Physics Letters A 373 (2009) 1762–1767

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Two mechanisms of spiral-pair-source creation in excitable media

Y. Biton [a], A. Rabinovitch [a,*], I. Aviram [b], D. Braunstein [c]

[a] Physics Department Ben-Gurion University, Beer-Sheva 84105, Israel
[b] Guest, Physics Department Ben-Gurion University, Beer-Sheva 84105, Israel
[c] Physics Department Sami Shamoon College of Engineering, Beer-Sheva, Israel

ARTICLE INFO

Article history:
Received 24 December 2008
Received in revised form 5 March 2009
Accepted 12 March 2009
Available online 19 March 2009
Communicated by C.R. Doering

PACS:
87.19.Hh
87.19.Ug
87.10.-e
87.15.A-
52.35.Mw

ABSTRACT

Two new modes of generating spiral pairs in an excitable medium have been found. They depend on a geometrical structure (GS) inside the medium. This may be formed e.g. as a result of scars or fibrosis in the heart tissue, or artificially built in a chemical reaction substrate. Both sources involve a GS composed of a circular "convergent lens" bounded by two opaque "walls". One mode can be induced by a single wave and behaves as a "flip–flop" type of a limit cycle. The other mode is generated by a train of plane waves impinging on the GS, and is created at the focus of the converging wave-fragments.

1. Introduction

Excitable media are important in many areas [1]: The safe propagation of electrical information in the heart (3D) [2], and in neurons and axons (1D) [3], the appearance of intricate and repeating patterns in chemical reactions such as the Belousov–Zhabotinsky [4], the use of chemotaxis by amoebae to form multicellular structure [5], the modeling of combustion propagation, oscillations and fluctuations in various media [6], ecological dynamics ([7] and references therein), bifurcations and hysteresis in electrical circuits [8].

They are characterized by the following: when not stimulated or when the stimulus is below a certain threshold they remain stationary. Stimulation above the threshold usually creates a single pulse which propagates through the medium without change of shape. Following that, the system returns to its stationary state. No additional waves are created.

There are only a few ways of creating an internal continuous source in an excitable medium. Thus a spiral wave source can be created by the reentry mechanism [9] and by a subsequence of two stimuli, the second appearing during a "vulnerable window" [10]. A spiral source can be created by the detachment effect [11,12]. A wave impinging on an opaque wall of finite length, instead of turning back along the wall, can, under some conditions, detach from it and form a spiral. A hole in an otherwise opaque wall can lead [13] in this fashion to a spiral-pair source. Another spiral pair source can be obtained by a pair of pulses having a specific geometrical shape [14]. Recently, mixed-mode temporal oscillations sources were realized by inhomogeneous boundary conditions [15] and a source of traveling waves was obtained by a resonance-inducement of a pacemaker [16].

Such "ectopic" sources, unwanted in this case, can appear for example, in scarred heart tissue [9], causing severe malfunctions in the natural heart electrical wave progress. The geometrical shape of the scar can have major significance in determining the disturbance type. A well-known case is a ring shaped scar which can lead to a spiral wave source by the reentry mechanism. And, as will be presently shown, a scar of a different special shape can induce, in two different ways, the creation of a spiral-pair source. Such a spiral pair in the heart is denoted by "figure of eight" and can lead to heart problems such as tachycardia and fibrillation (see e.g. [17–20] and references therein)

A different mechanism leading to a creation of different geometric structures in the heart is fibrosis [21]. Fibrosis is known to be conducive to various heart deceases, although the exact mechanisms of this arrhythmogenecity are not known [22]. We discuss here a special geometric structure (GS) which can lead to a spiral pair source creation thereby inducing such malfunctions.

In this study two new methods to create spiral pair sources are numerically simulated and discussed. It is demonstrated that such sources can be created in specific geometrical structures, under suitable excitability conditions.

