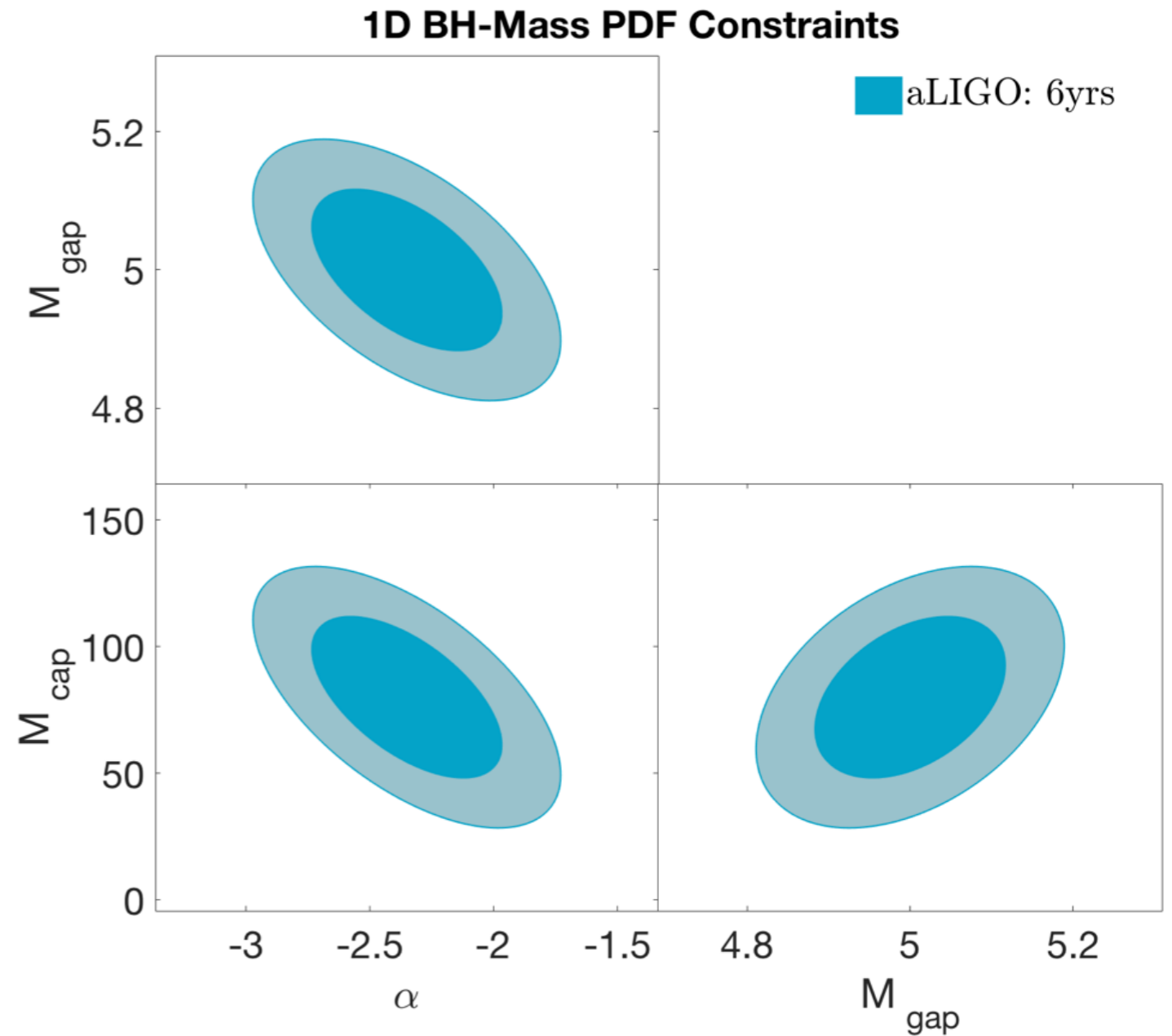
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Probing the mass function parameters:

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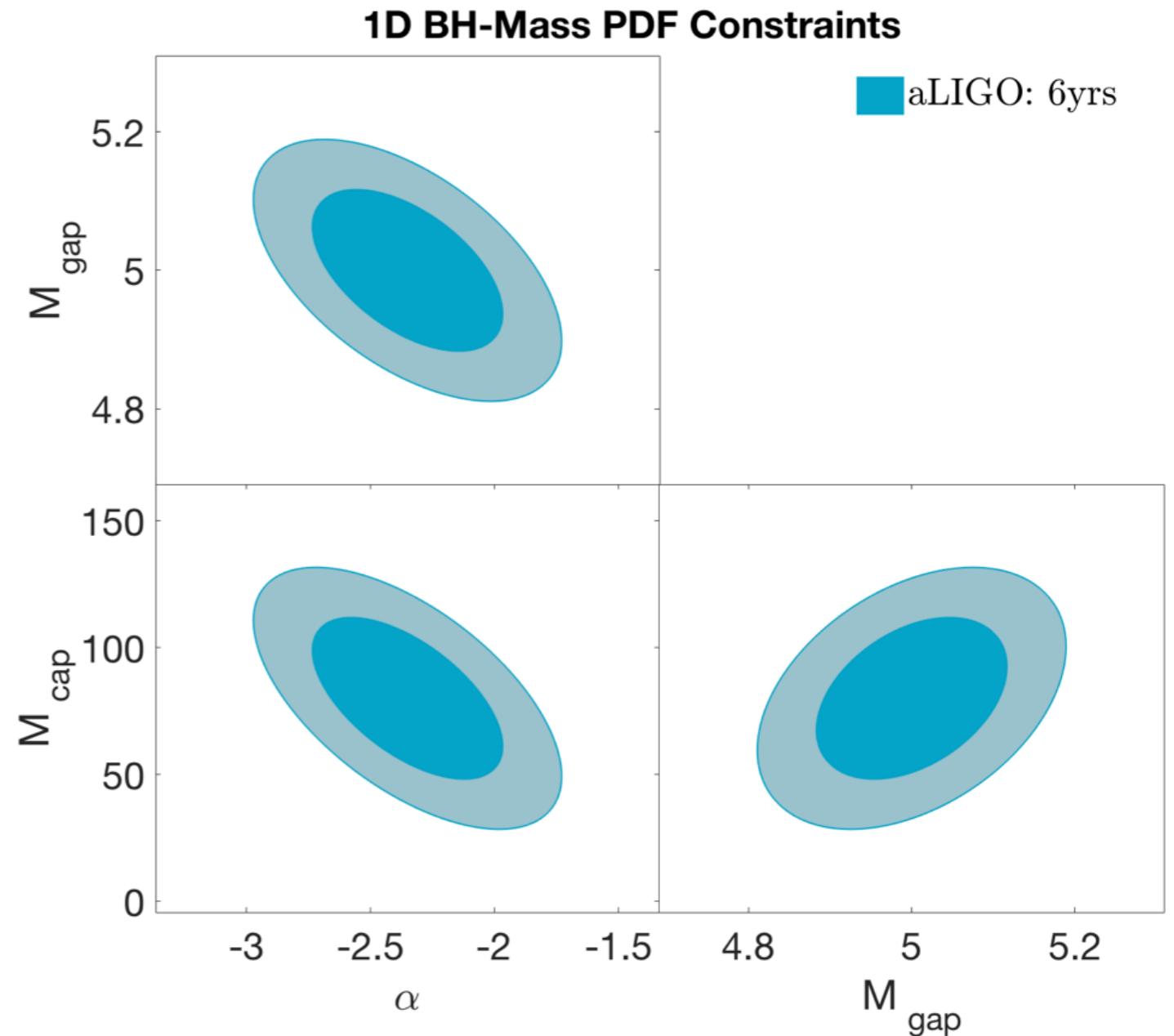


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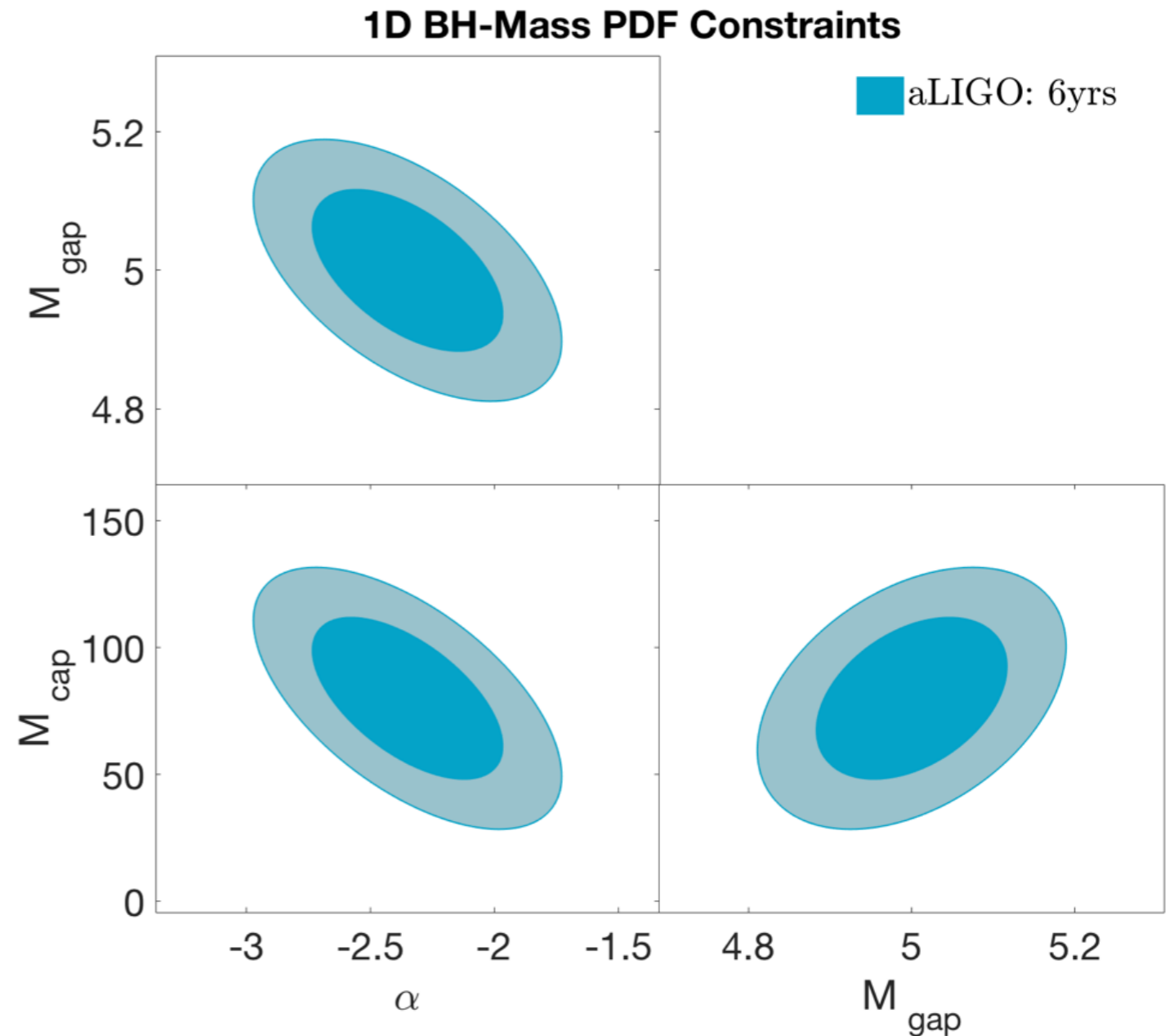
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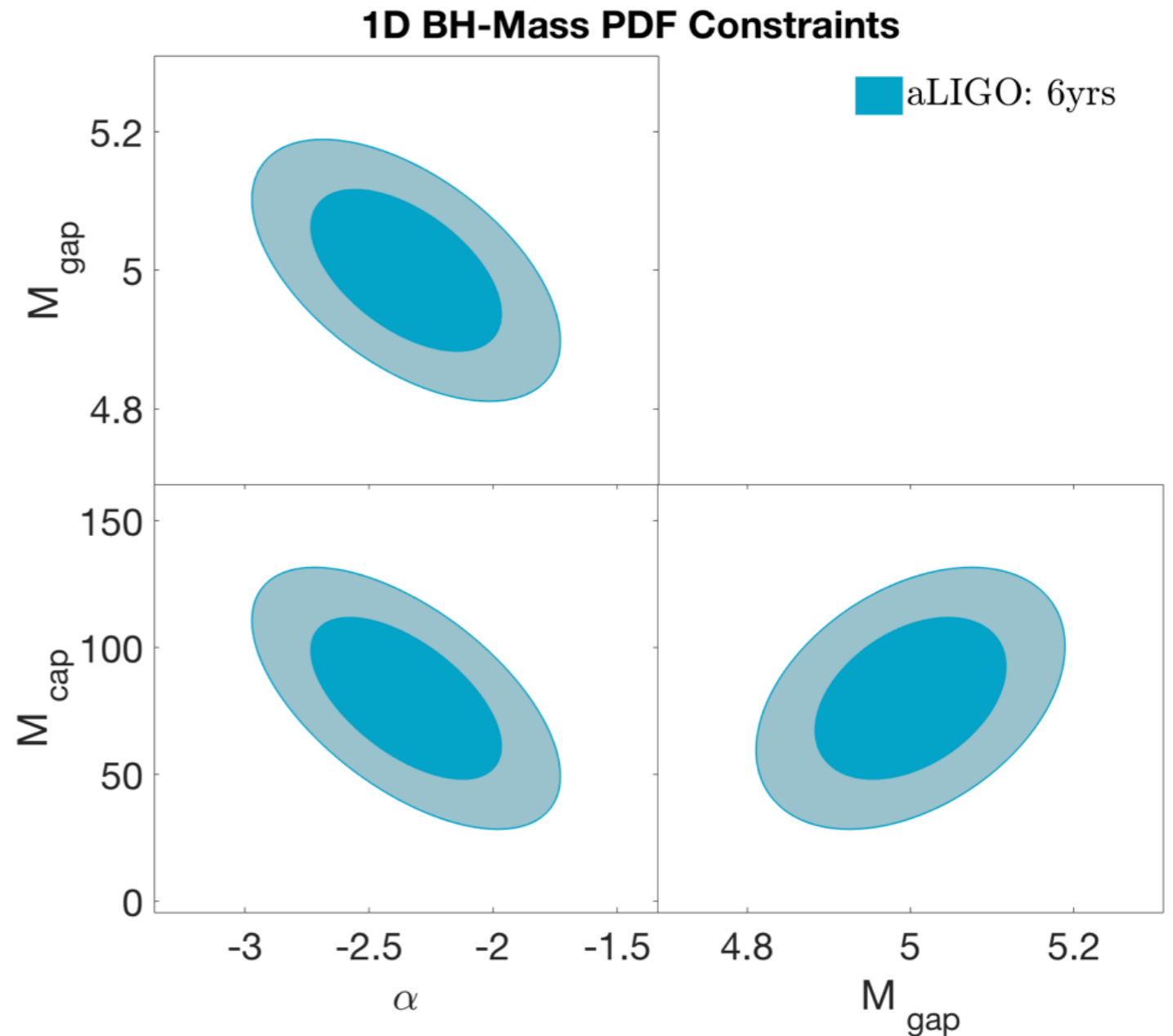
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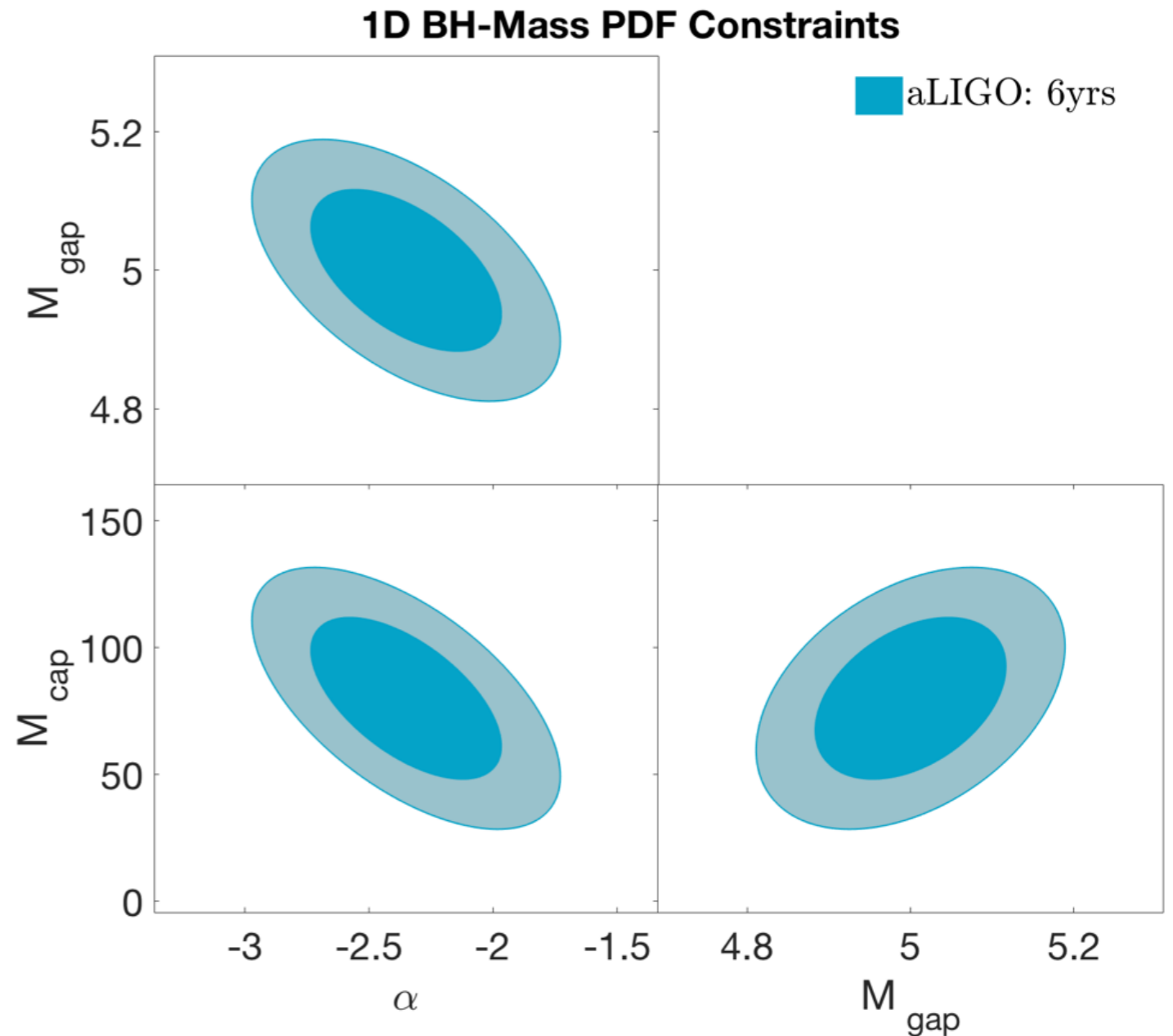
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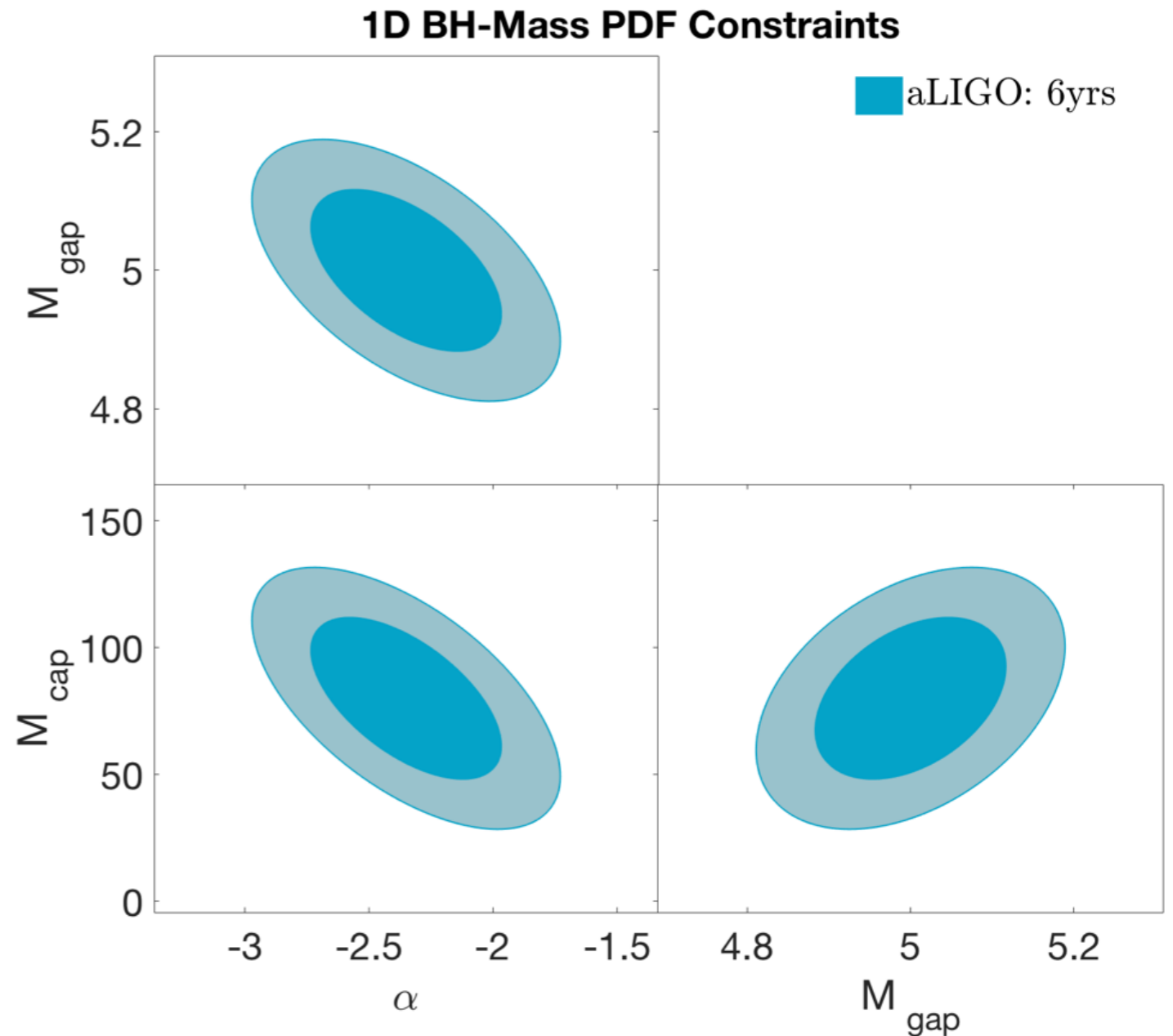
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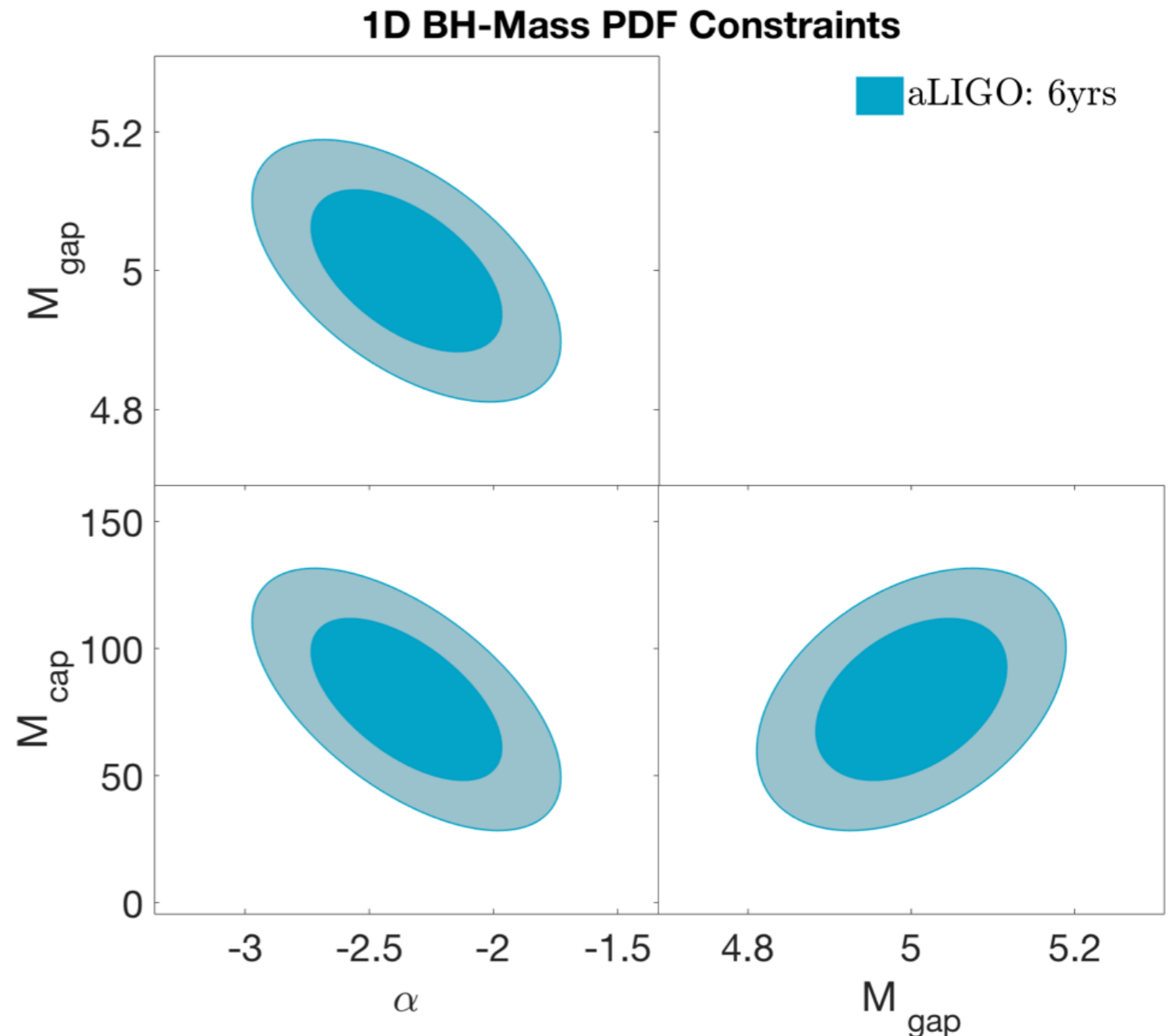
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- Mass cap:

→ need more data



# Outline

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- The Mass Function of Merging Black Holes

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- The Mass Function of Merging Black Holes

- The History of Black Hole Mergers

# Progenitors of Coalescing Binary Black Holes

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Progenitor models:

# Progenitors of Coalescing Binary Black Holes

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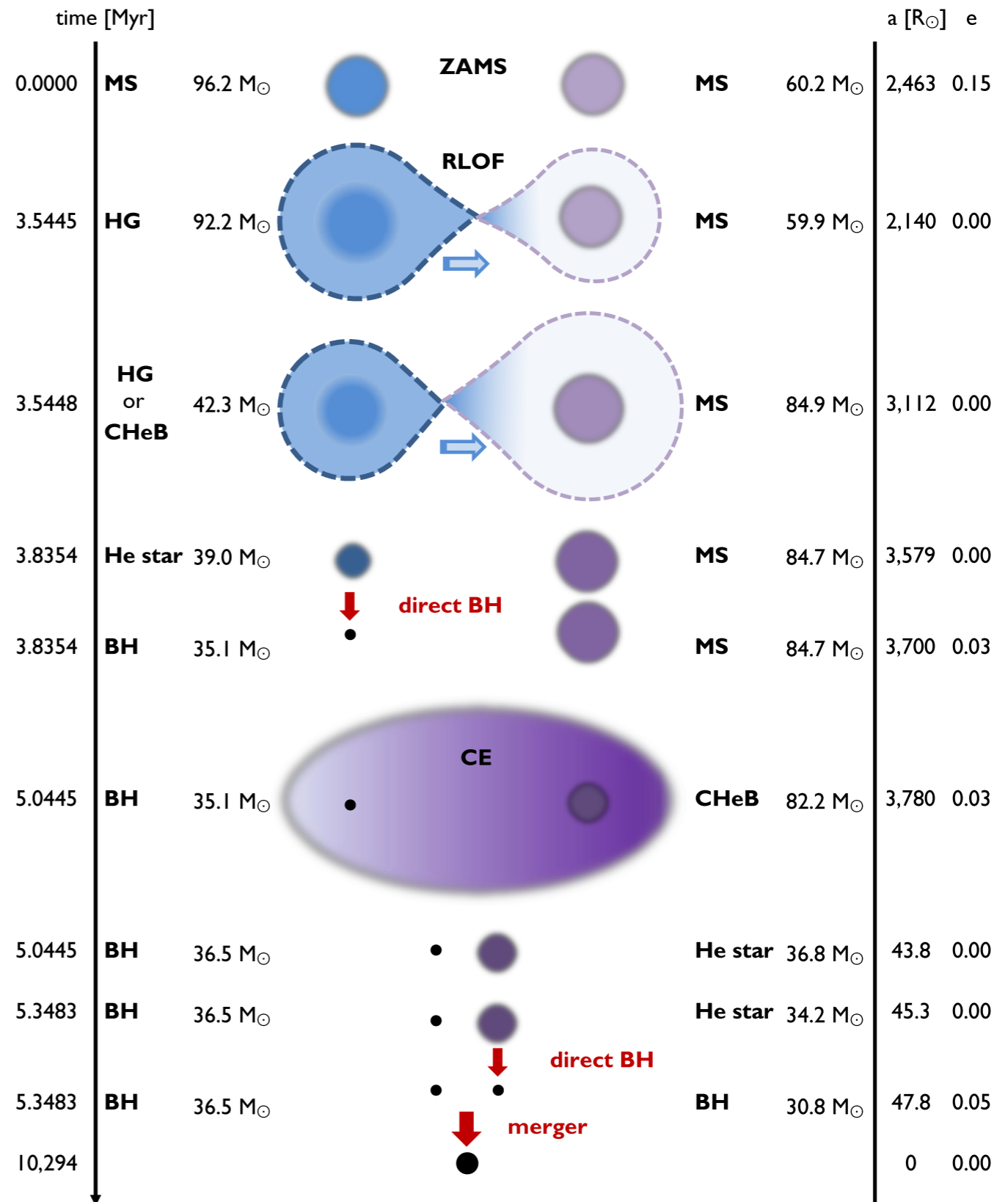
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Field (isolated binaries):

# Progenitors of Coalescing Binary Black Holes

Progenitor models:

Field (isolated binaries):



Source: LIGO

# Progenitors of Coalescing Binary Black Holes

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Progenitor models:

Field (isolated binaries)

Dynamical formation in GCs:

# Progenitors of Coalescing Binary Black Holes

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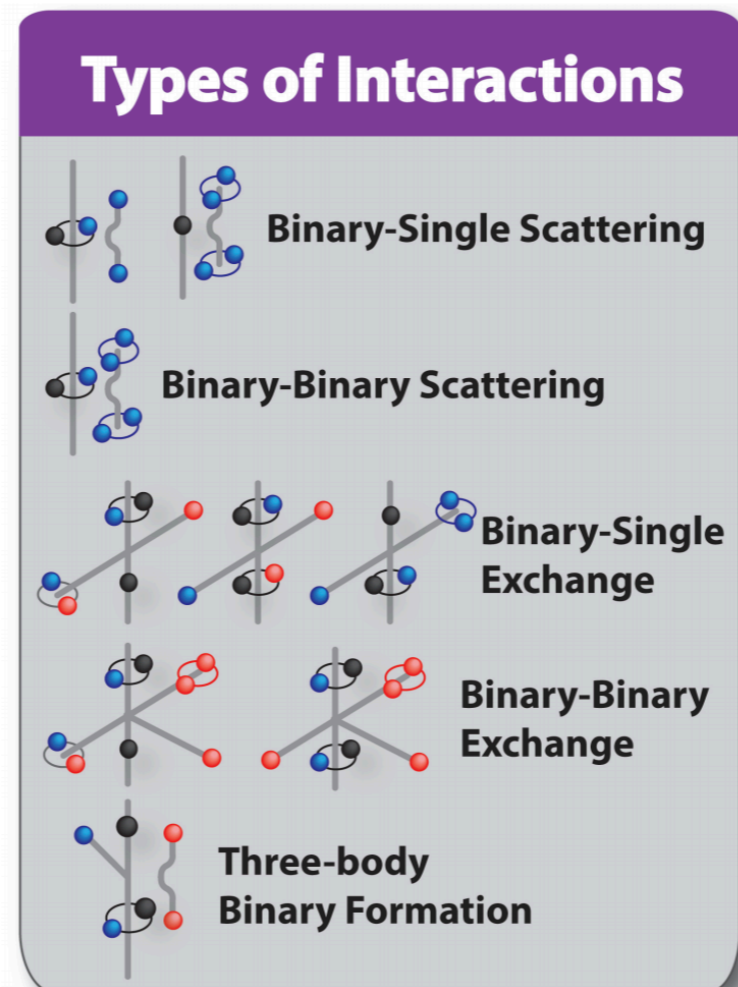
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Dynamical formation in GCs:



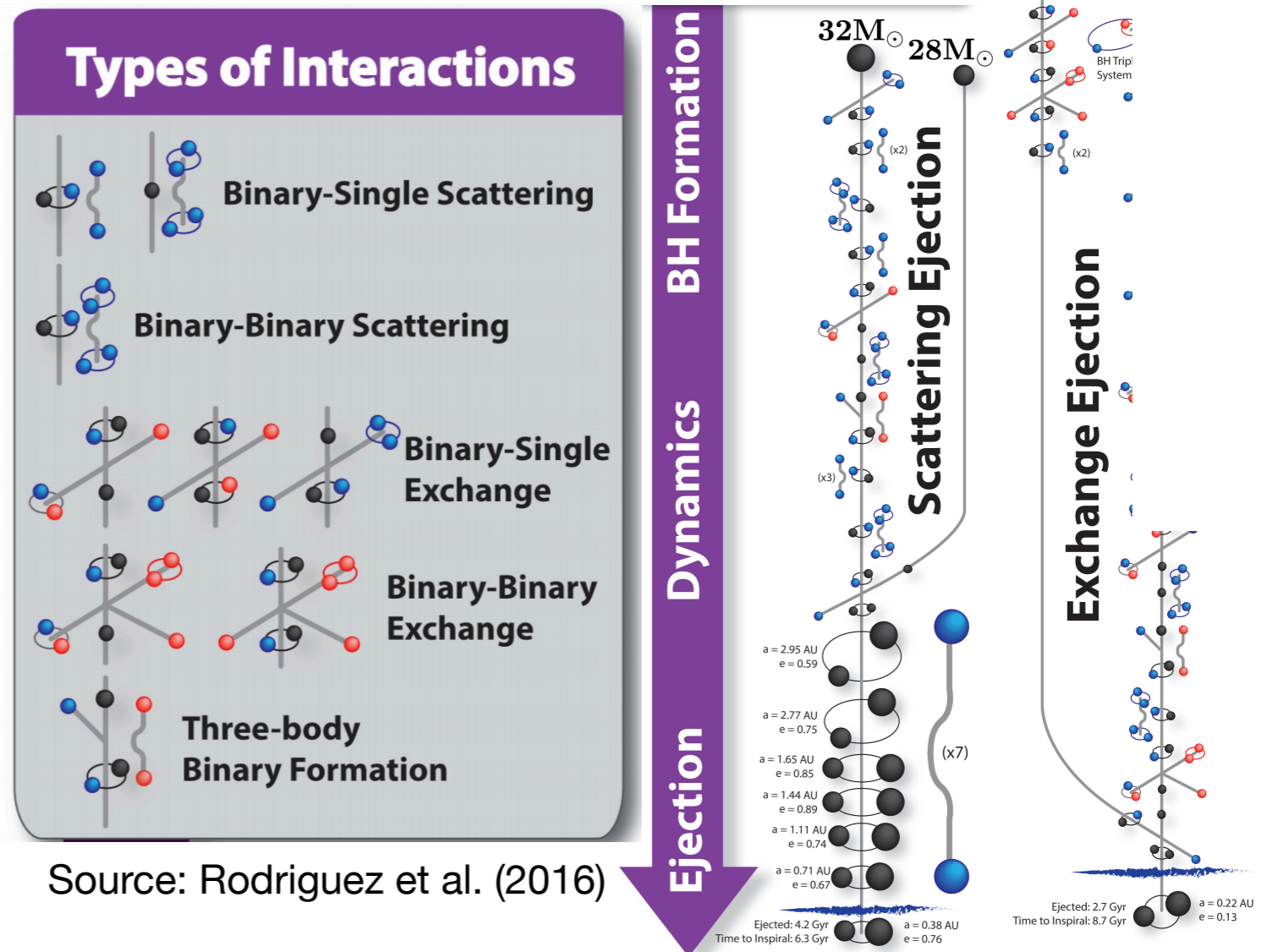
Source: Rodriguez et al. (2016)

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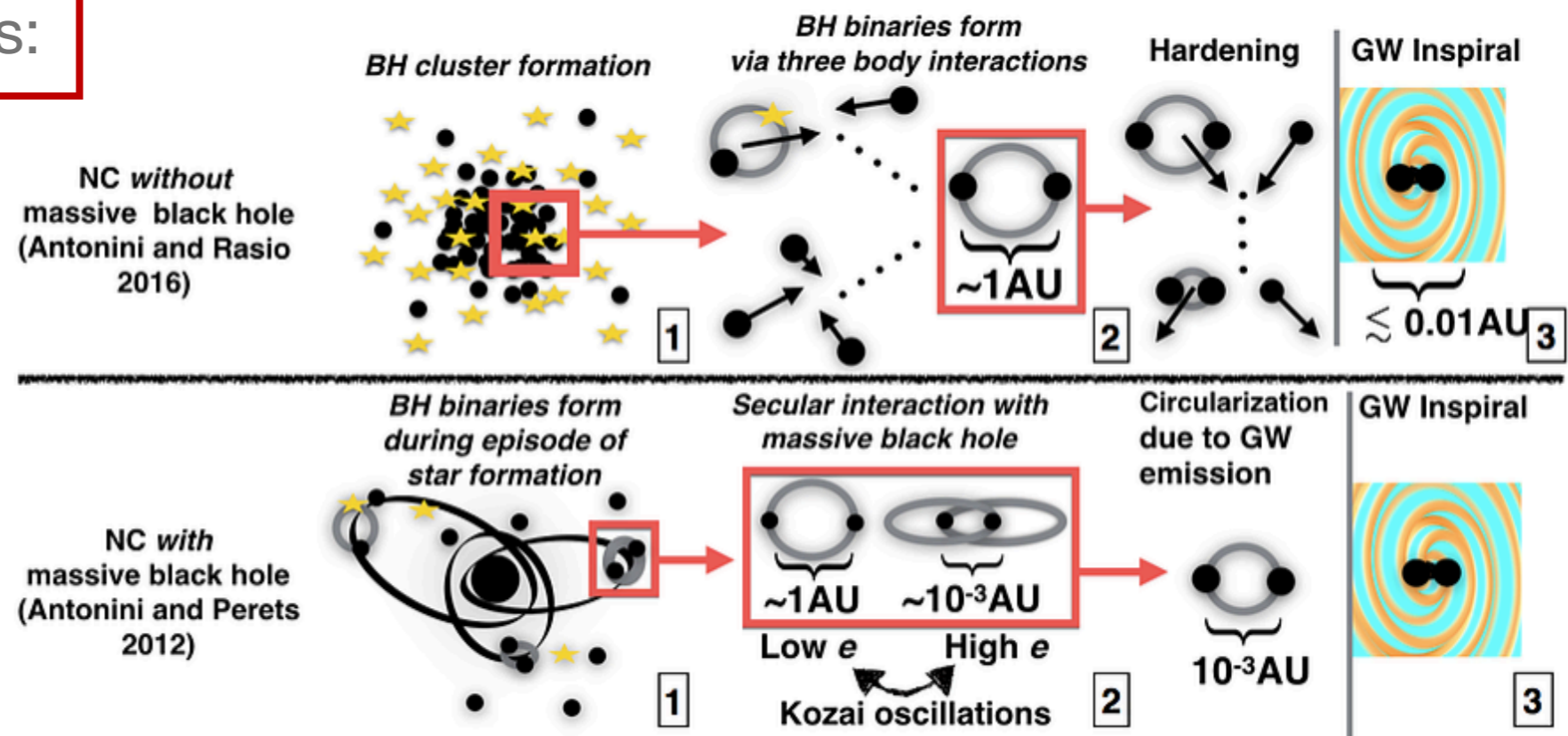
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Credit: F. Antonini

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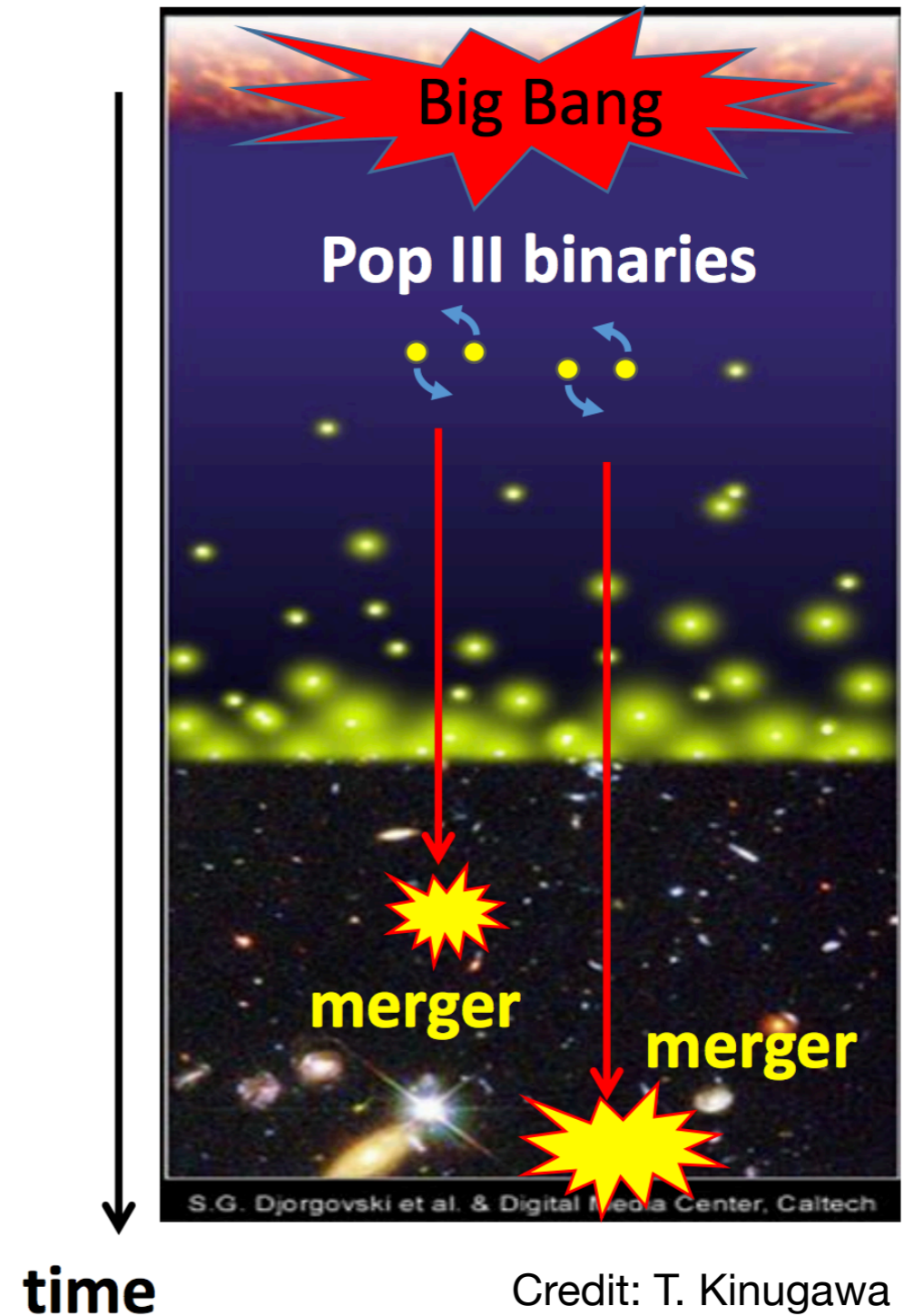
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Other ideas?

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(EDK et al., arXiv:1611.01157)

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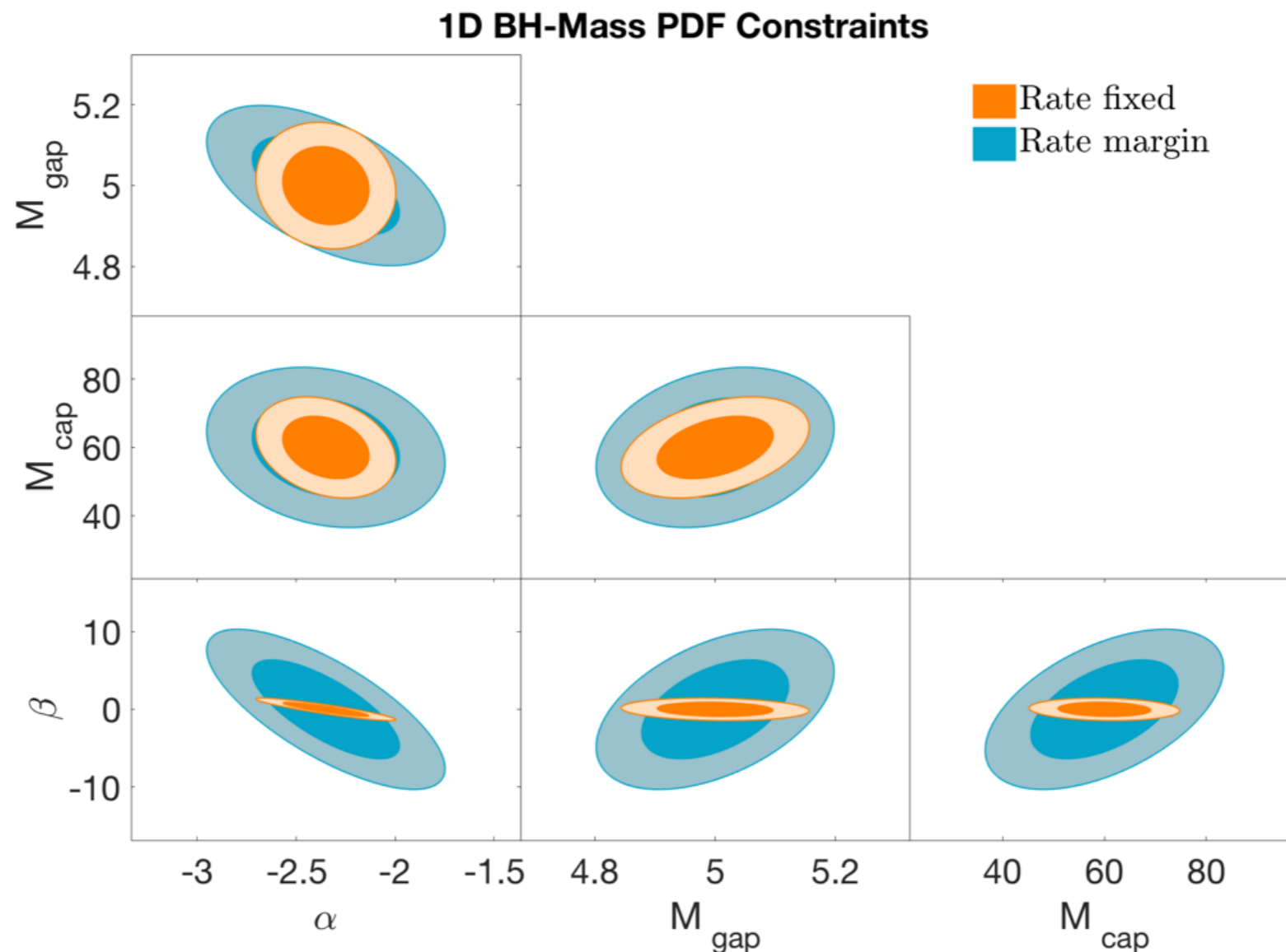
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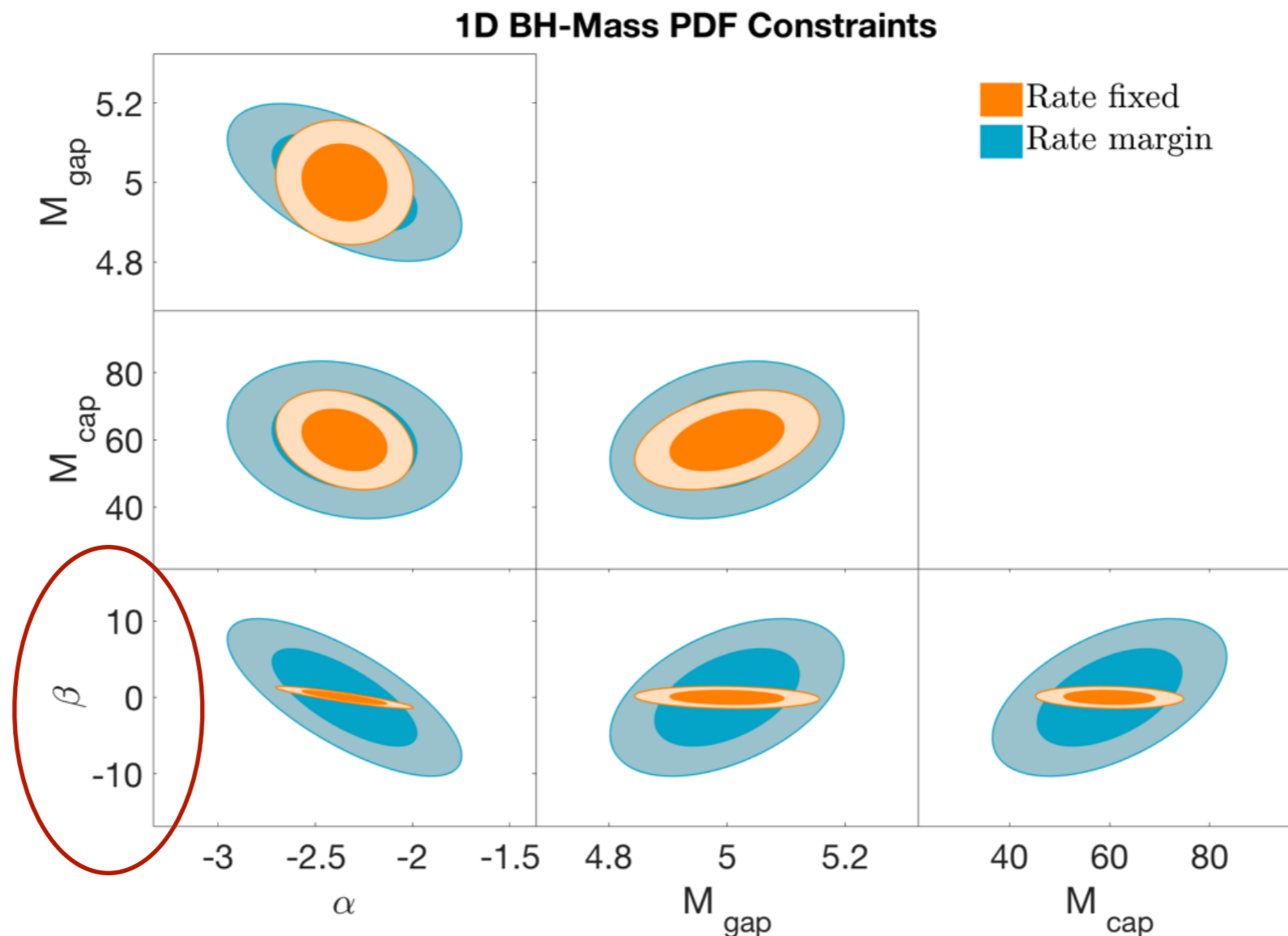
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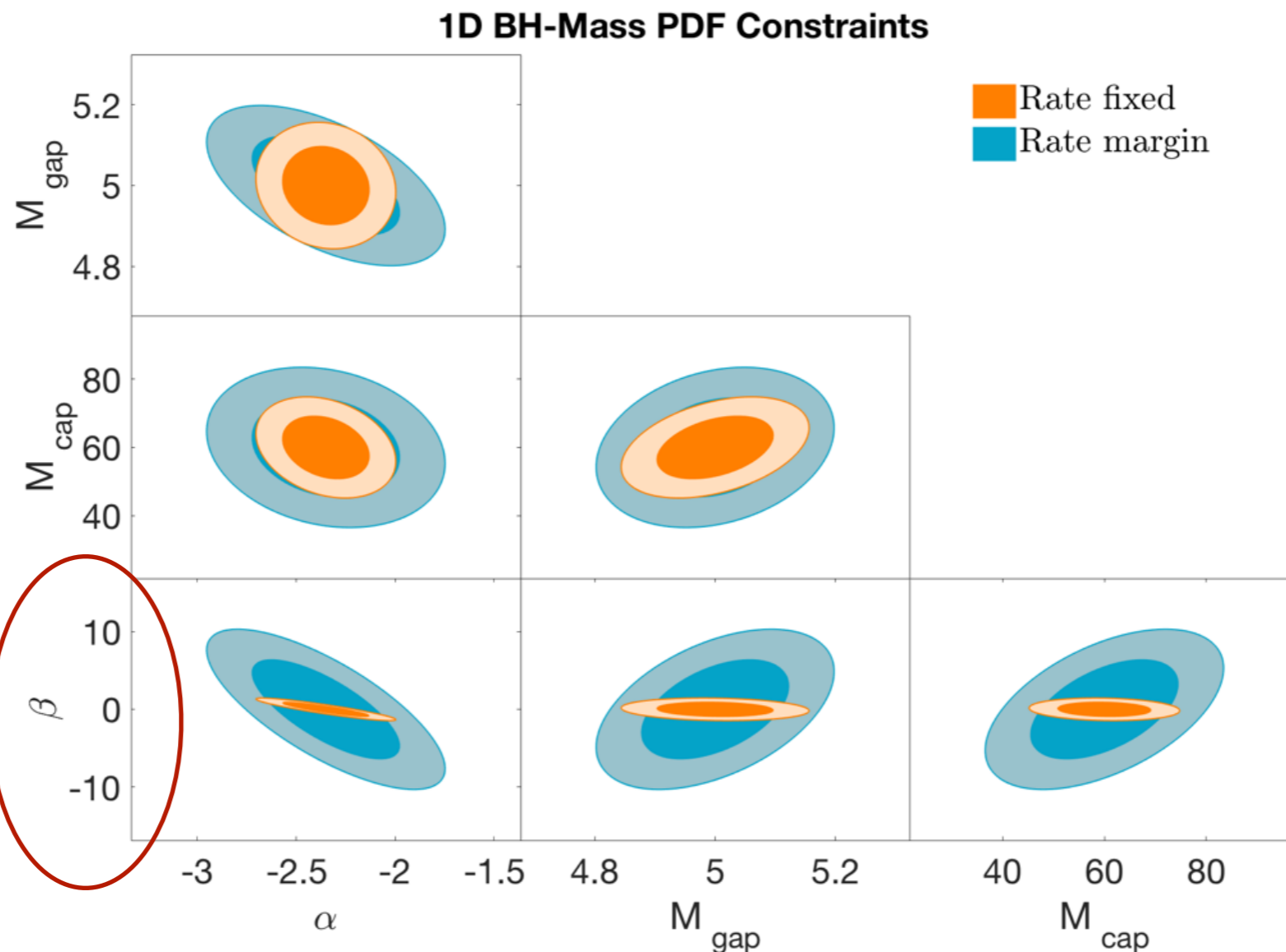
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More info  
in 2D MF!