* Corresponding author. Tel.: +97286461172; fax: +97286472903.
E-mail address: avinoam@bgu.ac.il (A. Rabinovitch).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2009.03.030

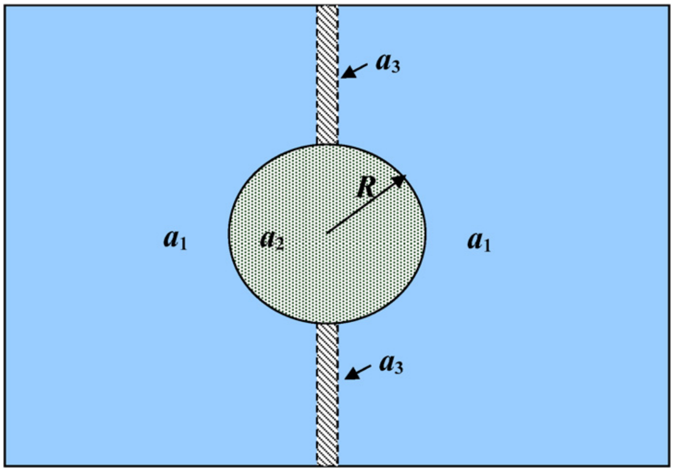

Fig. 1. The GS, and the corresponding a parameters: a central circle (CR), of radius R with $a = a_2$, is surrounded by a medium with $a = a_1$ ($a_1 < a_2$). Hatched gray areas designate the opaque walls with $a = a_3$ ($a_1 < a_2 \ll a_3$).

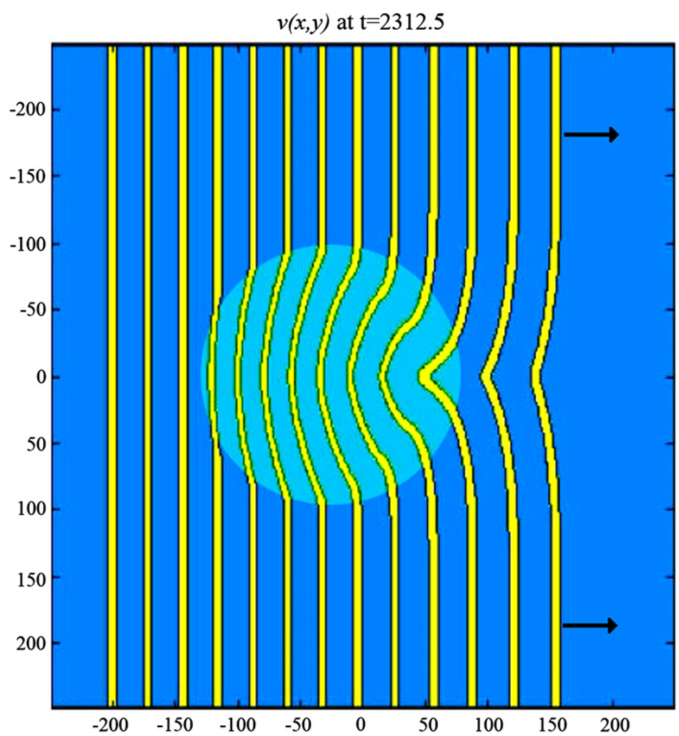

Fig. 2. A train of plane-waves passing through a higher a CR with *no* walls (a la Ref. [24]), $a_1 = 0.12$ and $a_2 = 0.16$. Note the curving of the waves, such that the rays perpendicular to them converge towards a focus ("lens" effect). Yellow color filled contours indicate the space where the wave's amplitude is larger than 0.1. Arrows indicate the wave propagation direction. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