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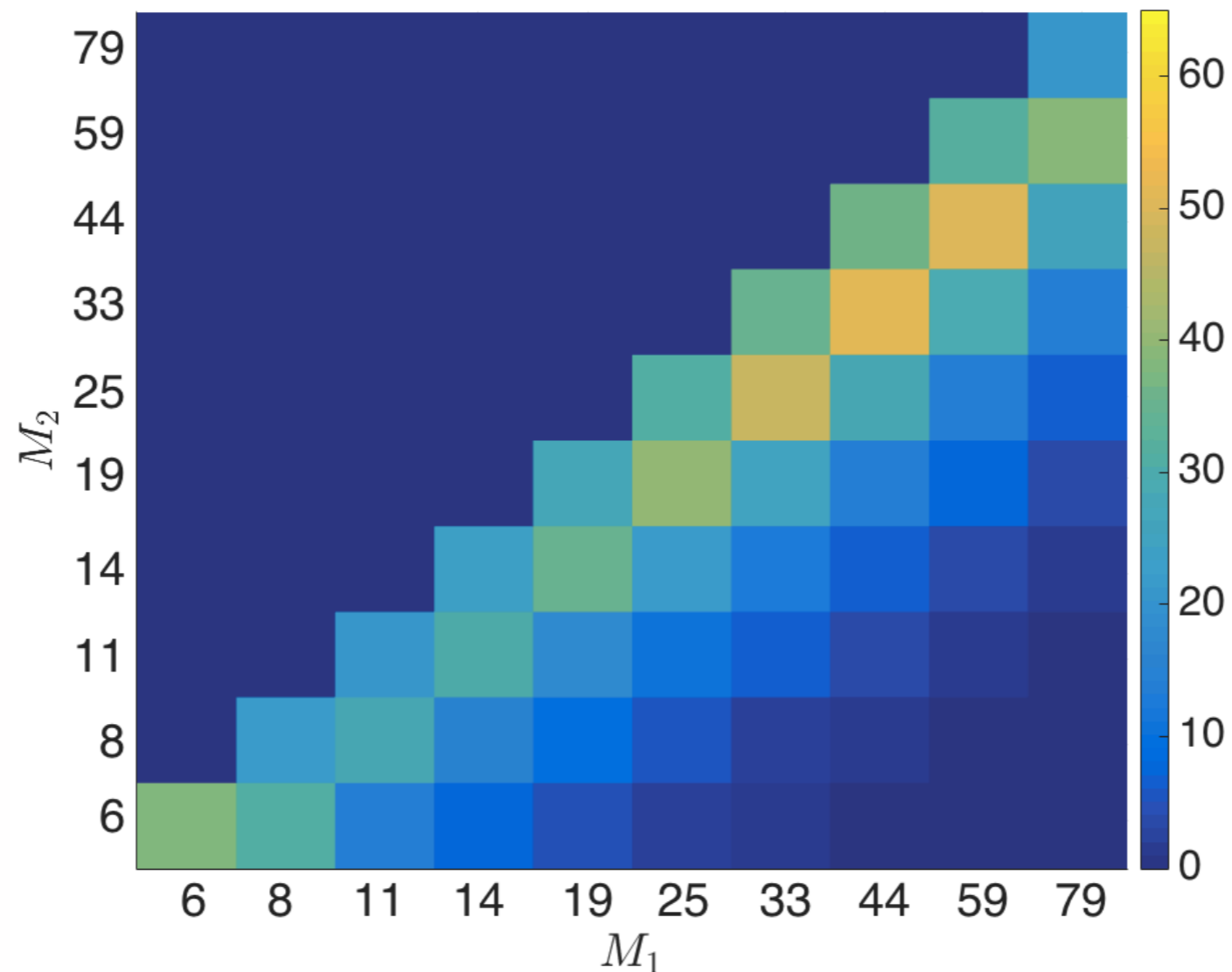
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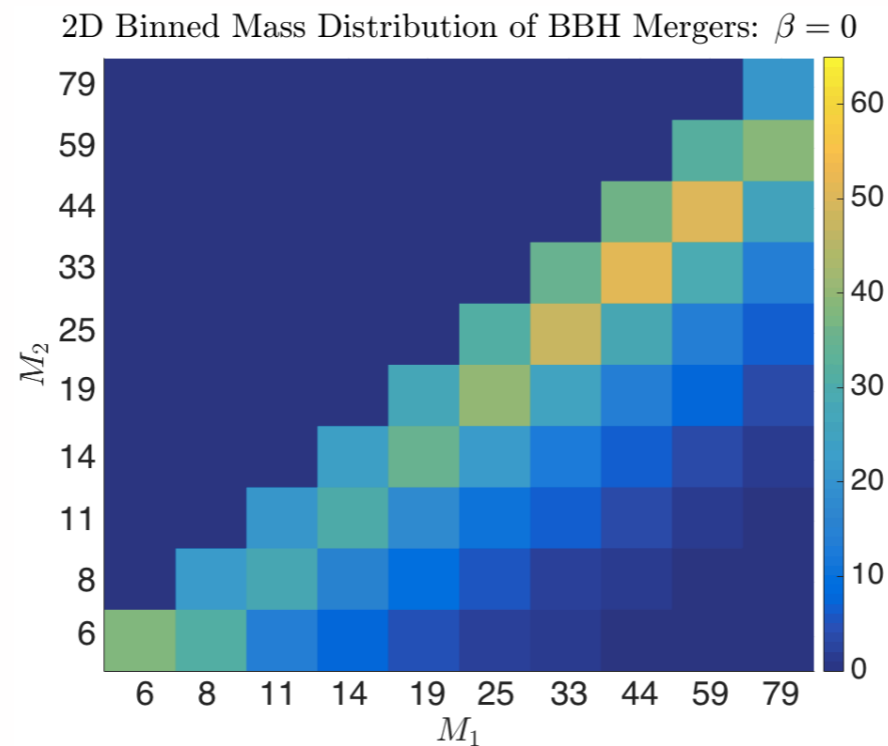
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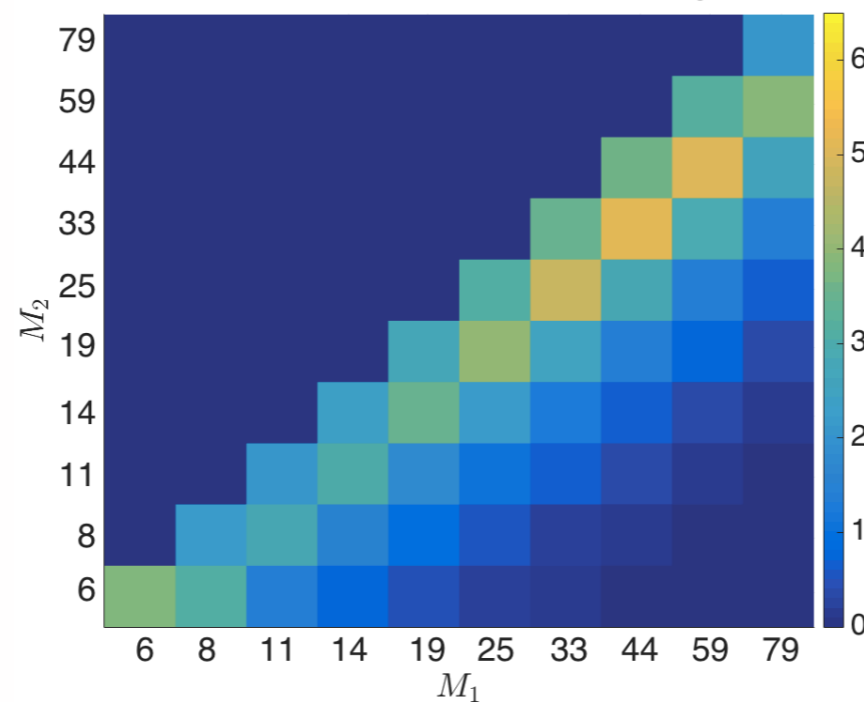
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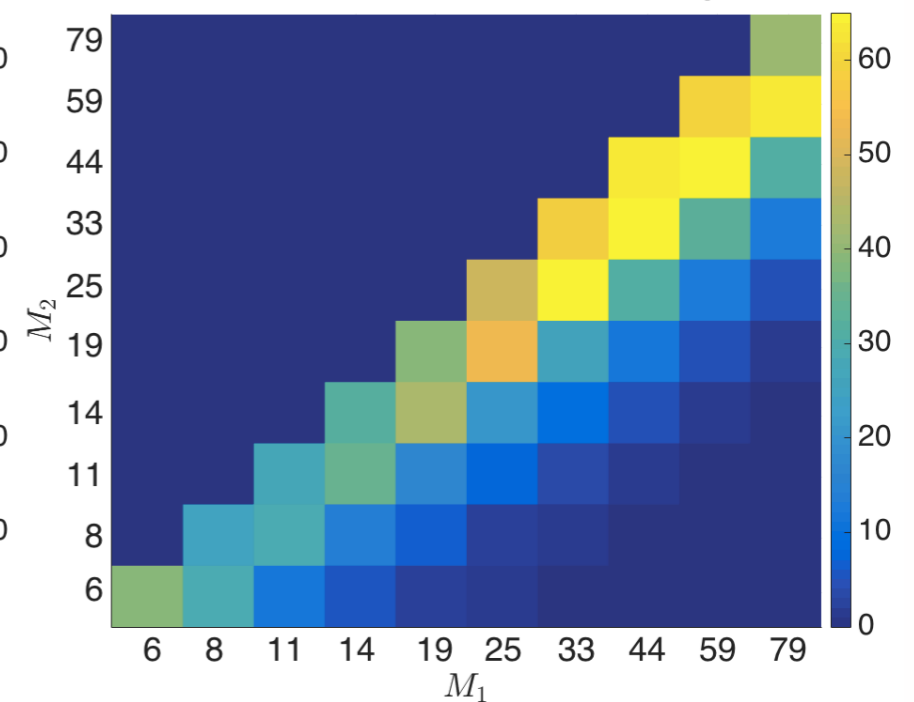
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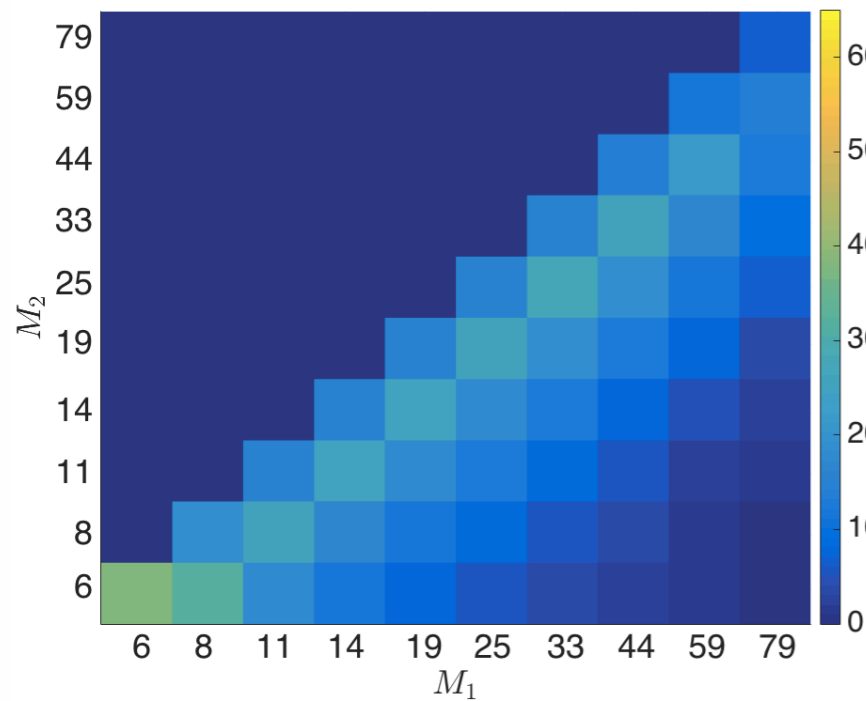
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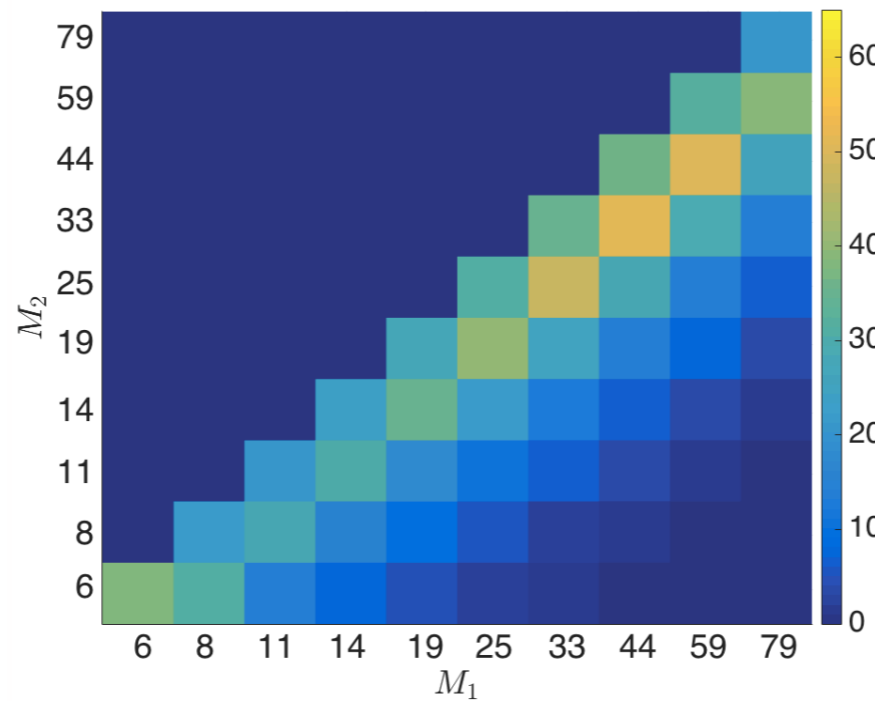
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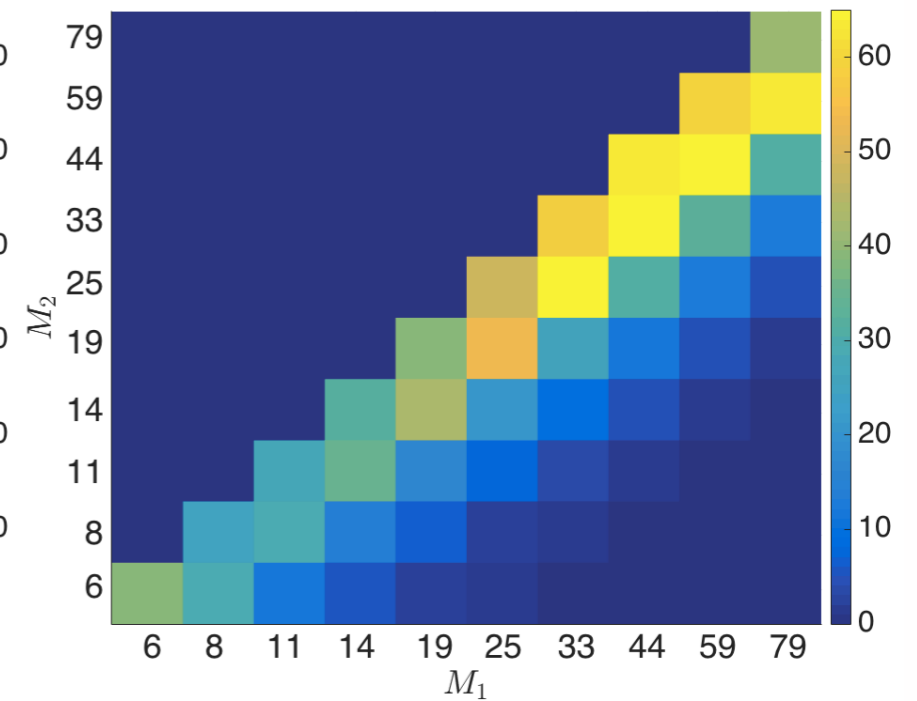
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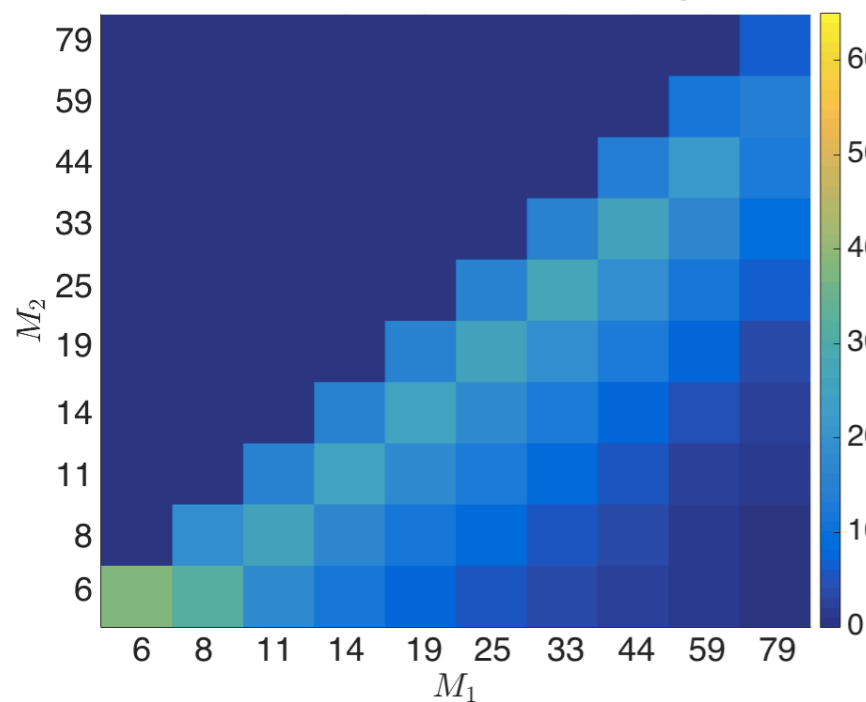
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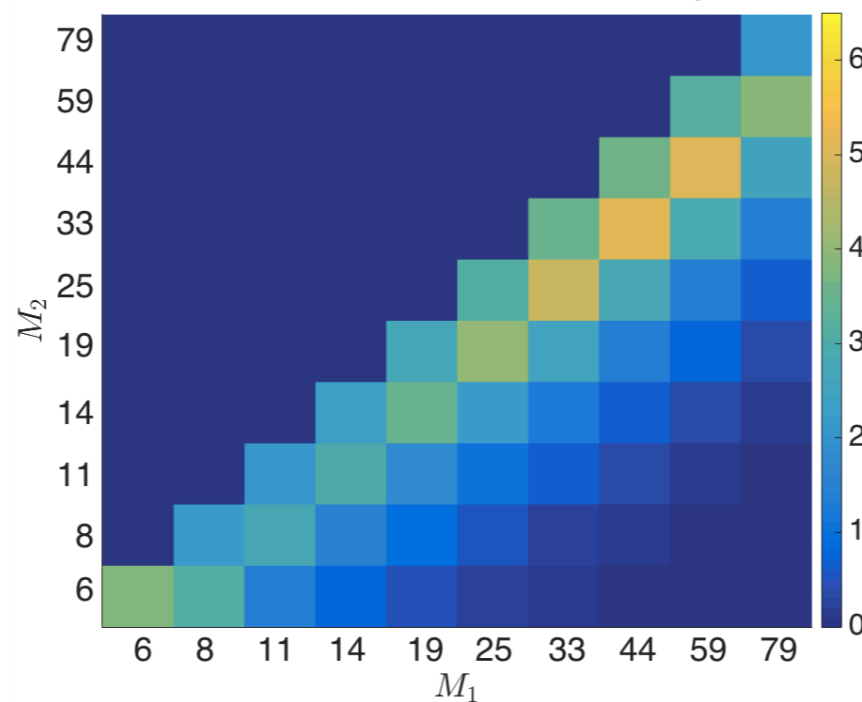
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The mass ratio is a sensitive probe of the progenitor model!

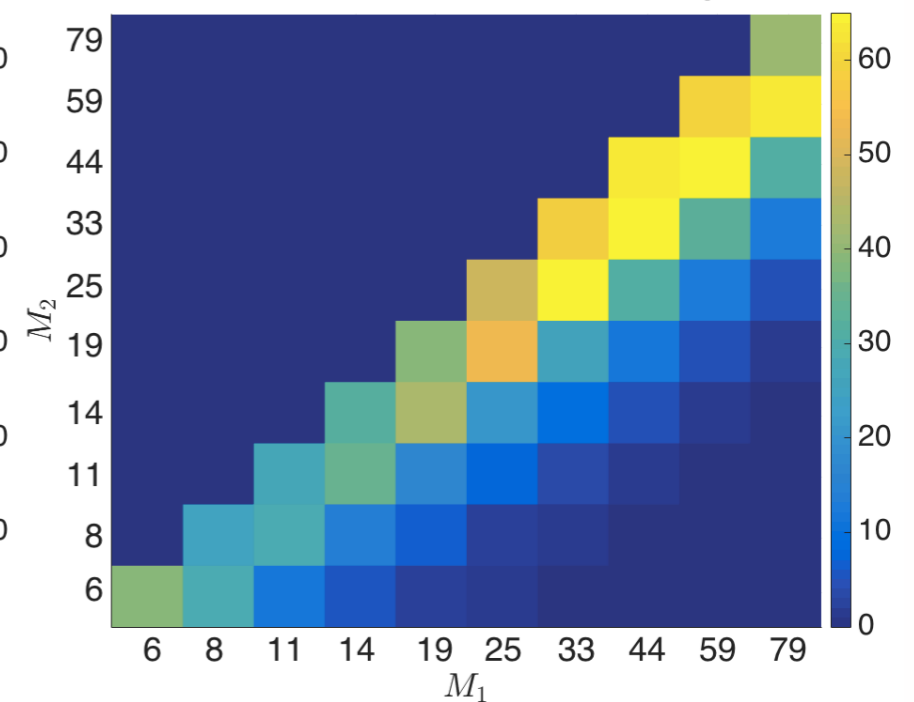
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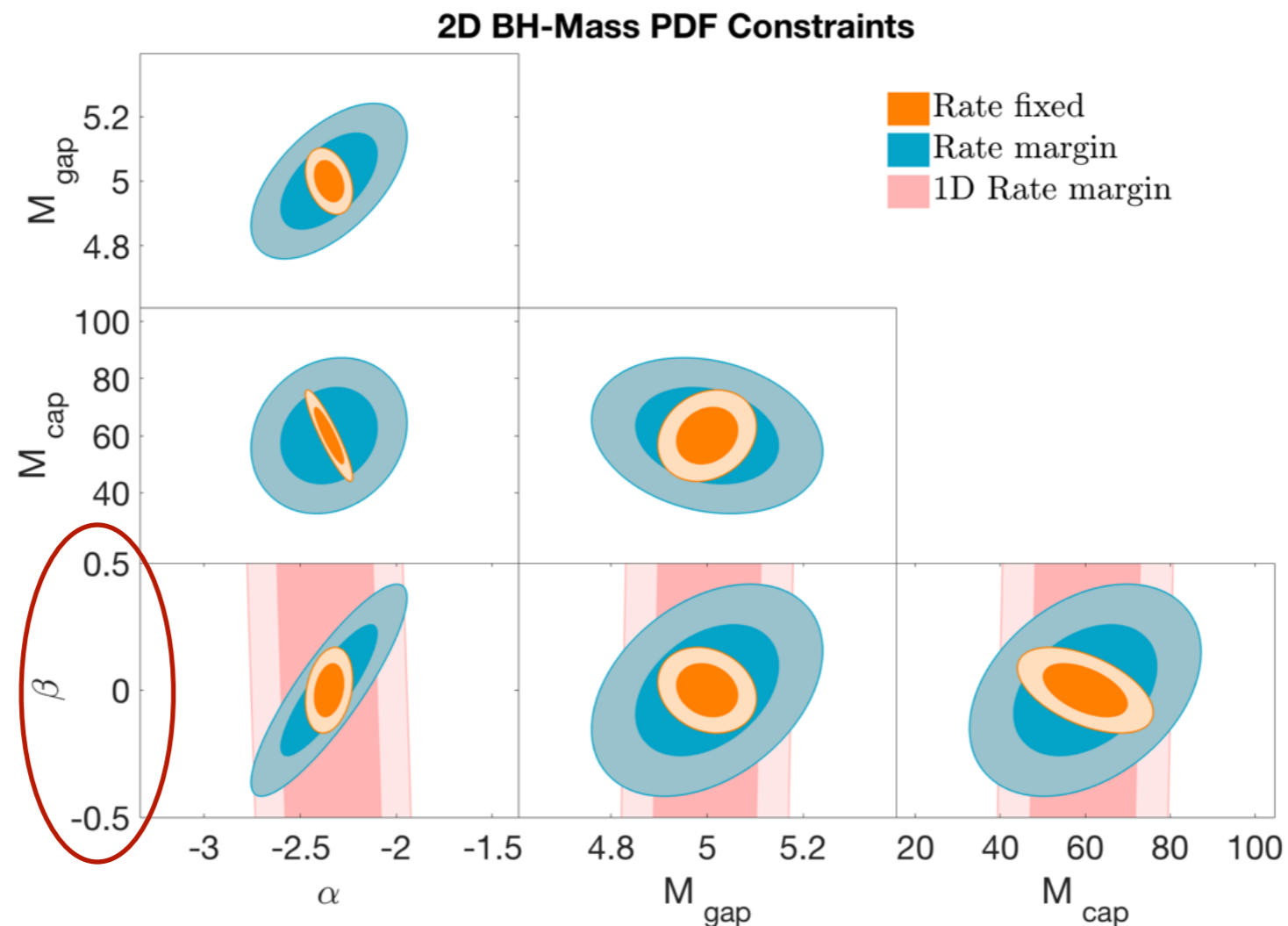
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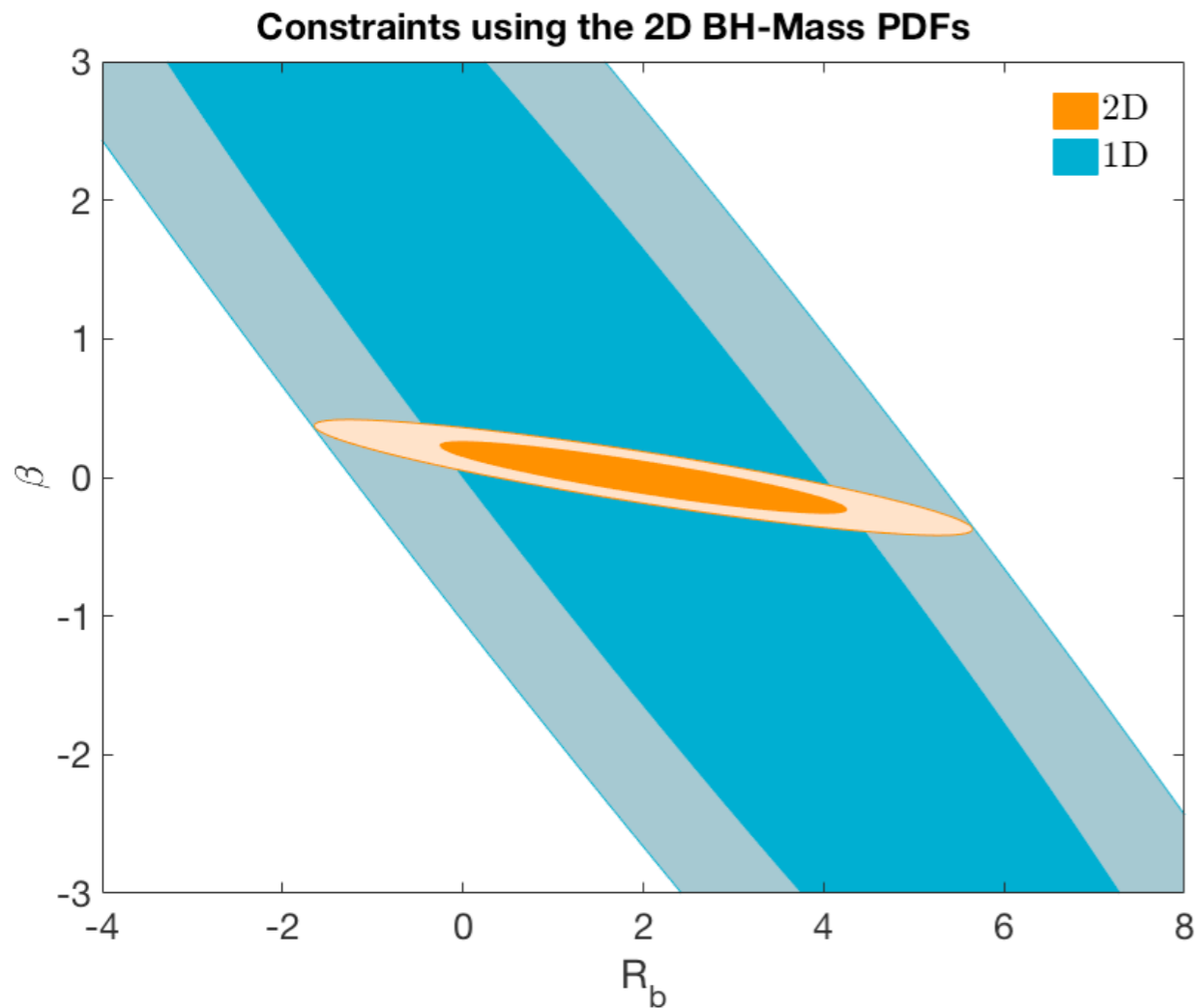
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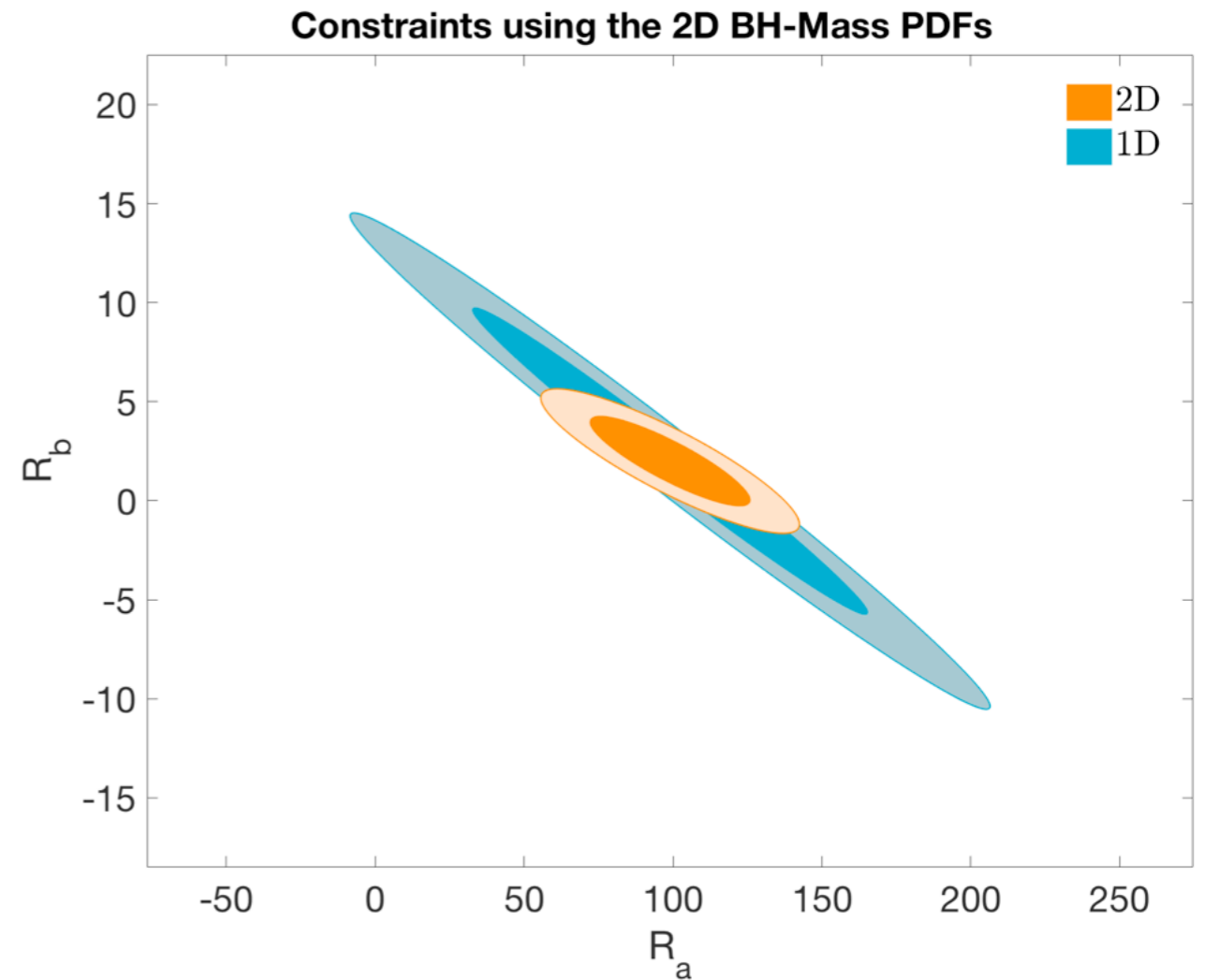
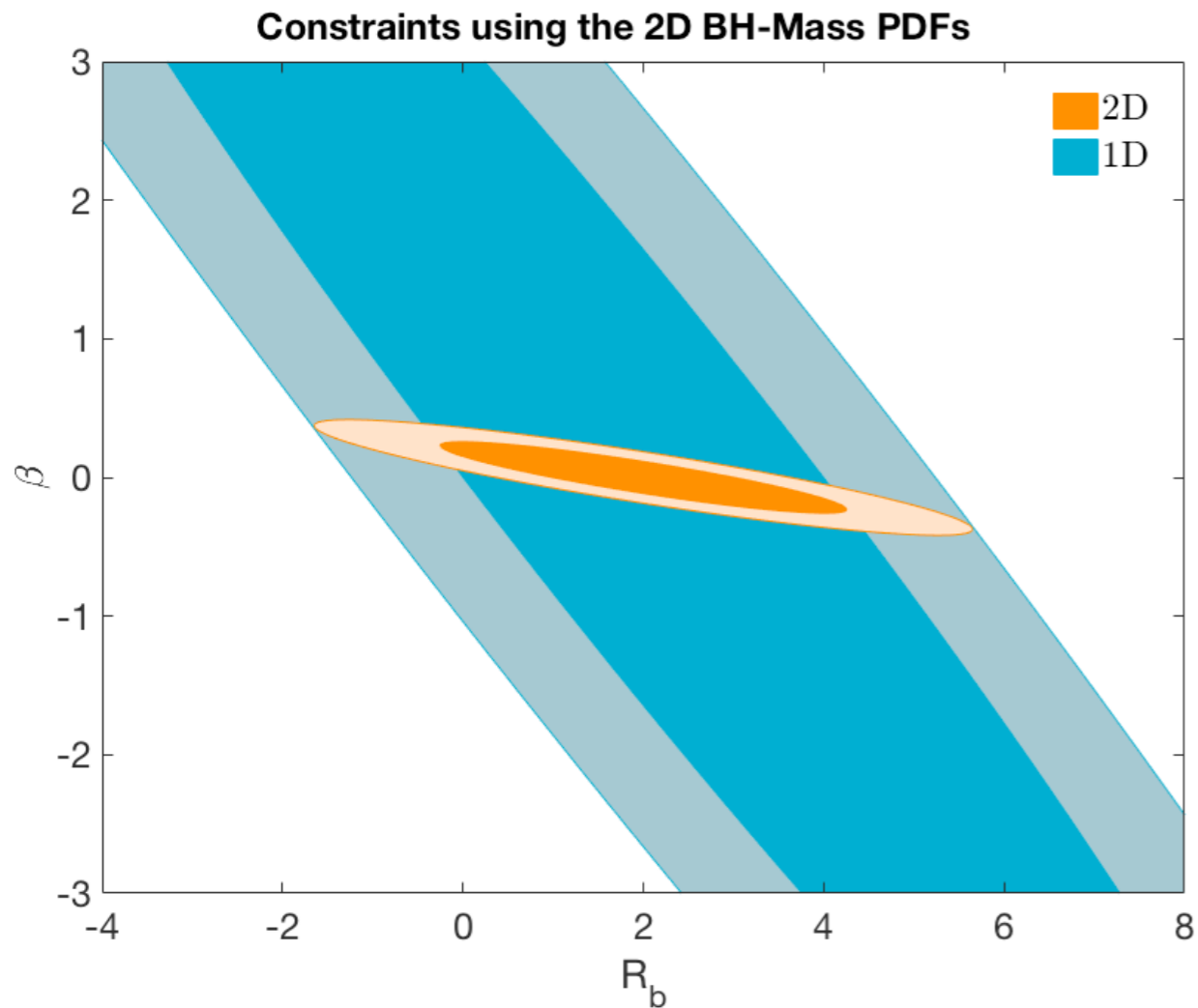


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# BHMF Constraints: Next Decade Forecasts

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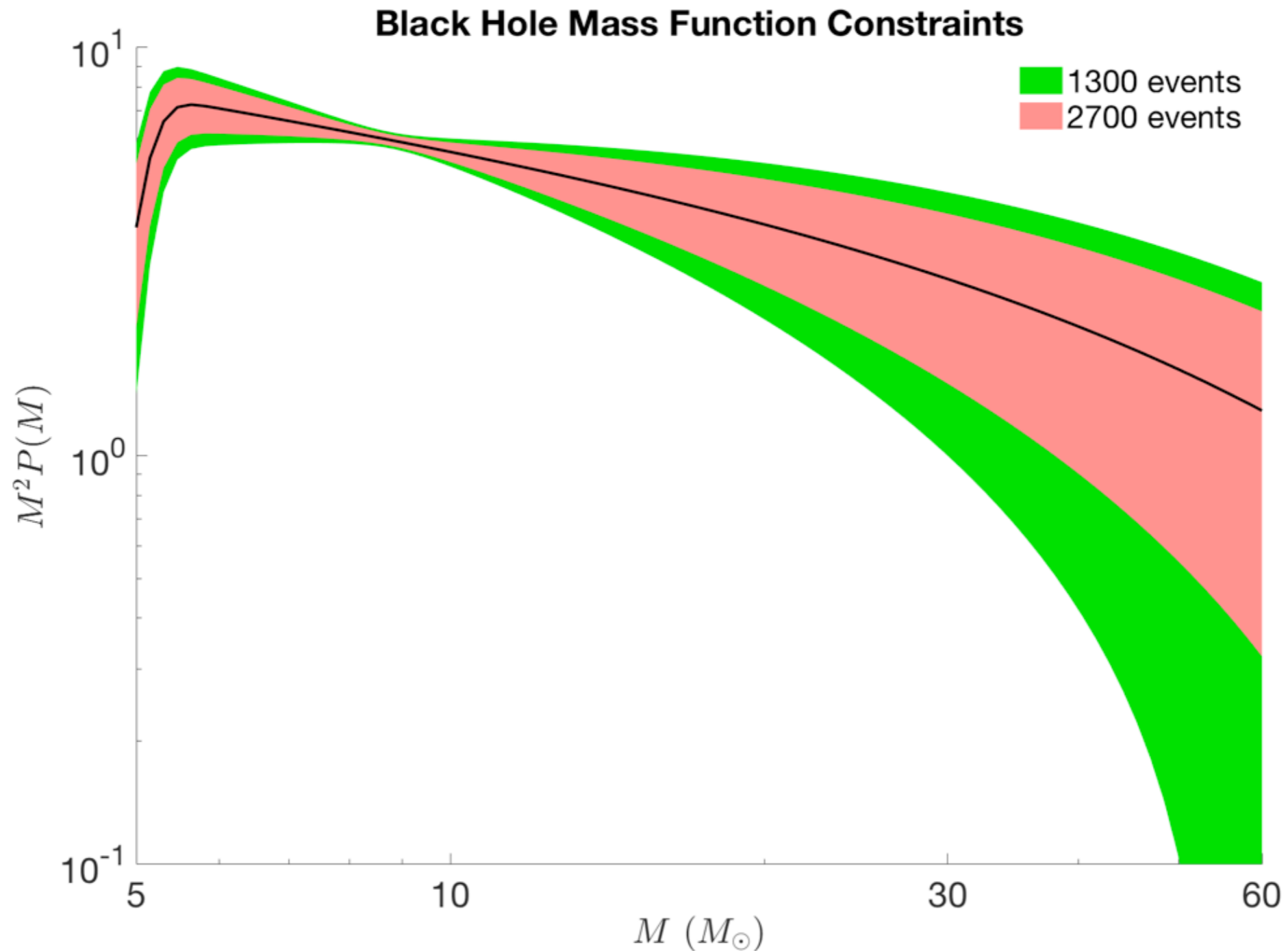
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$S/N > 8$ and 1 years	440	0.41	0.24	27.29	43.46	3.65	0.42
$S/N > 8$ and 3 years	1330	0.23	0.14	15.75	25.09	2.11	0.24
$S/N > 8$ and 6 years	2670	0.17	0.10	11.14	17.74	1.49	0.17
$S/N > 10/\sqrt{2}$ and 6 years	3790	0.14	0.08	9.43	15.01	1.13	0.14
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Reducing the S/N threshold is invaluable.

# More Tools to Determine Progenitors: Eccentricity

(Cholis, EDK et al., PRD 94 (2016))

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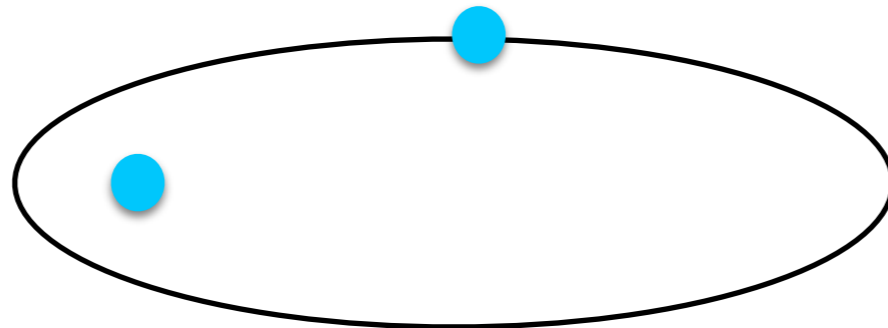
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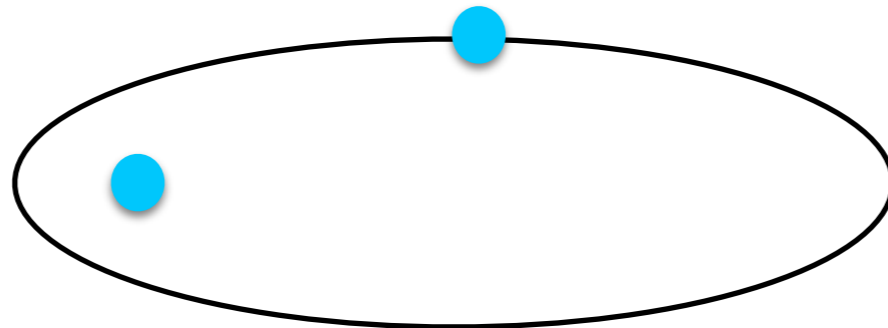


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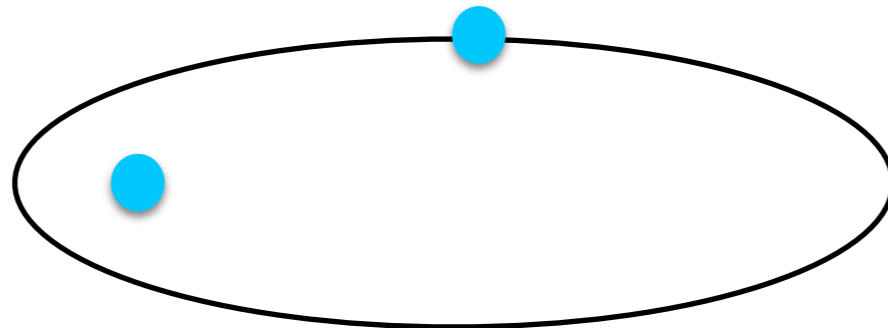
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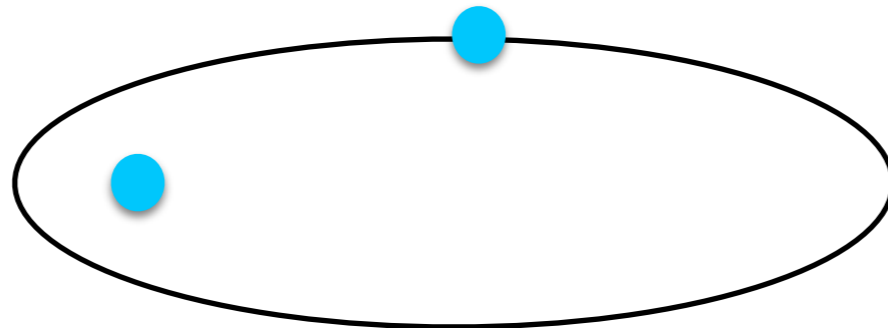
As the BHs coalesce, the orbit circularizes.