2. The model

In our two-dimensional (2D) simulations we consider the following FitzHugh–Nagumo (FHN) system [23]:

$$\frac{\partial v}{\partial t} = D\nabla^2 v + f(v, w),$$
$$\frac{\partial w}{\partial t} = g(v, w), \tag{1}$$

where all the variables are dimensionless. Here, v is an activator (the action potential in the heart tissue), and w is an inhibitory variable (refractivity). The functions $f(v, w)$ and $g(v, w)$ are given by:

$$f(v, w) = v(v - a)(1 - v) - w, \qquad g(v, w) = \varepsilon(v - dw). \tag{2}$$

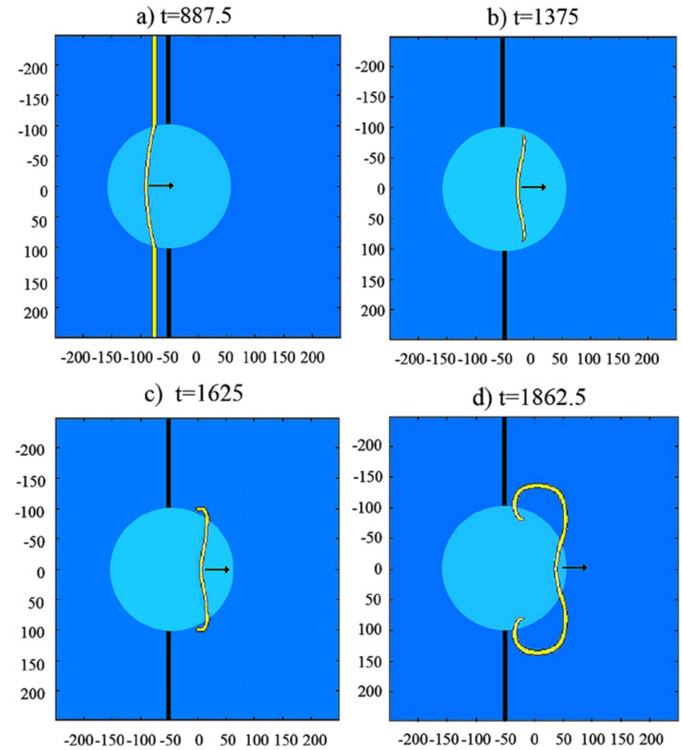

Fig. 3. The transient part of the flip–flop sequence. $R = 100$, $a_1 = 0.12$, $a_2 = 0.152$, $a_3 = 0.24$. All other parameters have the same values as those in Section 2. A single plane wave impinges on the GS from the left (a); Its outer parts are cut by the opaque walls and its shape curves inside the CR (b); Emerging from the CR, the free edges rotate backwards (c), creating a spiral pair (d).

The coefficients D, a, and ε are, respectively, the diffusion constant, the excitability parameter, and the ratio between the fast and the slow time constants (a small parameter). The constant d depends on the application of the model. It defines the slope of the w null-cline. An infinite d marks the Van-der-Pol system while a small enough d leads to a case of three fixed points, two of which are stable. We use the value $d = 3$, which keeps the system in the excitable regime. We solve the FHN system in a 250×250 point (x, y) plane grid using explicit finite-difference numerical method. The typical discretization step values were $\Delta t = 0.25$ in time and $\Delta x = \Delta y = 0.5$ in space. Recalculation with smaller steps showed the approximation error to be very small and the two sets of results were visually indistinguishable.

We solve the equation in excitable medium with different a values (see below) and with Neumann boundary condition. Fig. 1 displays the GS used here: a circular region (CR), with radius R and $a = a_2$. Two walls externally adjoin the circle, and have a very high value of $a = a_3$, making them 'opaque' to waves. The entire structure is embedded in a surrounding medium with $a = a_1$, where $a_1 < a_2 \ll a_3$. The other parameter values are: $D = 0.2$, $\varepsilon = 0.005$, $d = 3$. The CR constitutes a less excitable medium than the surrounding medium; the wave velocity there will be smaller. Consequently, the CR acts as a "convergent lens", i.e. (see Fig. 2) rays—lines orthogonal to the waves passing through it—converge towards a focus.

Fig. 2 displays the passage of plane-waves through a medium with a circular lens, *but no adjoining walls* [24] for $a_1 = 0.12$, $a_2 = 0.16$ and $R = 100$. Note the lens effect of the CR and the complete connectivity of the waves. The ratio between the wave velocities

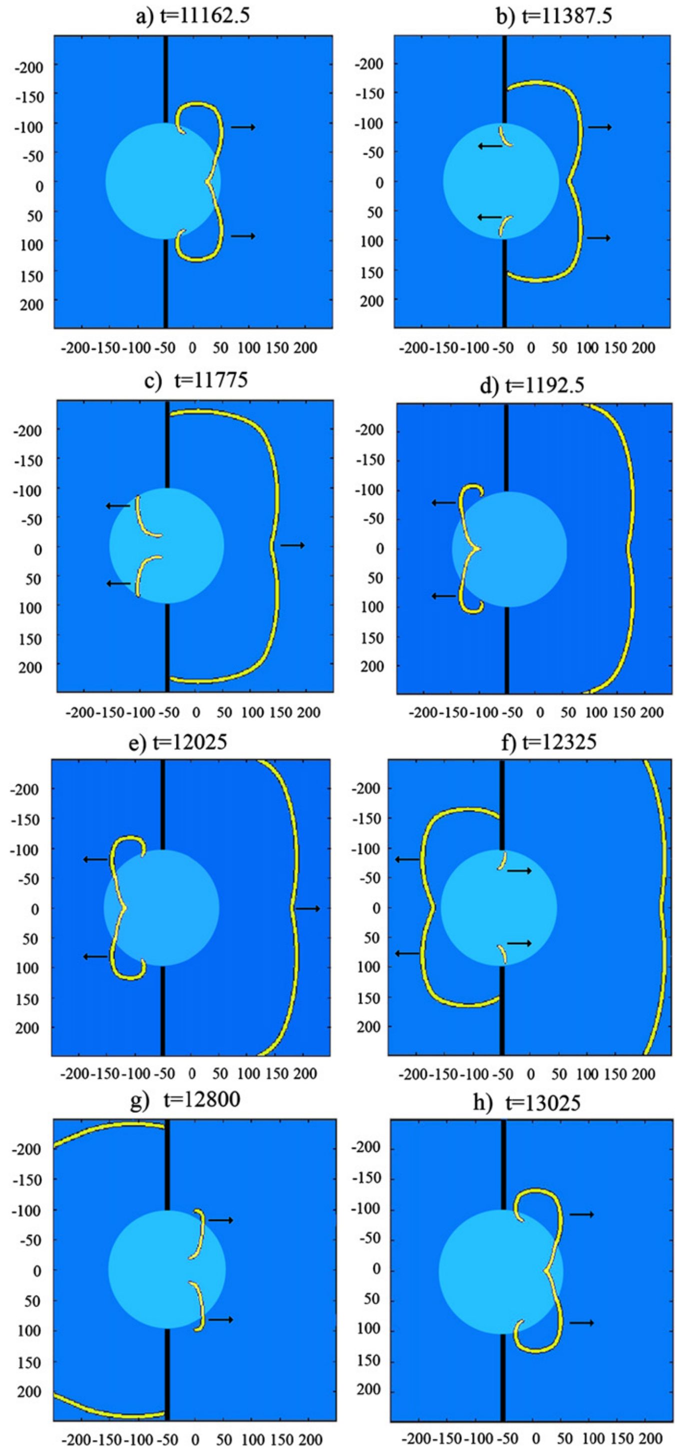

Fig. 4. The flip–flop type of spiral-pair source: A typical cycle. The arms of the spiral pair created by the wave (see Fig. 3) rotate backwards, encounter the CR (a) and the opaque walls (b) thus leaving behind two fragments, which are moving to the left while curving in the CR (c). The fragments coalesce (d), and create a spiral pair (e), which, while moving to the left, its arms rotate in the right direction, encountering the opaque walls and the CR (f). New fragments are thus created in the other side of the CR (g) returning to the initial structure (h).

outside and inside the CR is $V_{a1}/V_{a2} = 1.35$. This lens effect was theoretically obtained [24] and experimentally demonstrated [25] in chemical reactions.