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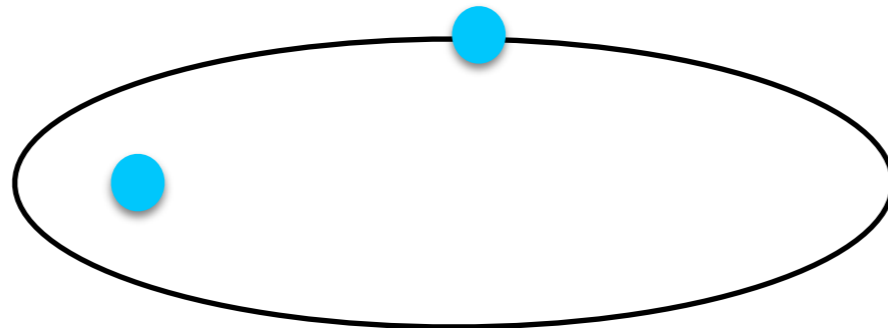
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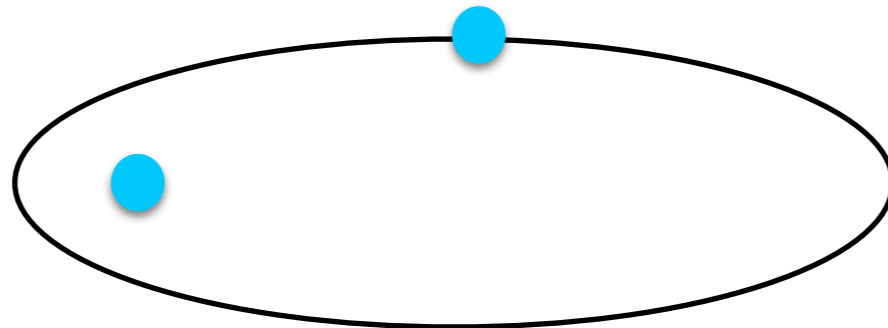
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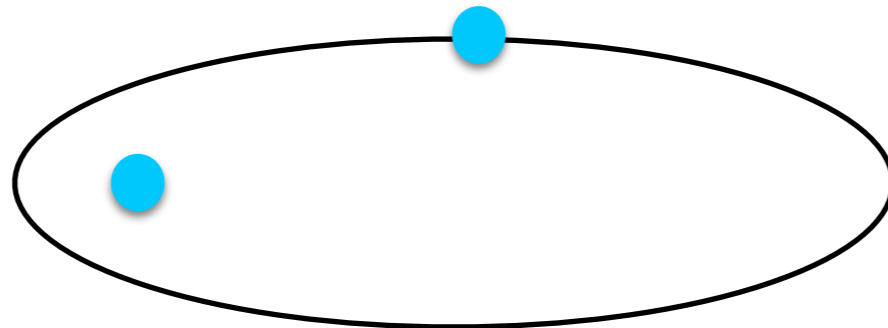
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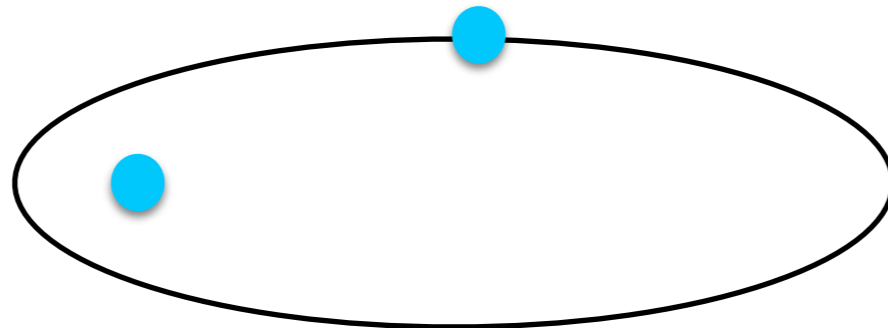
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Goal: detect deviation from  $e = 0$

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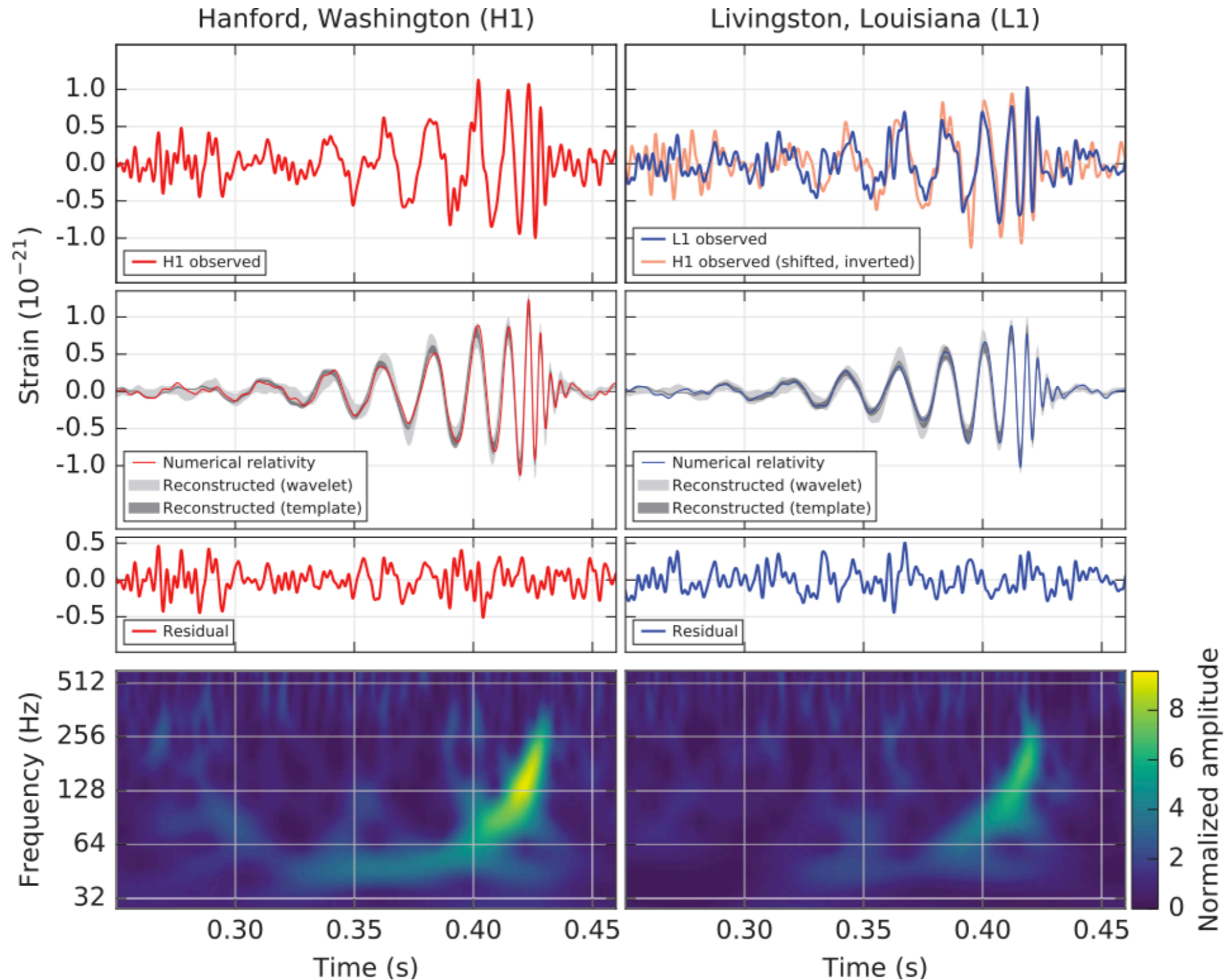
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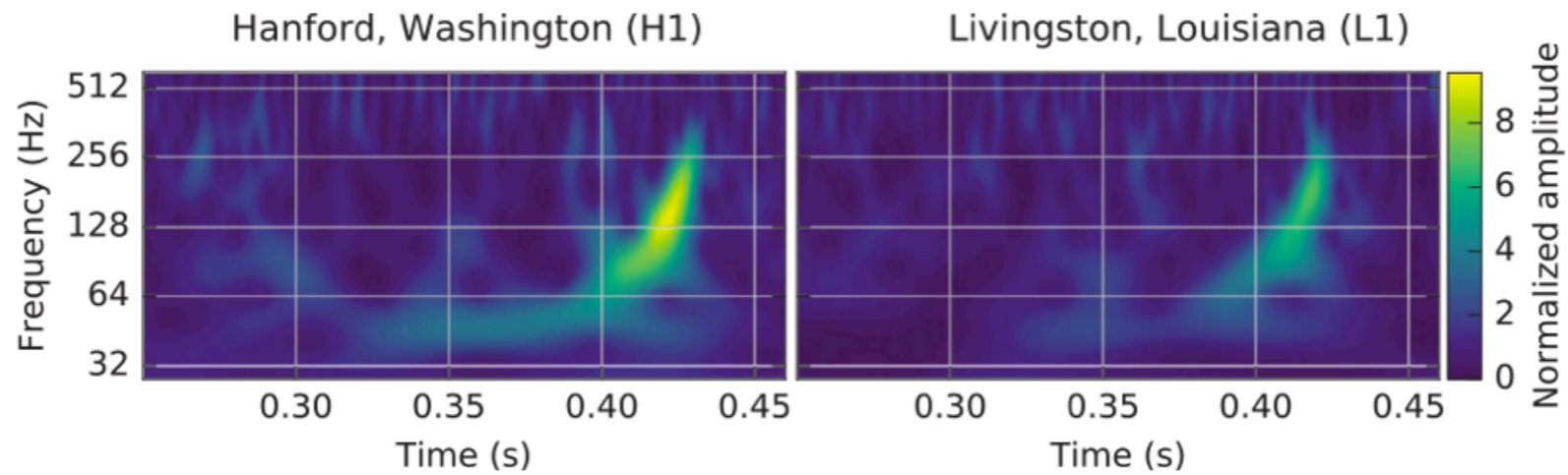


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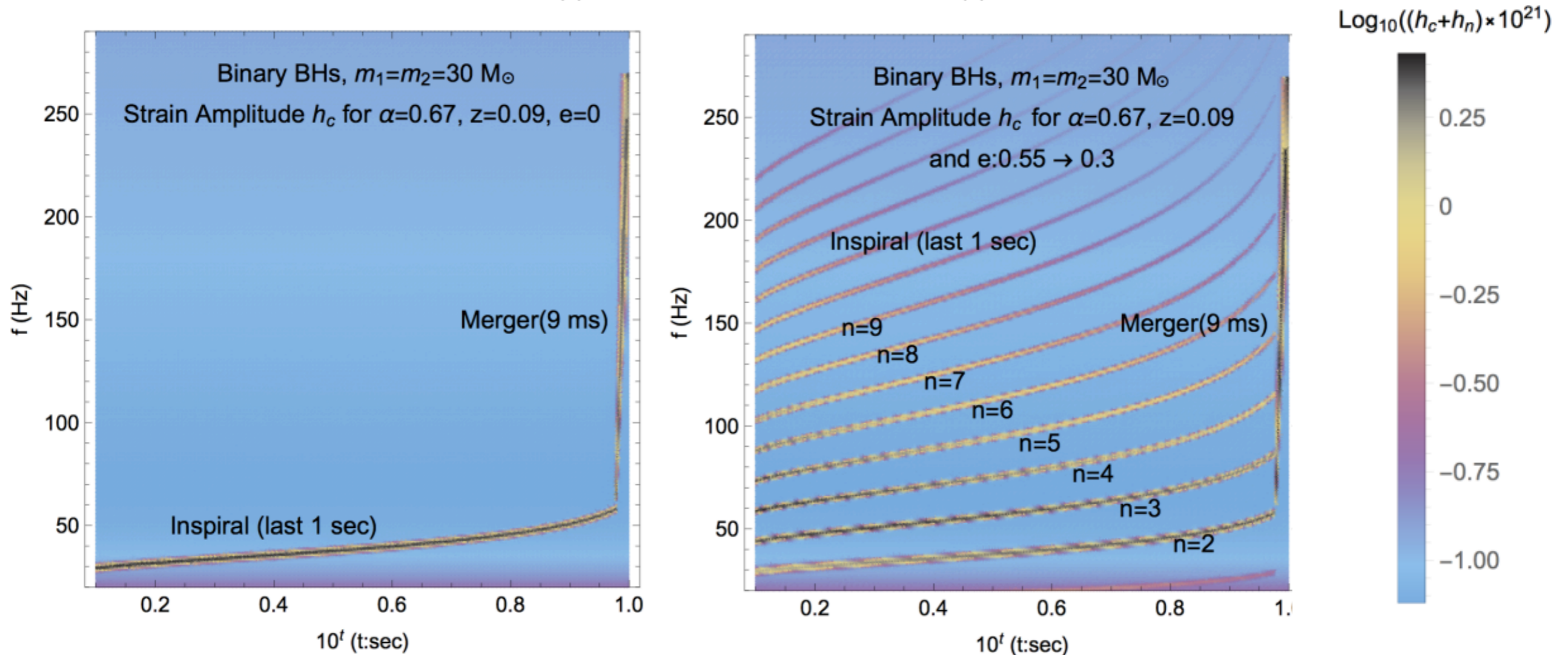
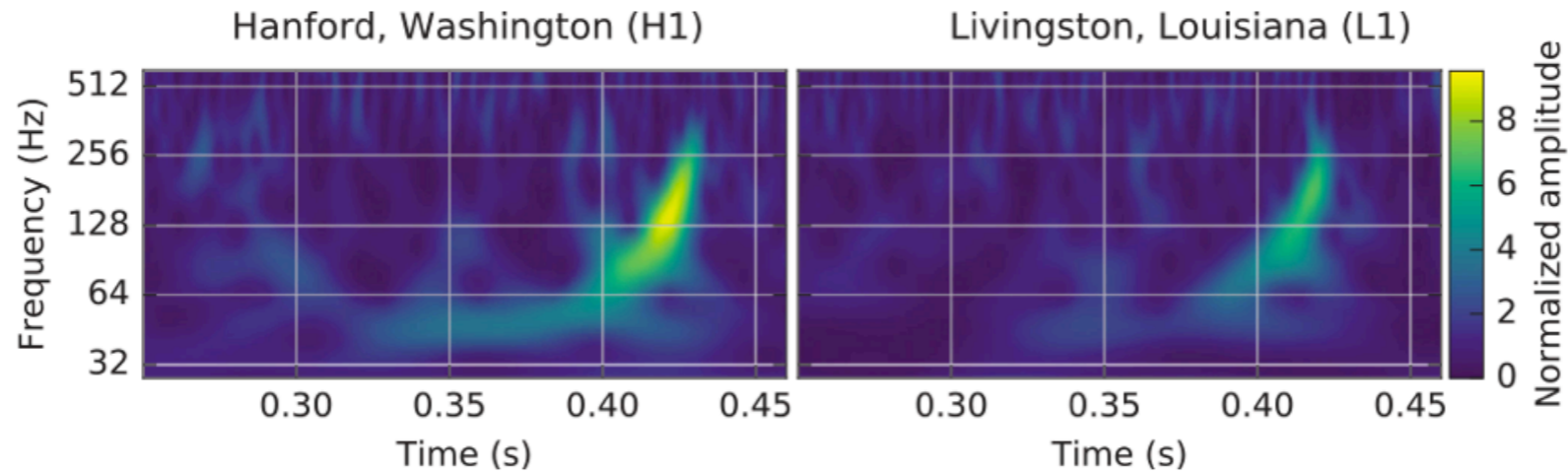
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# More Tools to Determine Progenitors: Eccentricity

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GW signal (+ noise):



# More Tools to Determine Progenitors: Spin

(Cholis, EDK et al., PRD 94 (2016))

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# More Tools to Determine Progenitors: Spin

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Spin aligned vs. non-aligned:

$$\chi_{1,2} = \frac{c}{Gm_{1,2}^2} S_{1,2} \cdot \hat{L}$$

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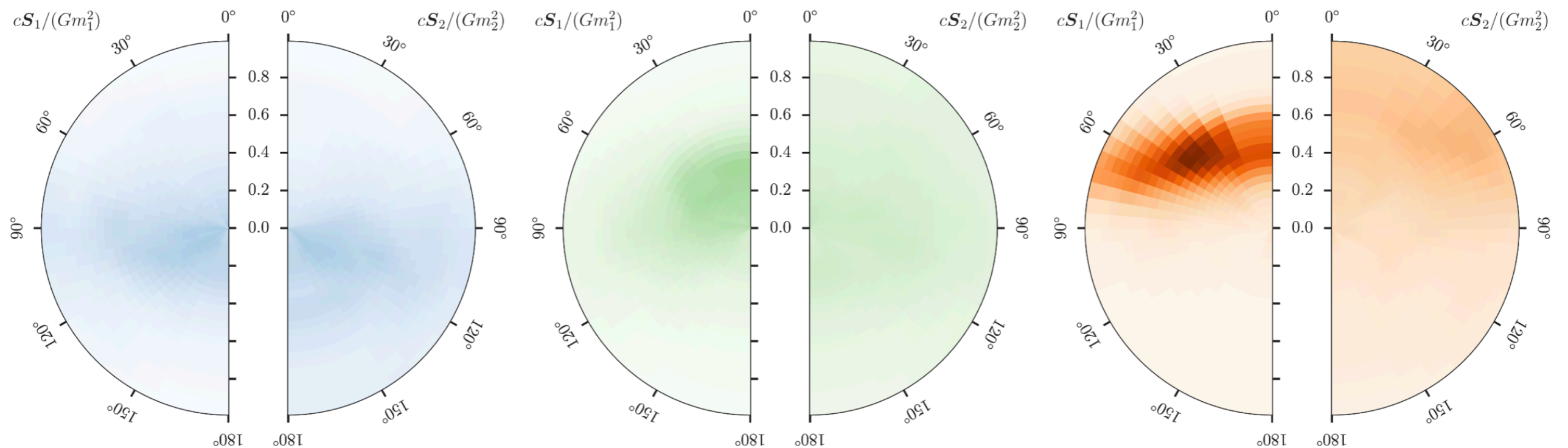
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Source: LIGO

# More Tools to Determine Progenitors: Localization

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Where do the GWs originate from?

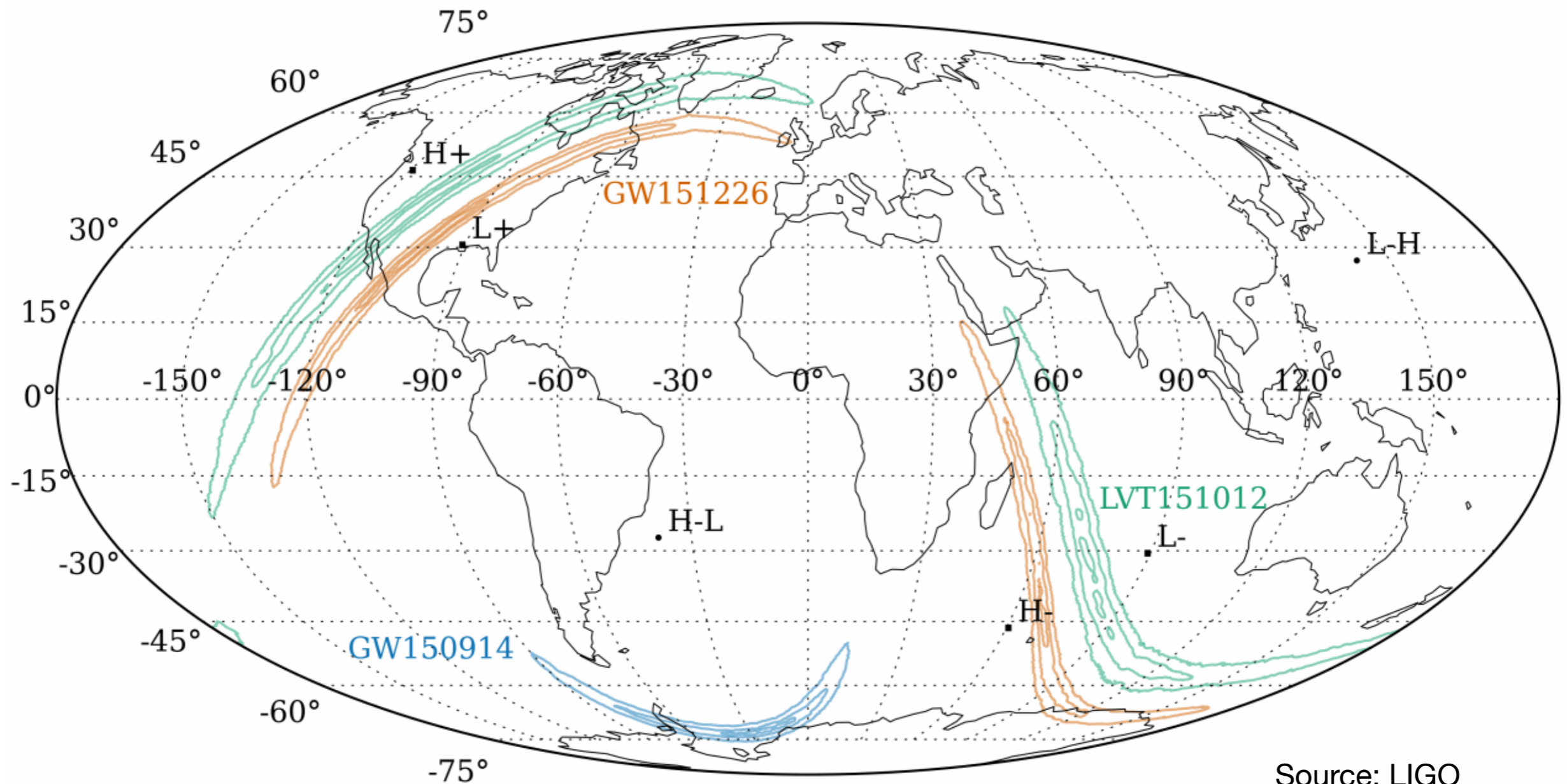
# More Tools to Determine Progenitors: Localization

Where do the GWs originate from?

GW150914: 230 deg<sup>2</sup>

LVT151012: 1600 deg<sup>2</sup>

GW151226: 850 deg<sup>2</sup>



Source: LIGO

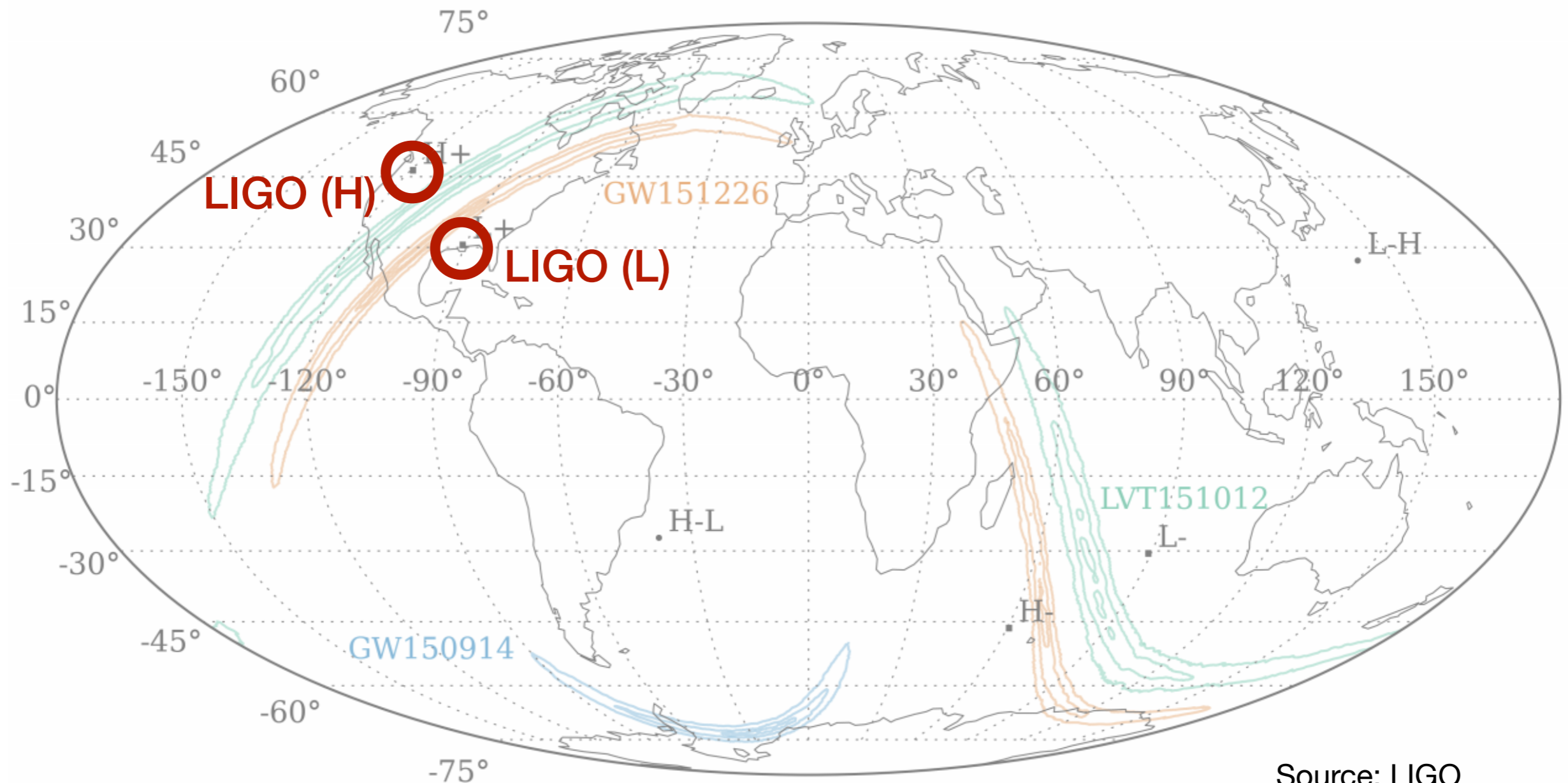
# More Tools to Determine Progenitors: Localization

Where do the GWs originate from?