It is precisely in order to avoid connectivity, and allow wave propagation only through the circular region, that the two walls with a high $a_3 = 0.24$ were added to the GS.

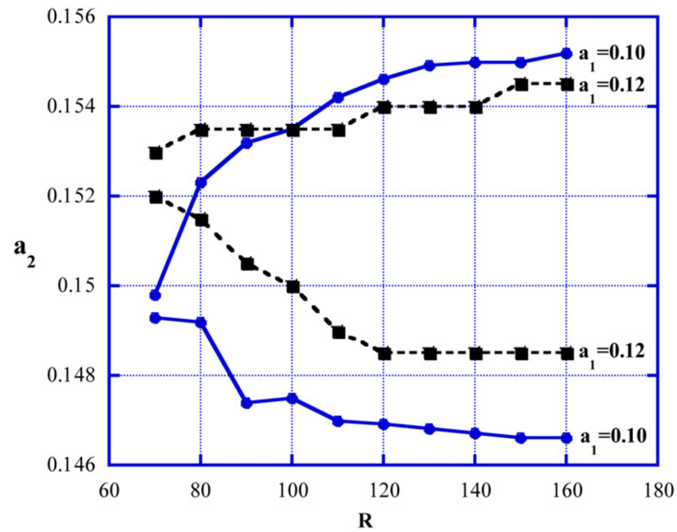

Fig. 5. A map of possible a_2 values generating a flip–flop behavior for $a_1 = 0.12$ (squares) and $a_1 = 0.10$ (circles). The Area between the two lines denoted by squares (by circles) is the region where a flip–flop can be created for $a_1 = 0.12(0.10)$.

3. Flip–flop induced spiral pairs

Figs. 3 and 4 describe the method to develop a periodic flip–flop type of spiral pair source. For this process, we used: $R = 100$, $a_1 = 0.12$, $a_2 = 0.152$ and $a_3 = 0.24$. All other parameters have the same values as those in Section 2. The sequence starts at $t = 0$ when a *single* plane wave is launched at $x = -240$. This wave propagates to the right with a constant velocity, until reaching the left edge of the CR. Inside the circular region the wave slows down since this is a less excitable medium ($a_2 > a_1$), while outside it reaches the opaque walls (Fig. 3a). The outer parts of the wave are then cut off. Fig. 3b shows that the wave segments close to the free tips are ahead of the median segment. This is due to the fact that they have traveled a shorter distance inside the slow medium than the latter. The free tips themselves start spiraling backwards *before* crossing into the fast medium ([26,27] and see a discussion of the function of the walls in the last section of this work). The entire wave segment however remains *connected* (Fig. 3c). The wave then gradually takes the form of a "digit 3" figure, and develops until (Fig. 3d) its ends penetrate back into the slow CR region. This is the end point in time of the transient sequence, when the basic wave shape has formed. Call this the "flip" position. From here on, one witnesses a periodic sequence, as explained below.

Fig. 4 exhibits a typical cycle of the periodic part, picked up some 9300 time steps later, starting at a flip position, when the digit 3 figure moves outwards (Fig. 4a) until its backwards spiraling branches encounter the opaque walls. Partial annihilation occurs at the contact, leaving three separate fragments: a median fragment propagating outwards to the right, and two small fragments which enter the CR, Fig. 4b. These small fragments expand, approach each other and join together while propagating to the left (Fig. 4c), and eventually form an "epsilon"-shaped wave segment (Fig. 4d). Next, the free ends of the epsilon leave the CR, while spiraling backwards relative to the motion of the full segment (Fig. 4e)—the "flop" position. Further on, the epsilon undergoes a similar process of fragmentation as the digit 3, i.e. the median segment continues its motion to the left, while the end segments penetrate back into the CR (Fig. 4f). The next two Figs. 4g and 4h show the reconstruction of the initial digit 3: cycle completed. The period duration is $\sim$1860 time units.