GW150914: 230 deg<sup>2</sup>

LVT151012: 1600 deg<sup>2</sup>

GW151226: 850 deg<sup>2</sup>



Source: LIGO

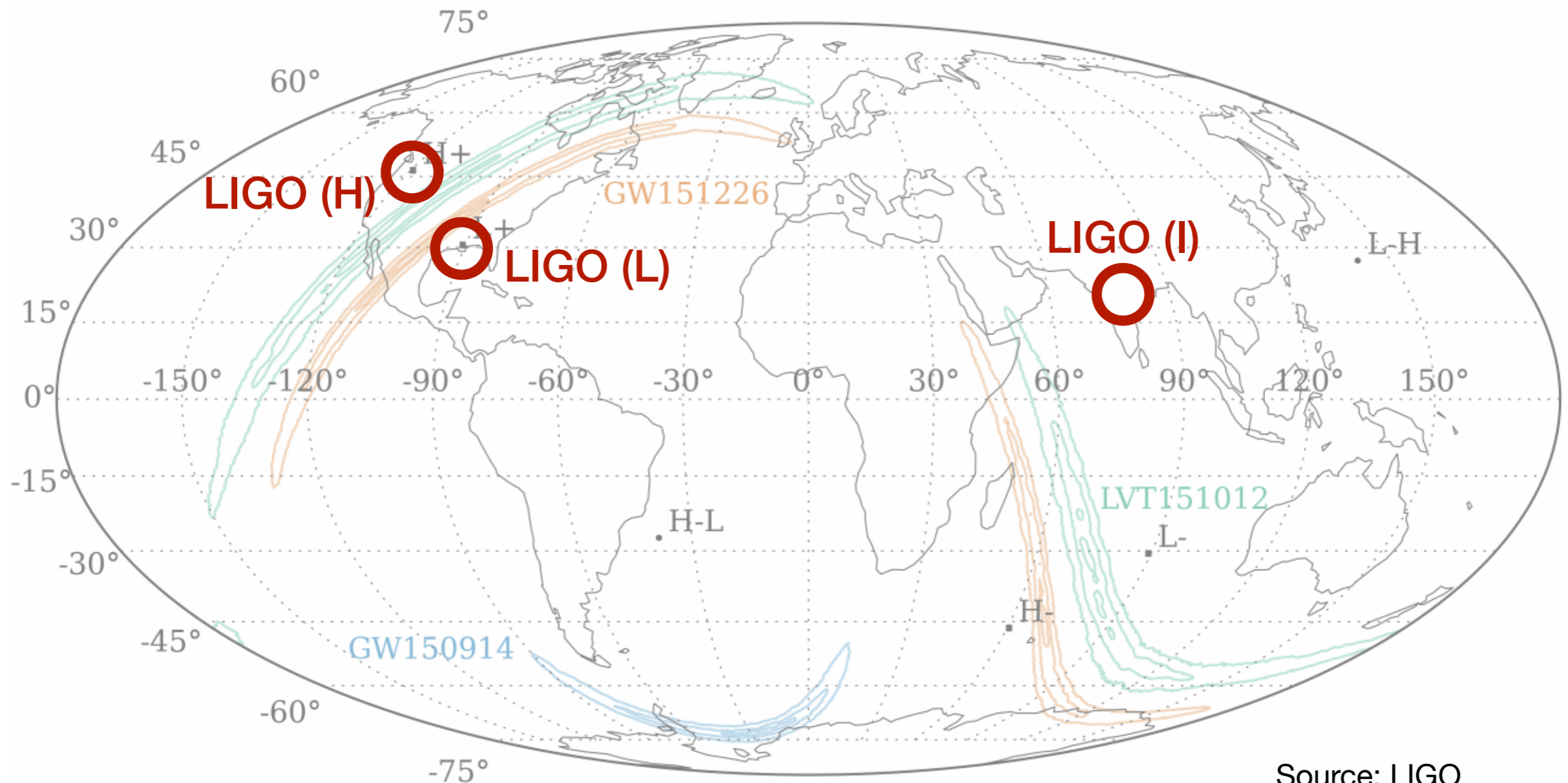
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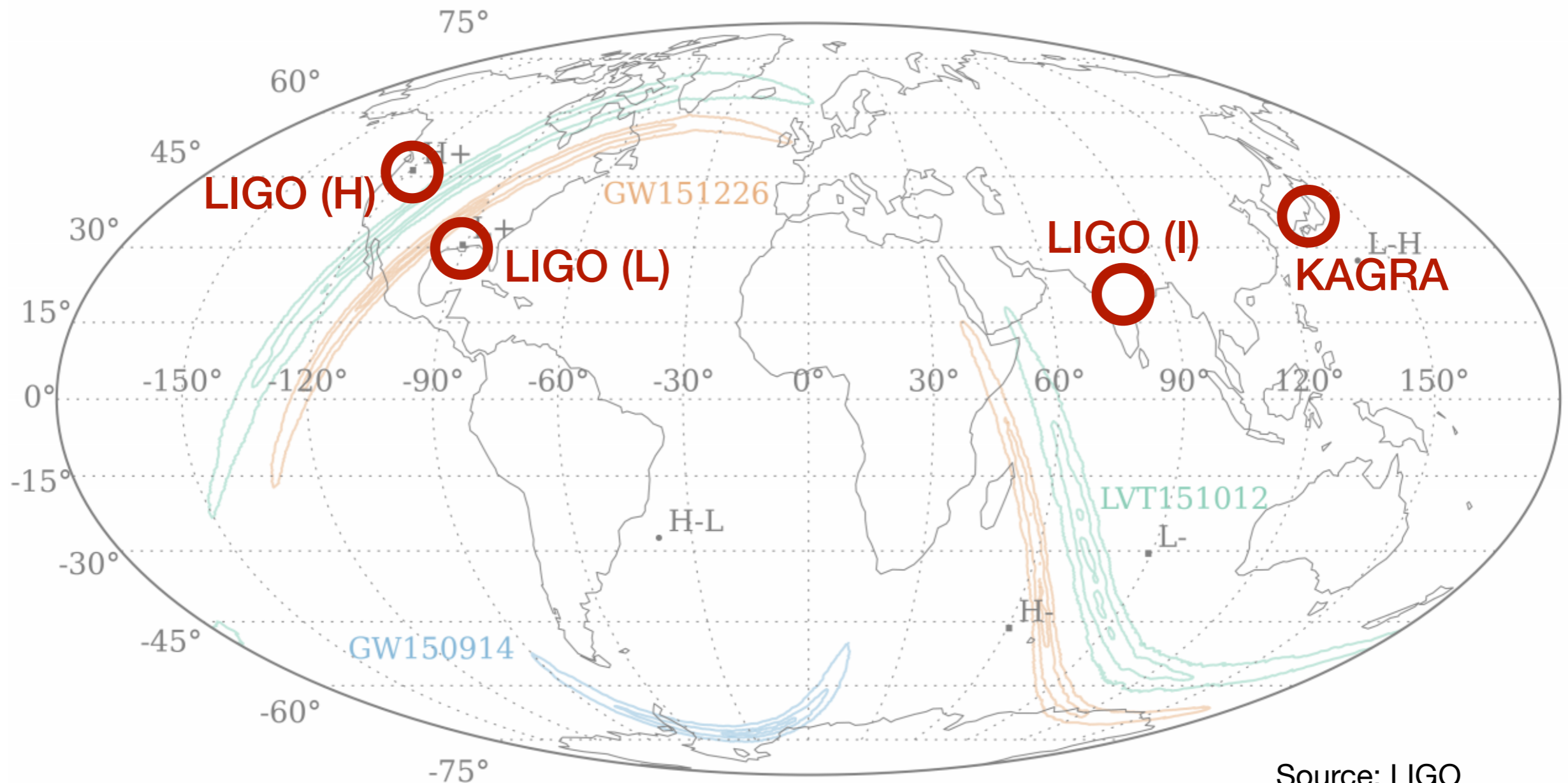
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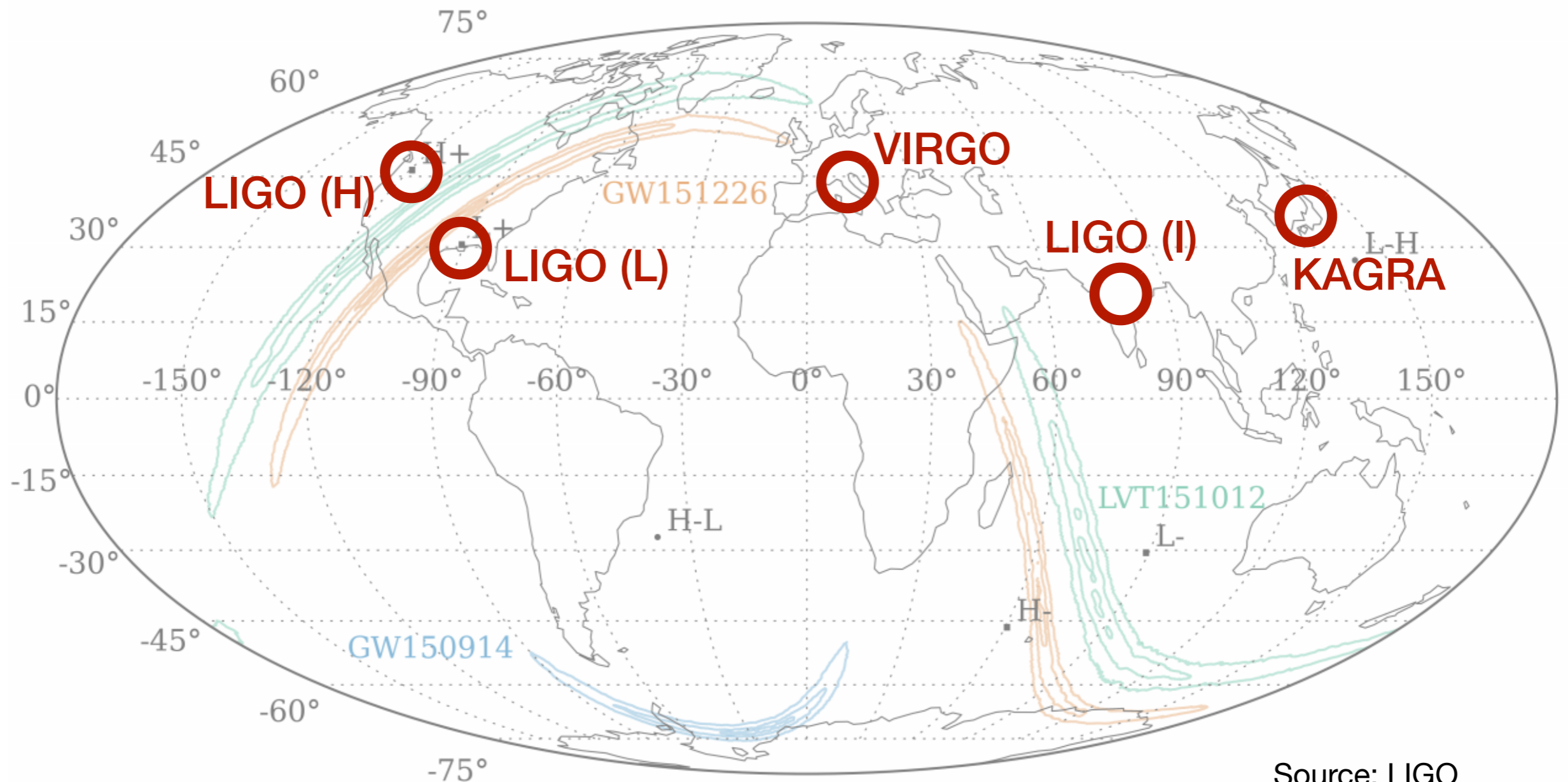
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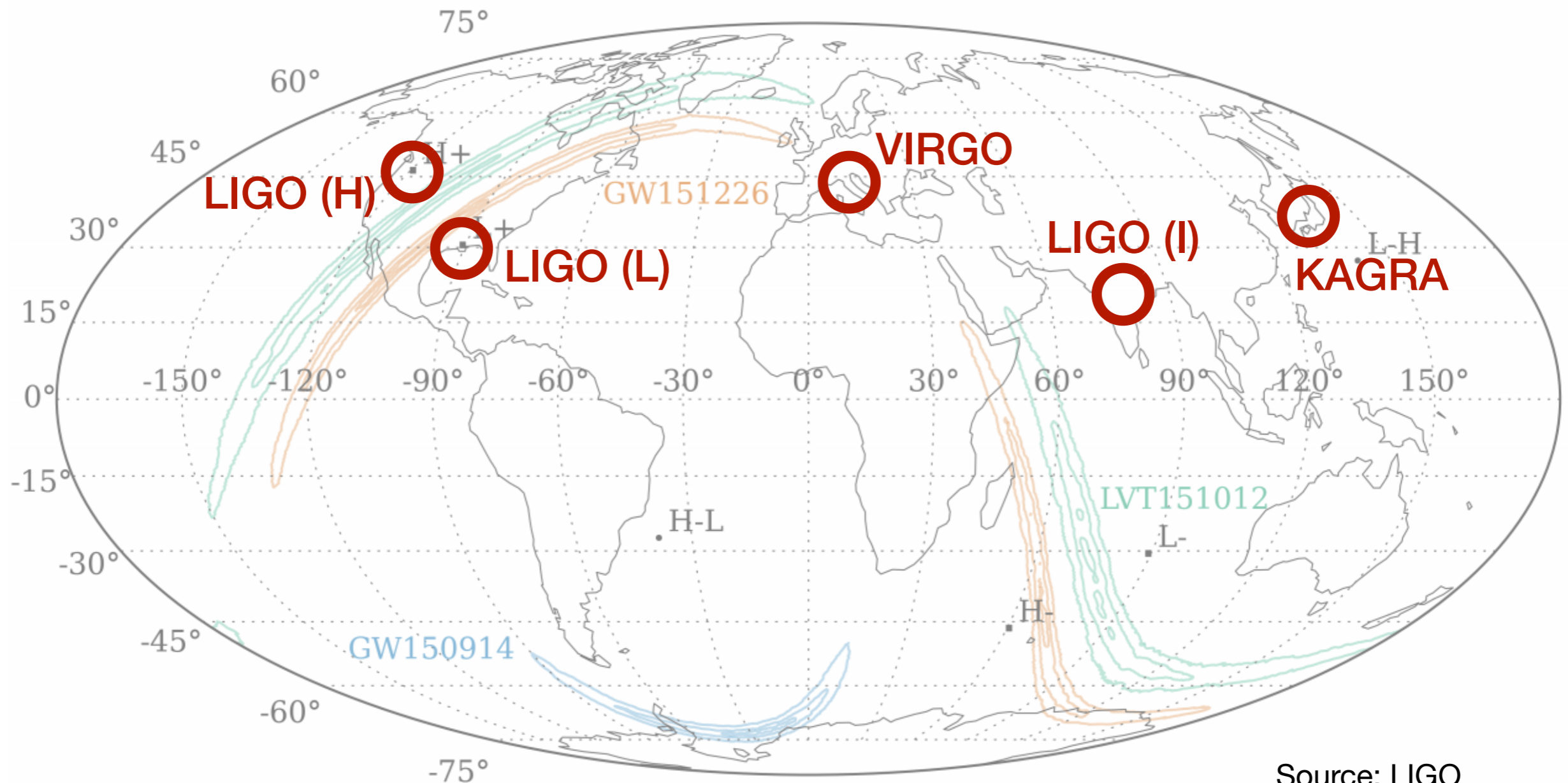
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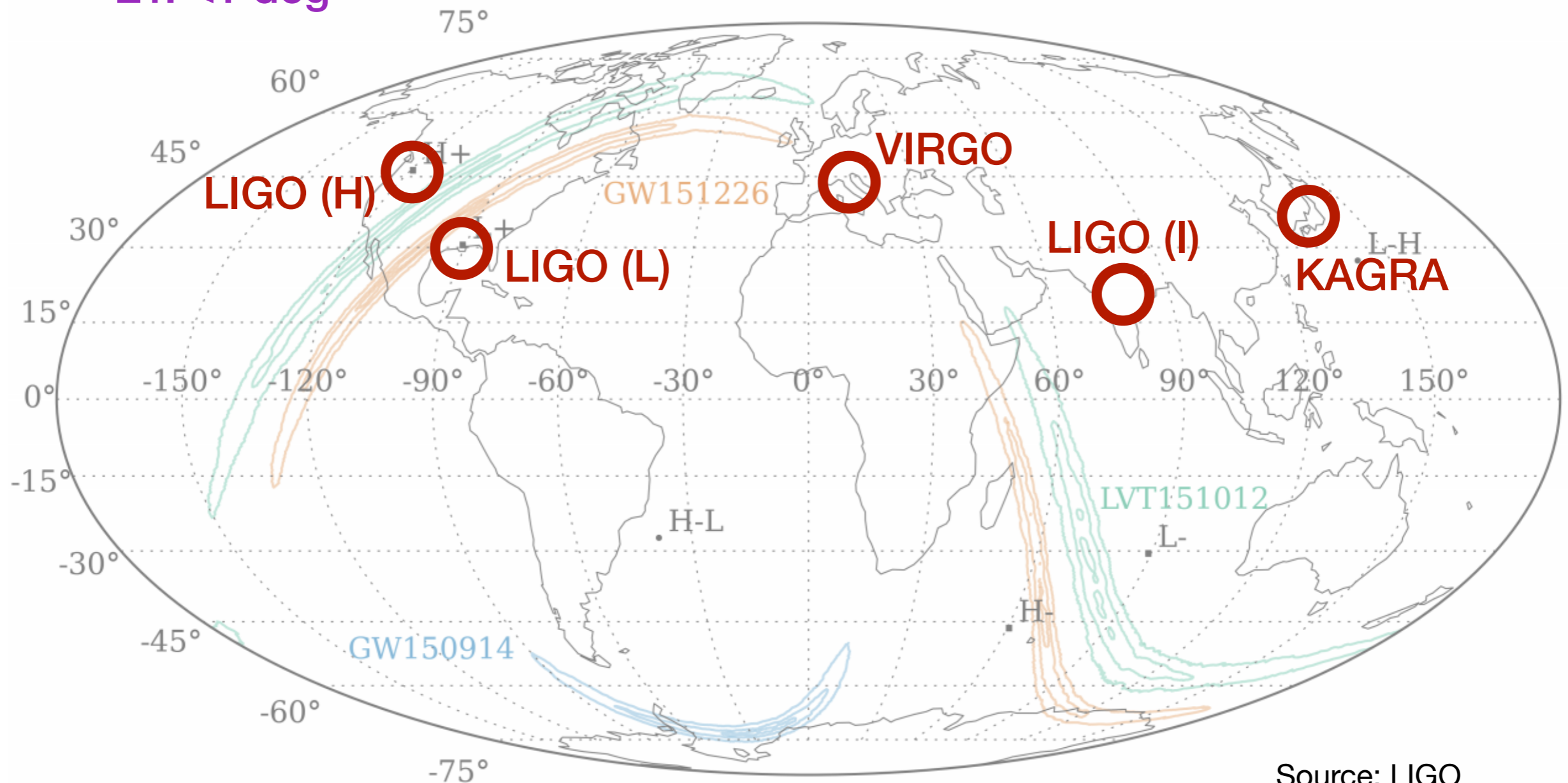
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# Cross-Correlation of GW map with Galaxy Surveys

(Raccanelli, EDK et al., PRD 94 (2016))


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Angular cross-correlation:  $C_\ell^{\text{GW,gal}} = \langle a_{\ell m}^{\text{GW}} a_{\ell m}^{\text{gal}} \rangle = 4\pi \int \frac{dk}{k} \Delta^2(k) W_\ell^{\text{GW}}(k) W_\ell^{\text{gal}}(k)$

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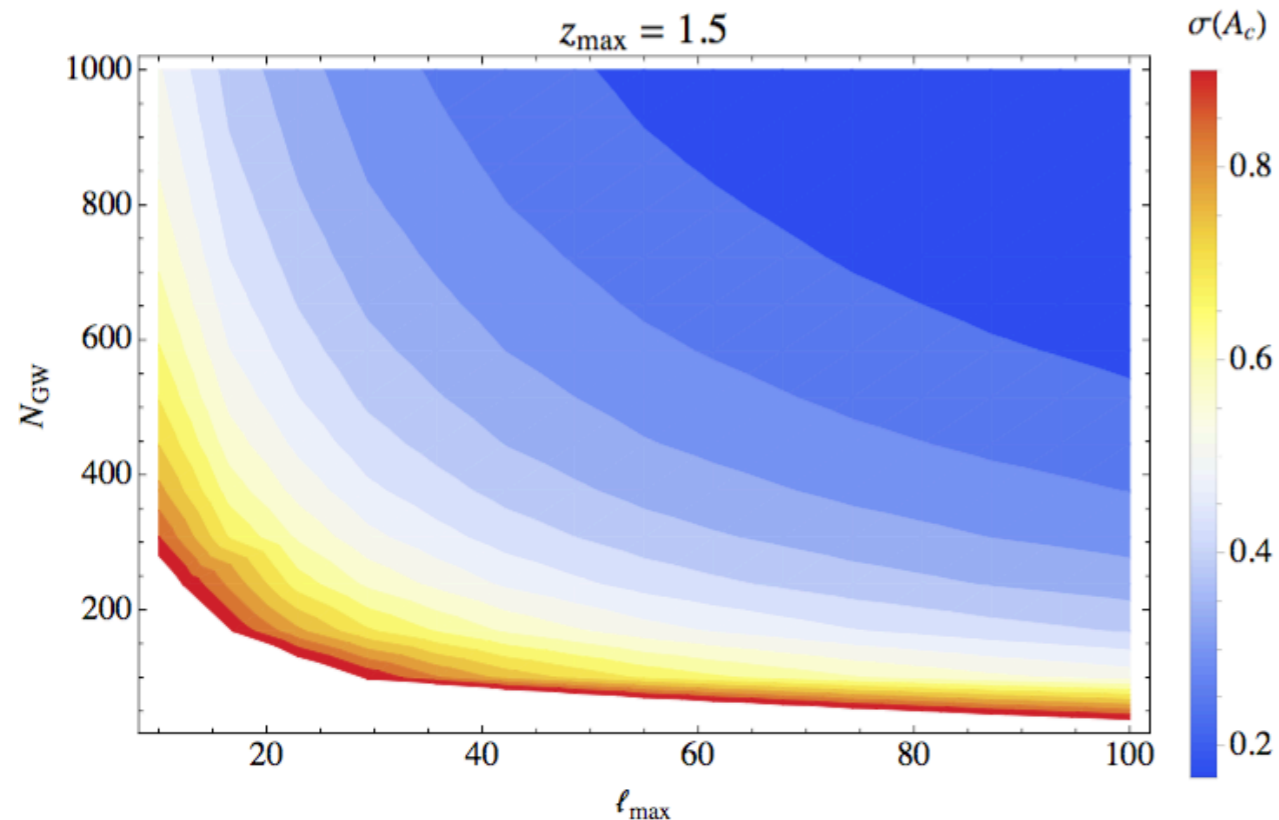
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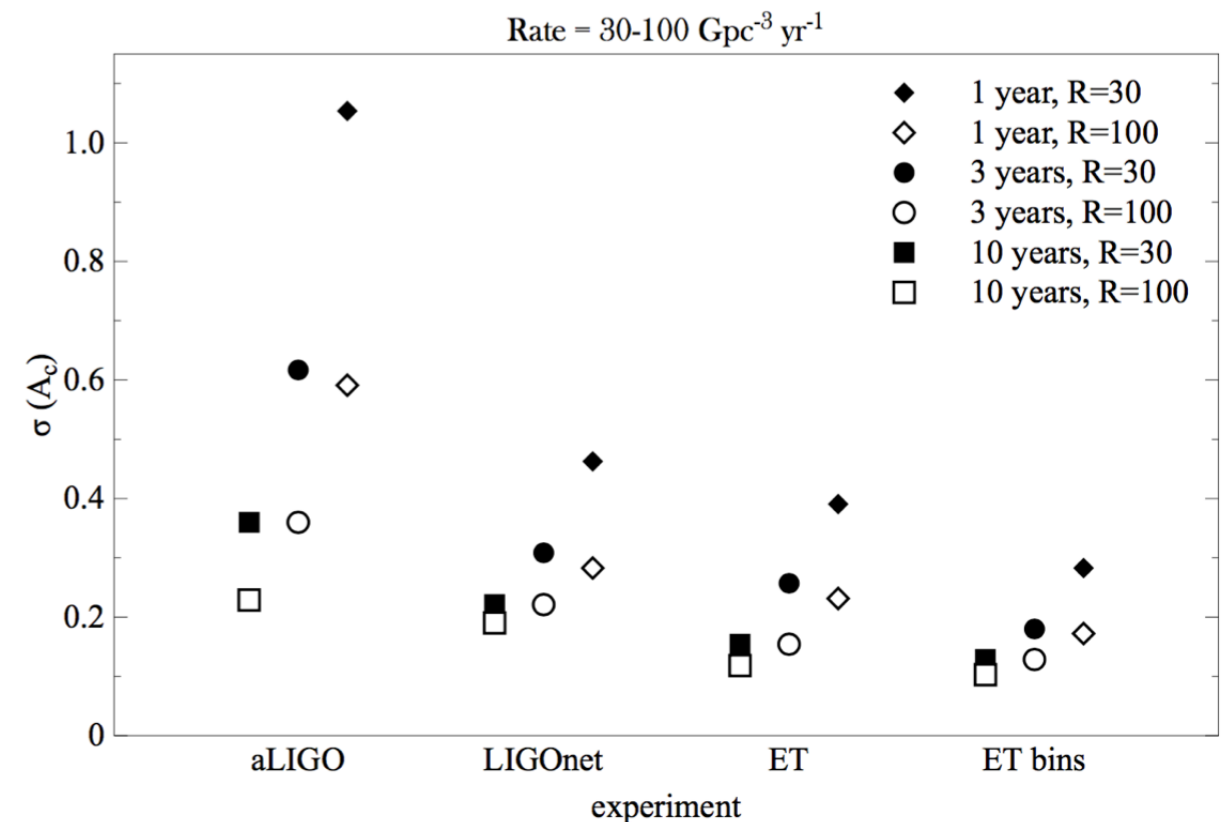
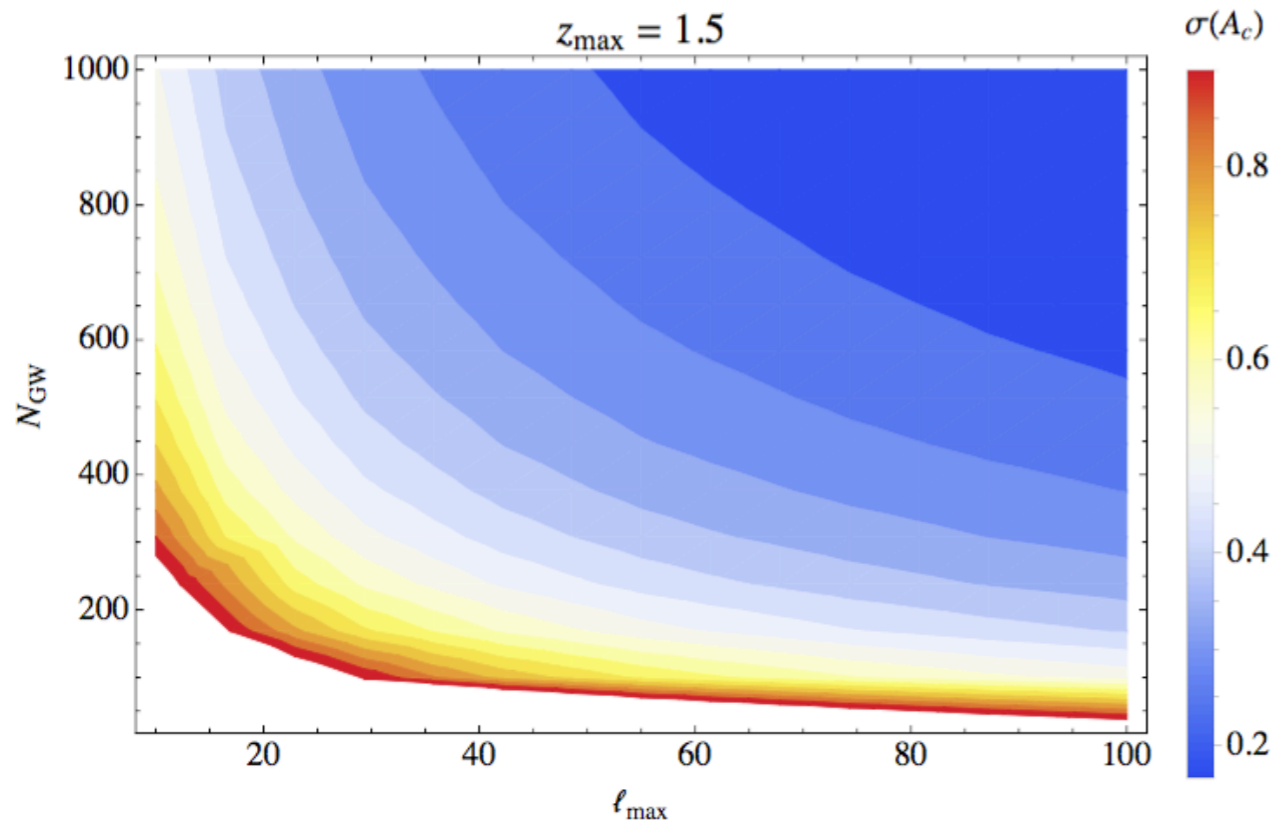
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# Outline