The flip–flop phenomenon is highly dependent on the set of chosen parameters, especially on the values of a and R. Fig. 5 dis-

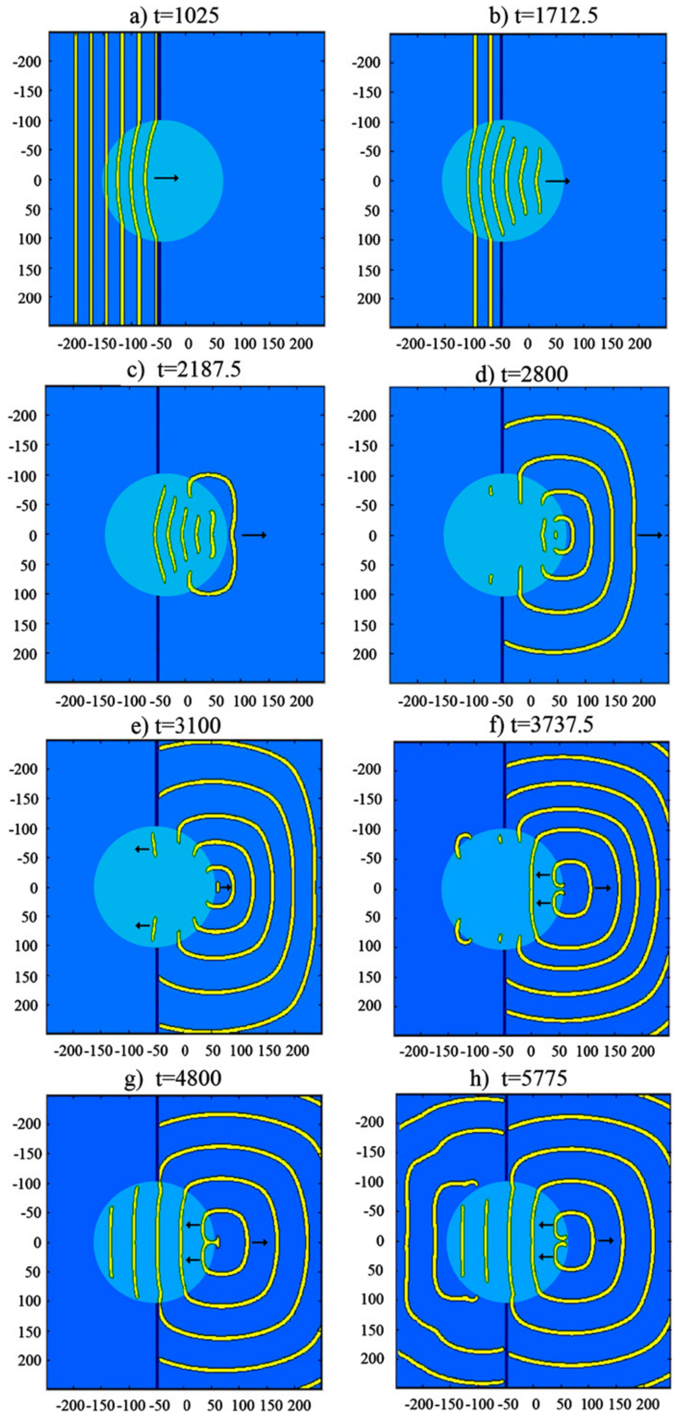

Fig. 6. Convergent lens source of spiral pairs. $R = 100$, $a_1 = 0.12$, $a_2 = 0.155$. A train of six plane waves impinges on the GS from the left (a). The opaque walls cut the outer parts (b). Spiral pairs appear on the right side (c, d). On encountering the walls and the CR, fragments are created on the left side (e, f). Only the sixth wave causes fragments strong enough to generate new spiral pairs (g, h).