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- The Mass Function of Merging Black Holes
- The History of Black Hole Mergers

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- The Mass Function of Merging Black Holes
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- Summary and observational outlook

# Observational Outlook

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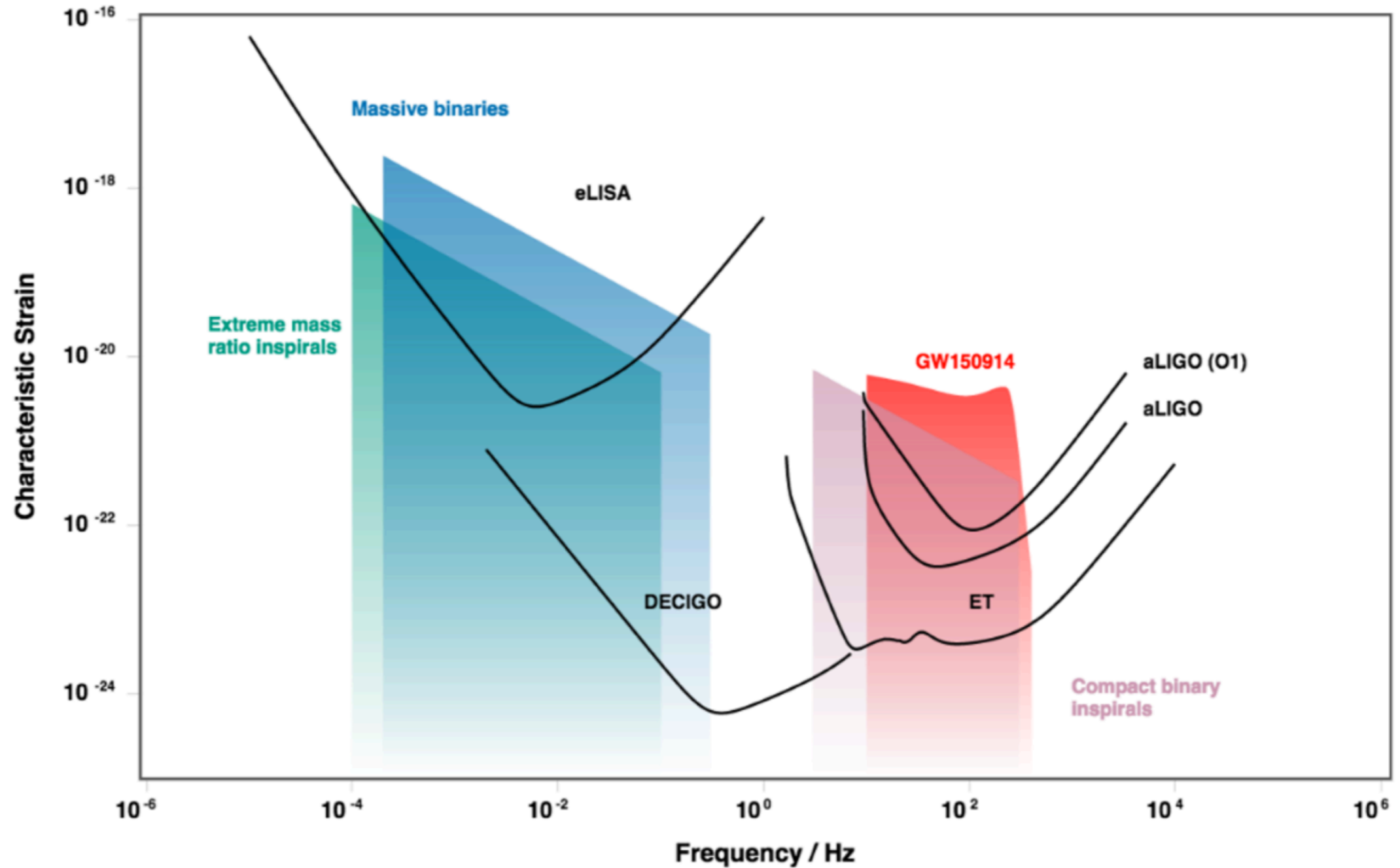
# Observational Outlook

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Gravitational waves:

# Observational Outlook

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# Synergy between Different GW Probes

(In progress, with J. Muñoz)

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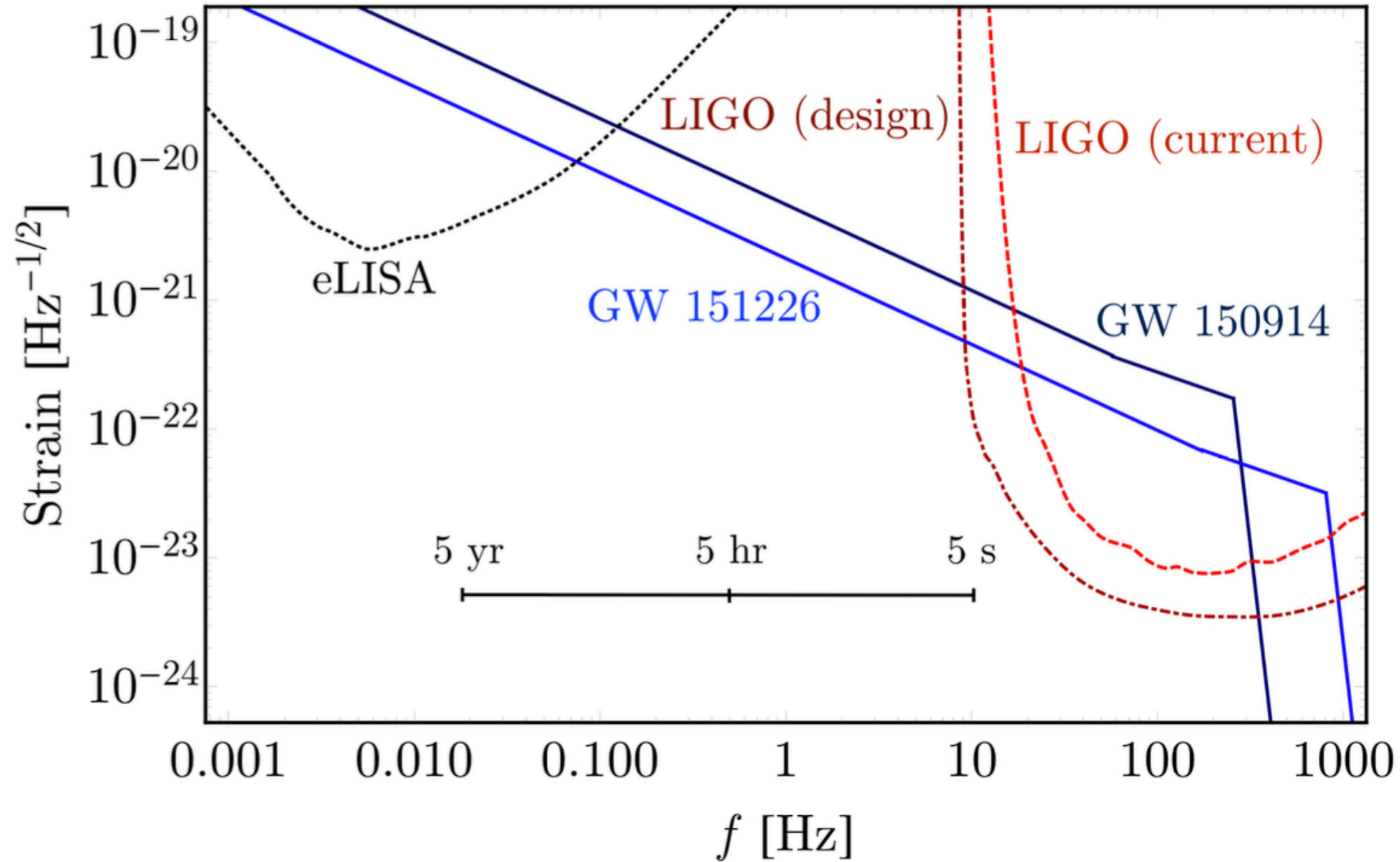
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In the future:

# Synergy between Different GW Probes

(In progress, with J. Muñoz)

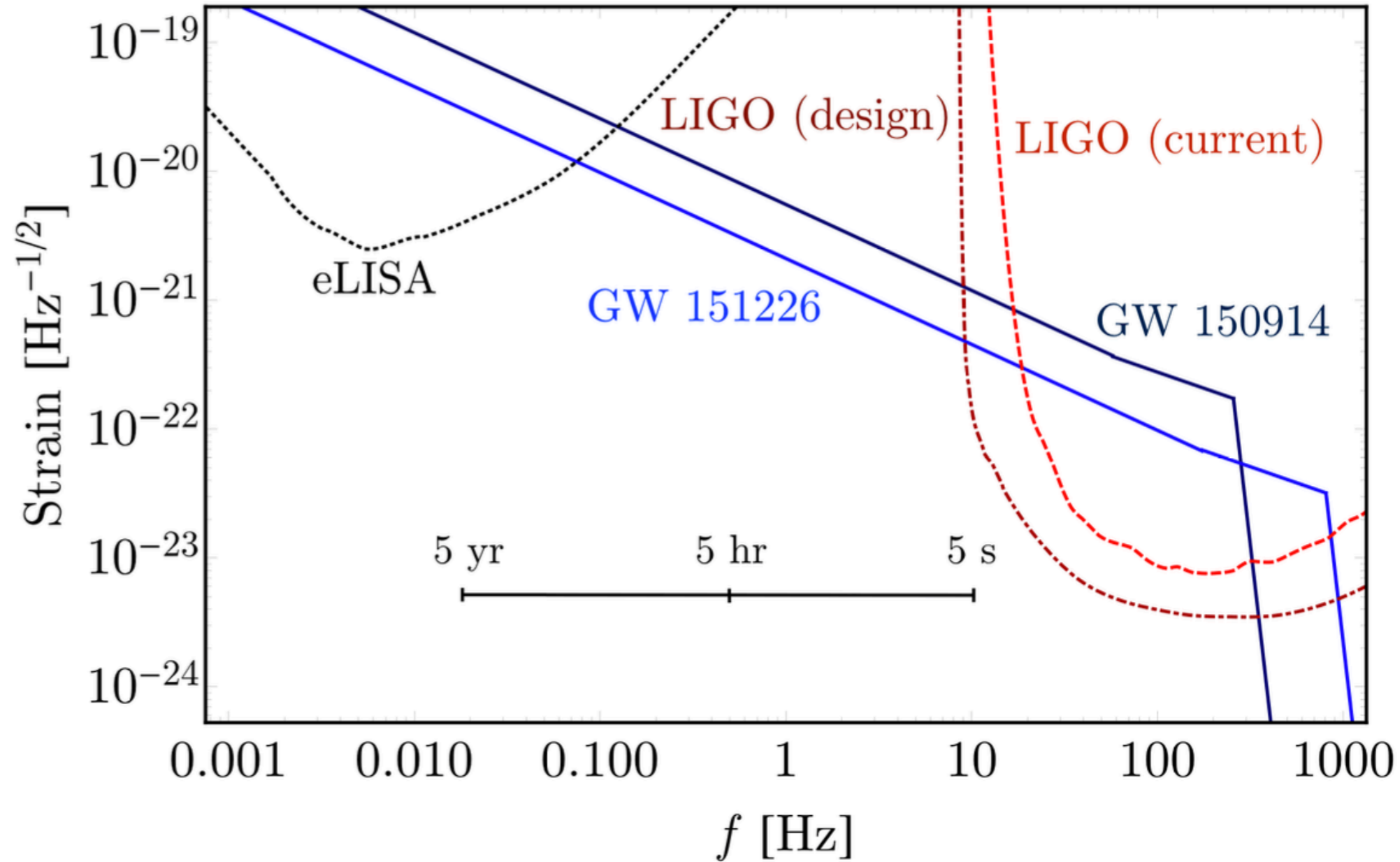
In the future:



# Synergy between Different GW Probes

(In progress, with J. Muñoz)

In the future:



We will be literally probing the history of the mergers!

# Cosmology with merging Black Holes?

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Standard sirens?

# Cosmology with merging Black Holes?

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Standard sirens?

SMBHs? counterparts? delensing?

# Observational Outlook: Experiment Timeline

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Experiment

2015

2020

2025

2030

beyond →

# Observational Outlook: Experiment Timeline

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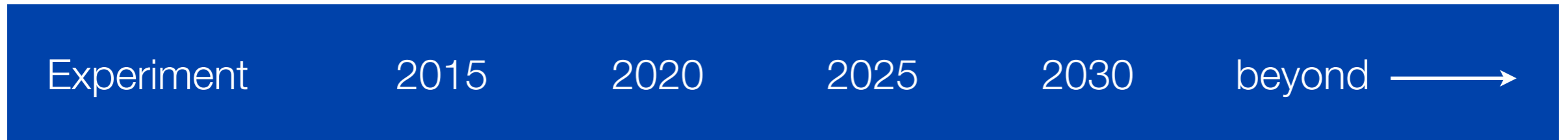


**aLIGO (O1+)**



# Observational Outlook: Experiment Timeline

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aLIGO (O1+)

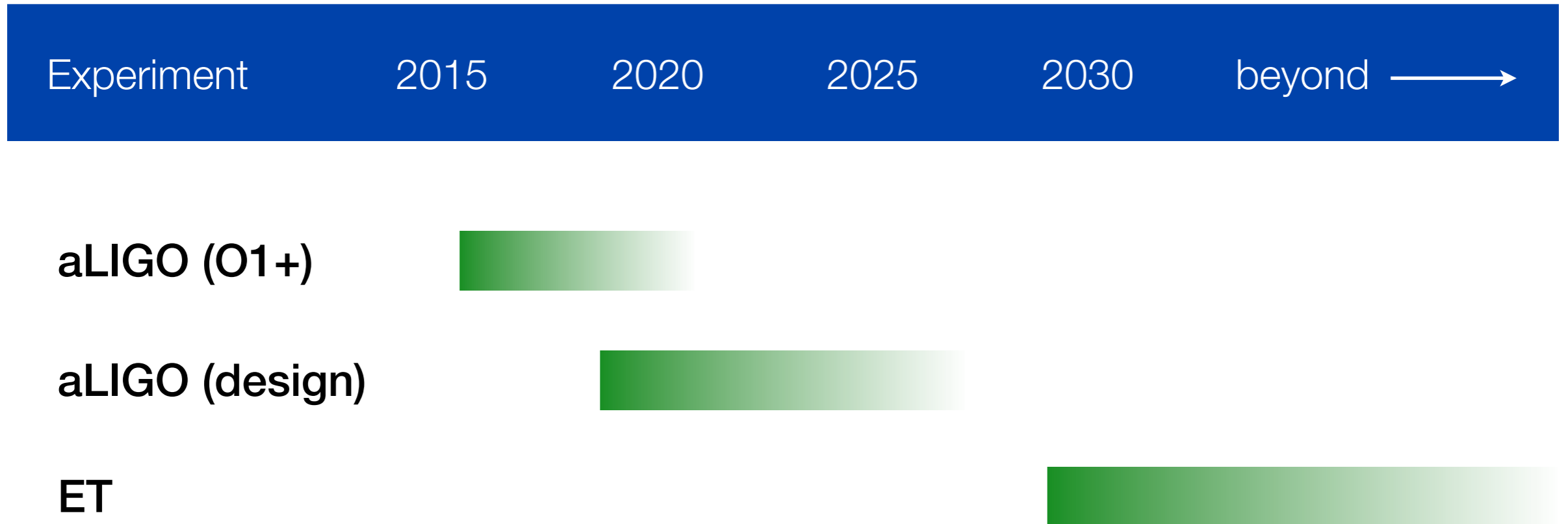


aLIGO (design)



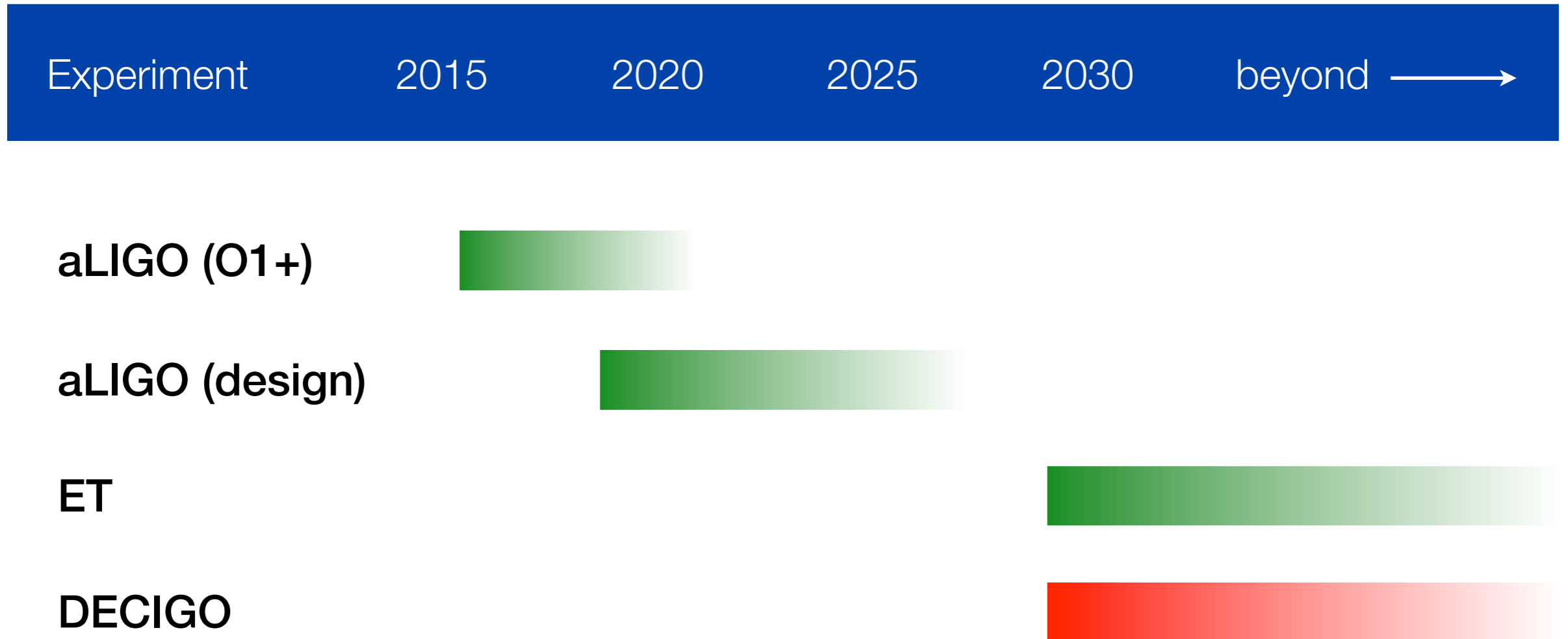
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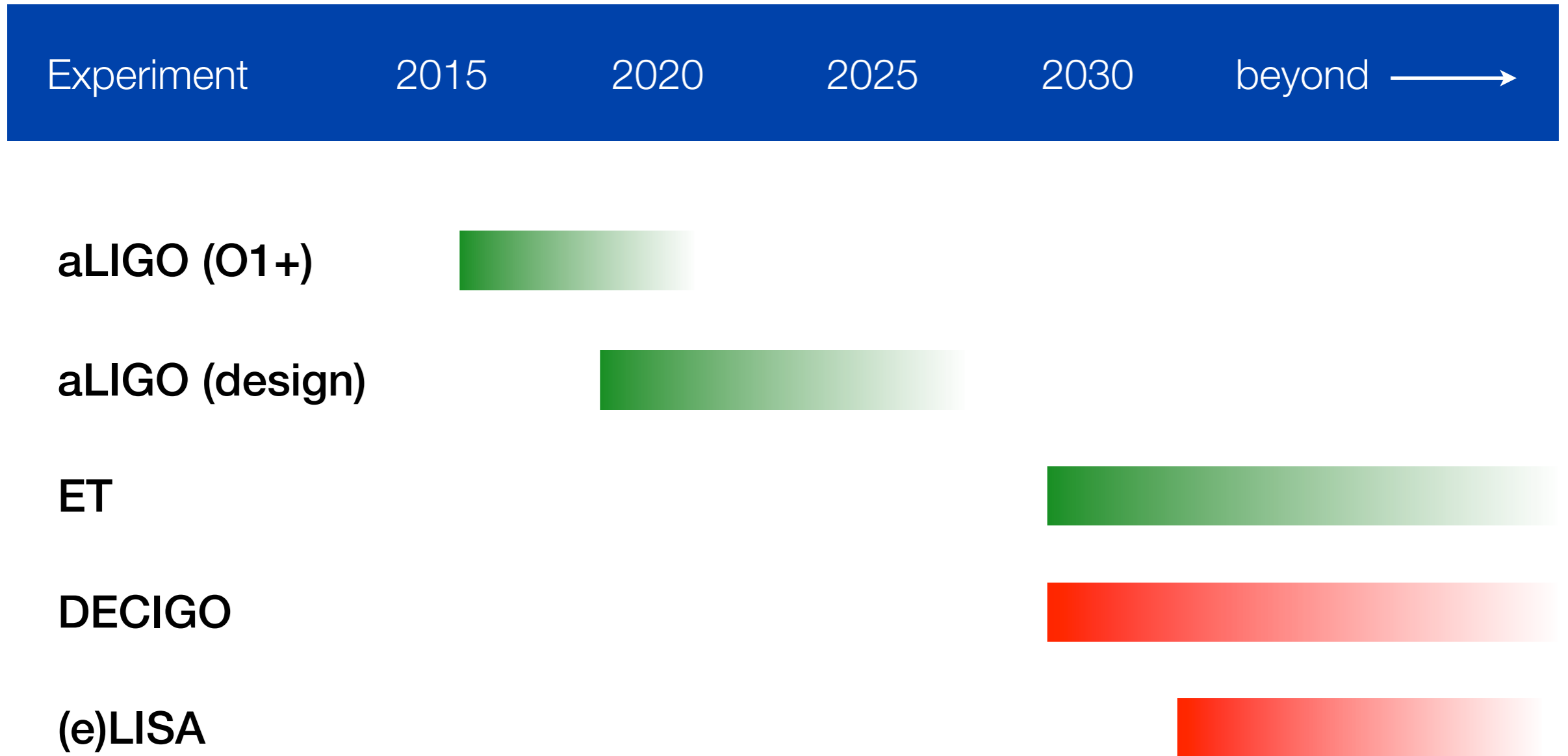
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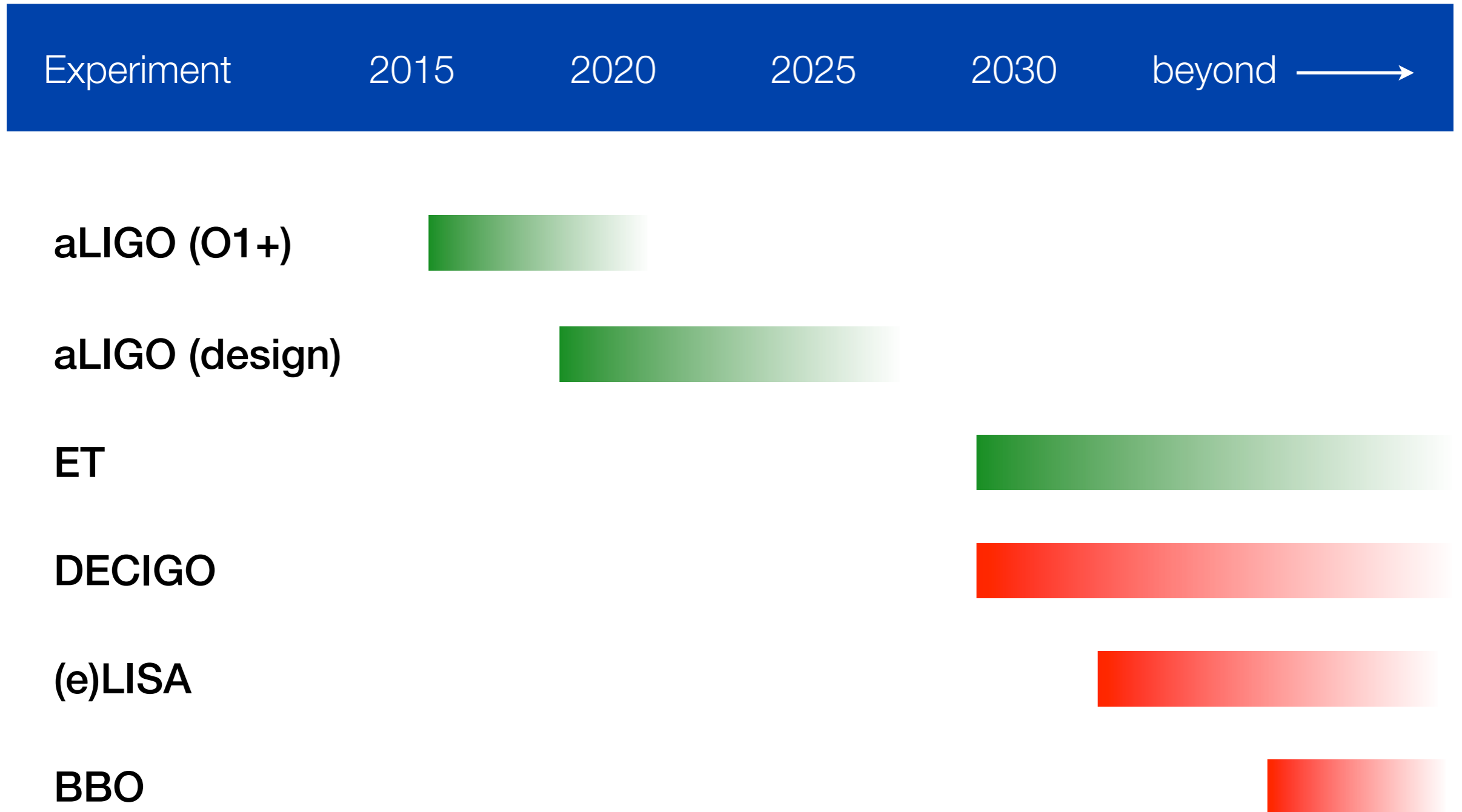
# Observational Outlook: Experiment Timeline

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# Observational Outlook: Experiment Timeline

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# Conclusions

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Merging black holes provide unique and ... tool to probe astrophysics,  
..... copy from paper

Lots of tools at our disposal: Mass, eccentricity, spin, points of origin

We can learn about:

- Stellar physics: black hole formation, stellar winds....
- Galaxy evolution: star-formation history, metallicity,
- Binary black hole progenitors: isolated, dynamical....
- Hubble expansion.....

# Teaser:

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# Teaser: Did LIGO Detect Dark Matter?

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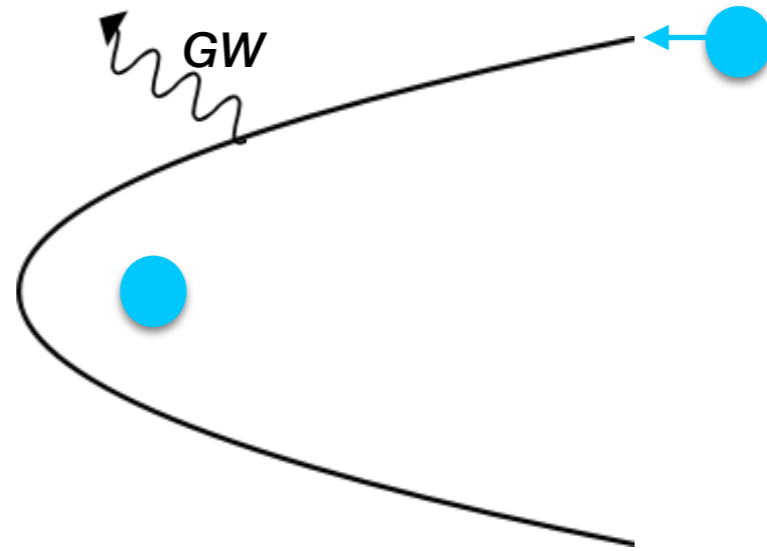
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**Was GW150914 a merger of Primordial Black Holes?**

# Teaser: Did LIGO Detect Dark Matter?

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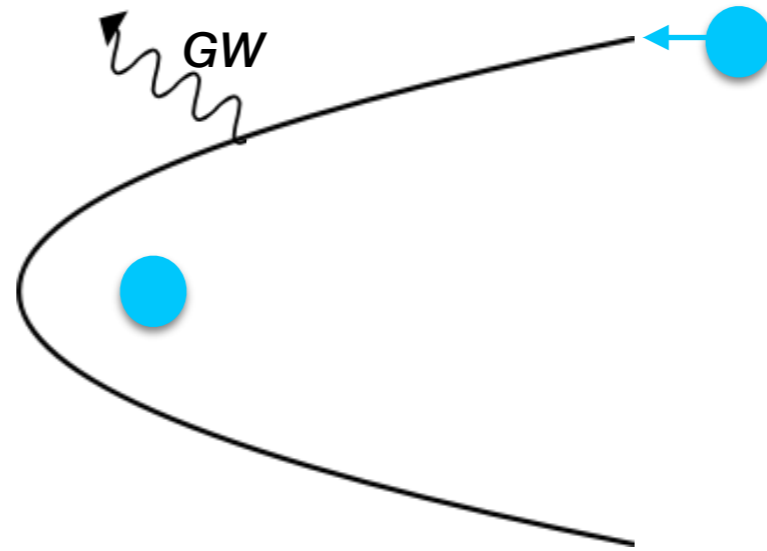
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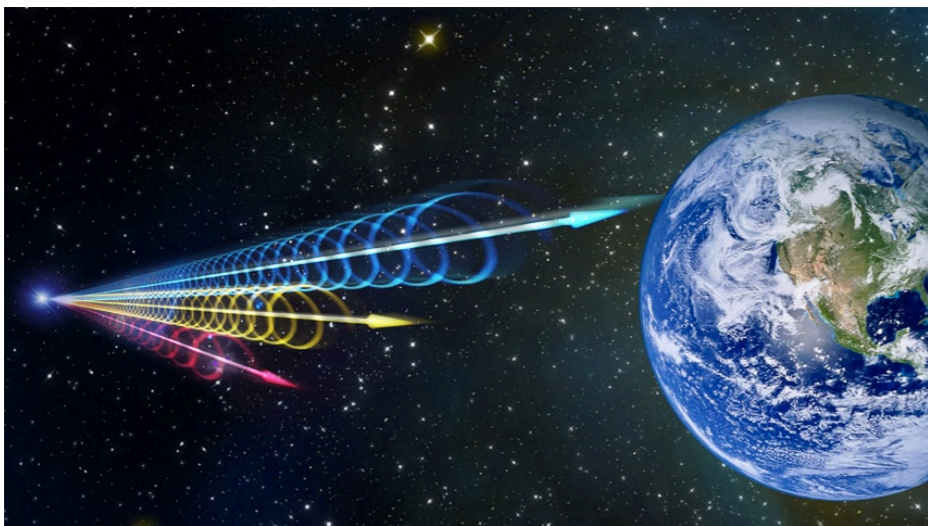
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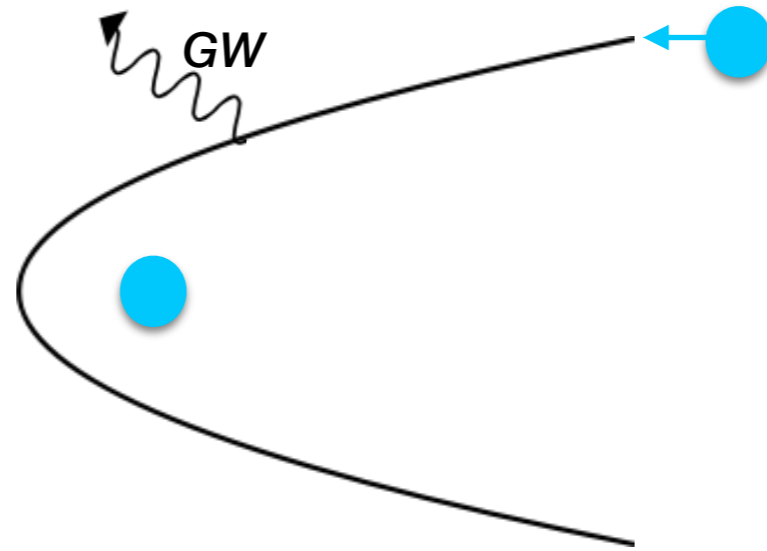
What could this have to do with Fast Radio Bursts?



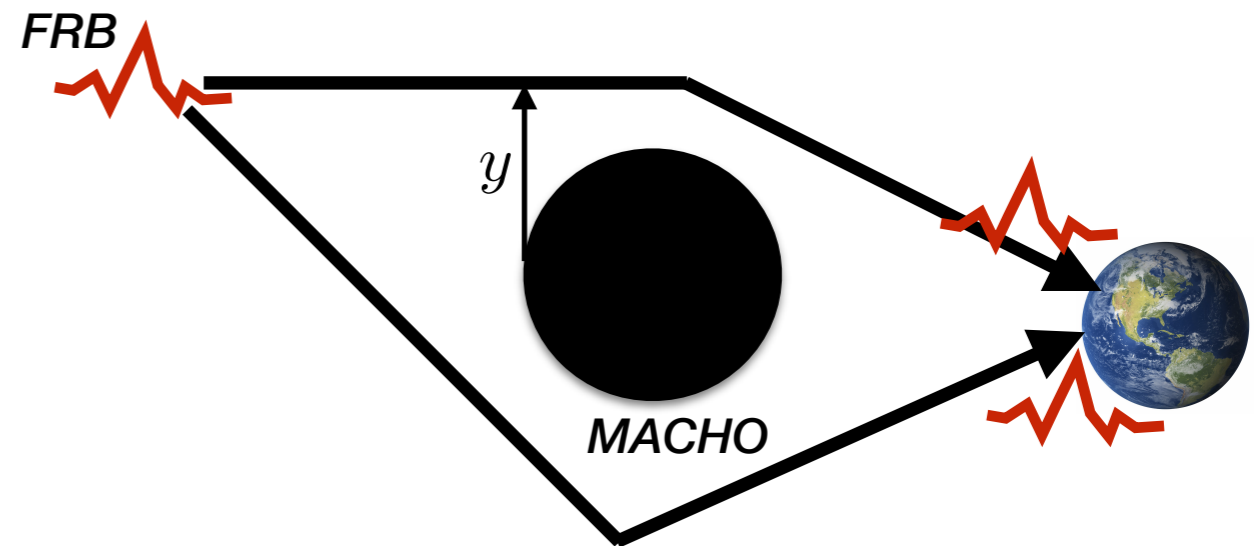
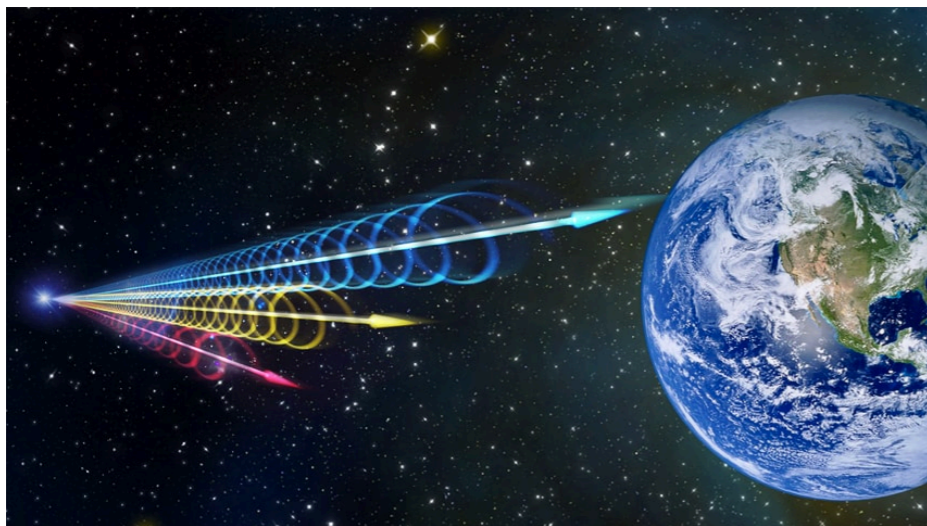
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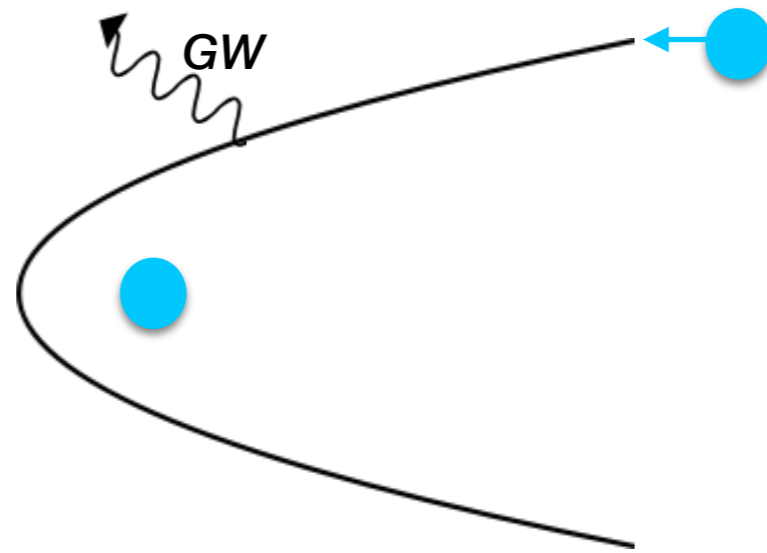
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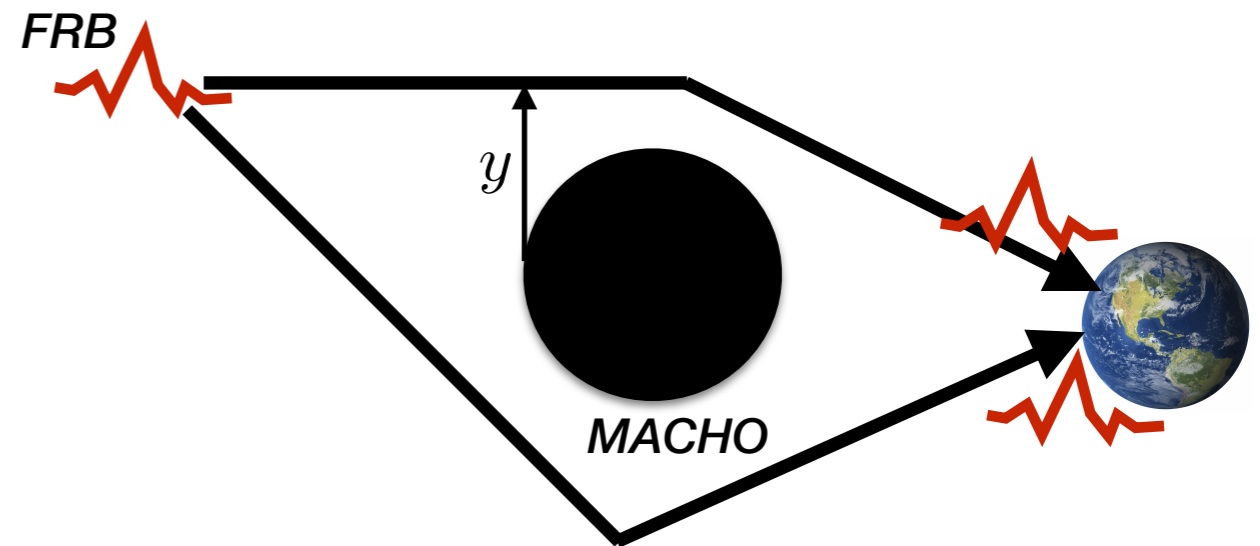
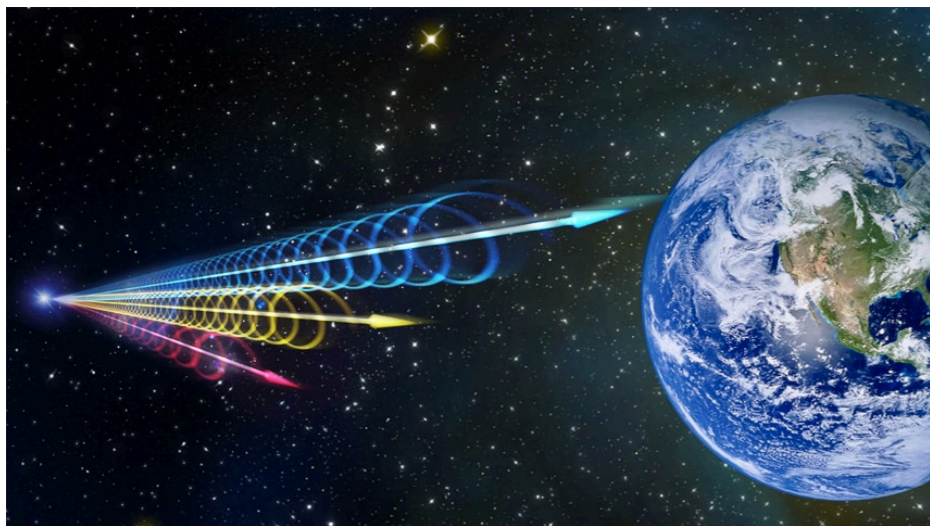
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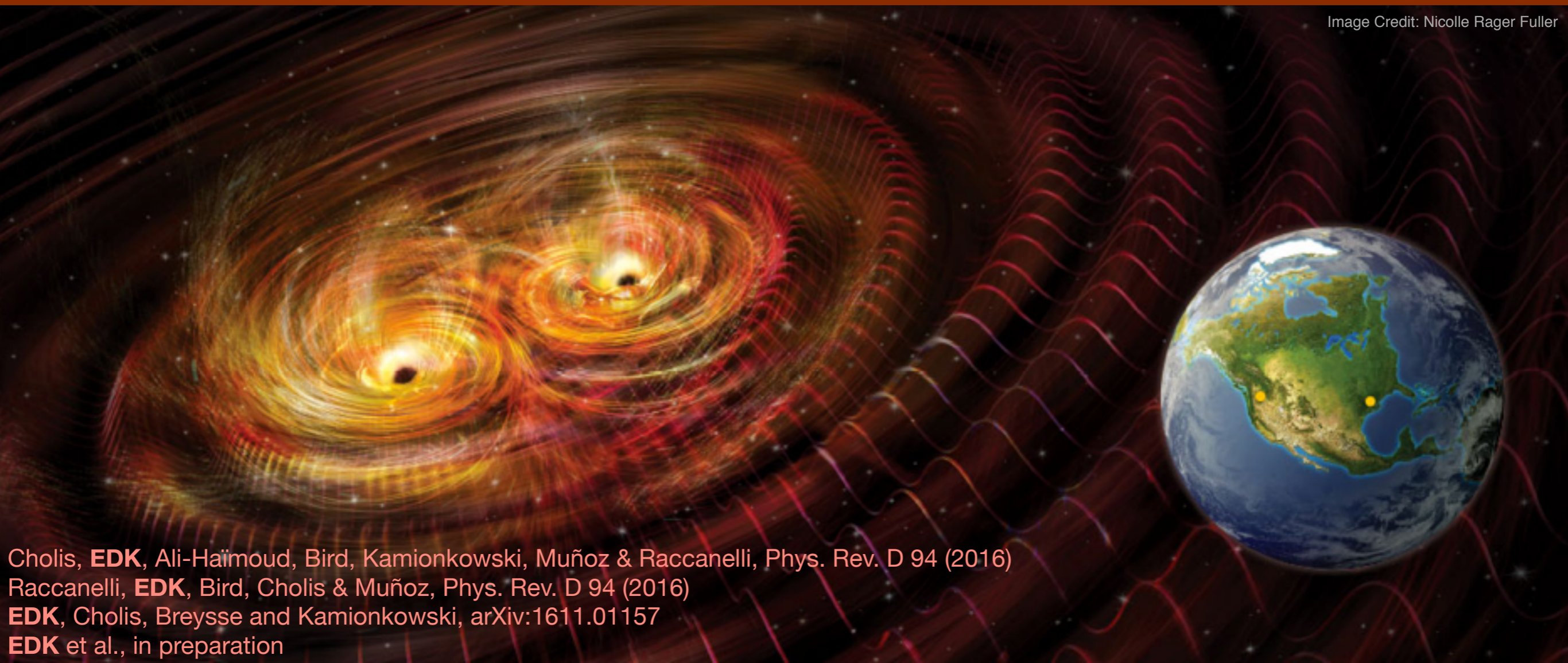
Tomorrow, 10:00am @Holzblatt Auditorium

# Thank you!

**Ely D. Kovetz**

Johns Hopkins University

Image Credit: Nicolle Rager Fuller



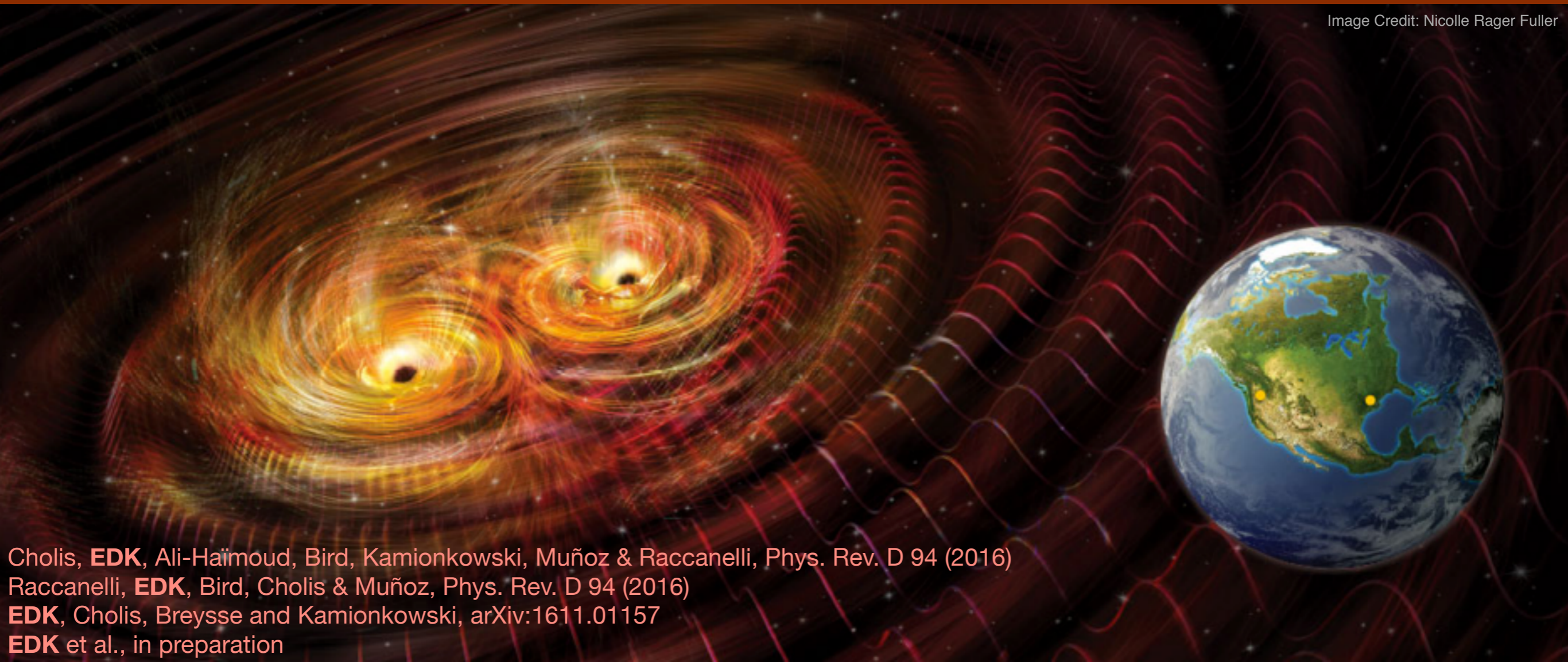
Cholis, **EDK**, Ali-Haïmoud, Bird, Kamionkowski, Muñoz & Raccanelli, Phys. Rev. D 94 (2016)  
Raccanelli, **EDK**, Bird, Cholis & Muñoz, Phys. Rev. D 94 (2016)  
**EDK**, Cholis, Breysse and Kamionkowski, arXiv:1611.01157  
**EDK** et al., in preparation

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