plays a bifurcation map of possible (a_2, R) values generating a flip–flop behavior for $a_1 = 0.12$ (squares), and $a_1 = 0.10$ (circles). As can be seen, there exists a minimal radius $R \sim 70$ below which no flip–flop generation is possible. Moreover, the range of a values producing the phenomenon is quite limited: $0.1485 < a_2 < 0.1545$ for $a_1 = 0.12$, and $0.14566 < a_2 < 0.1555$ for $a_1 = 0.10$. We checked the stability of the flip–flop phenomenon by adding random noise to the system. This was carried out by adding to the right side of the first equation in Eq. (1) a term $B \cdot \mathrm{rand}(t) \cdot \mathrm{rand}(x, y)$, where

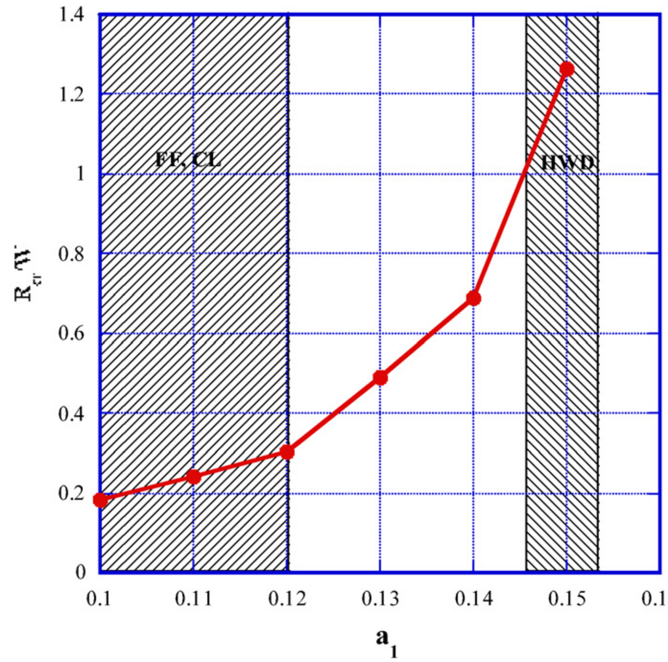

Fig. 7. The ratio R_{cr}/W, where R_{cr} is the critical (minimal) radius of a circular hole through which a plane wave can pass and W is the width of this wave, as a function of a, and the ranges of a where the hole in the wall (HWD), flip–flop (FF) and the convergent lens (CL) mechanisms occur.

rand is a random function between $+1$ and -1 and B is a multiplication constant. Changing B. It was seen that for a B value of 0.01 or less the flip–flop was retained and only for larger values of noise the configuration became distorted and eventually disappeared.

4. Convergent lens source of spiral pairs

The GS here is the same as shown in Fig. 1. For this process, however, a *different set of parameters* is used: $R = 100$, $a_1 = 0.12$, $a_2 = 0.155$ and $a_3 = 0.24$. All other parameters have the same values as those in Section 2.

In order to create the source here, a single wave is insufficient. The reason for that will presently become clear. For the chosen set of parameters, six consecutive initiating waves, or more, are needed. Fig. 6a describes a train of six right moving plane-waves created at the left side of our domain ($x = -240$), with an inter-wave time interval of 160 time units. As the waves pass through the center of the lens, *only the central parts continue to travel*, while the remainder is blocked by the walls (Fig. 6b). This behavior is different from that shown in Figs. 2 and 3, in that the segments shrink and loose contact with the CR perimeter. Note the effect of the lens depicted in the monotonous decrease in segment lengths as they pass through the CR. Similar to the behavior in Fig. 3, each segment tip starts to bend backwards ultimately creating a spiral-pair (SP) motion (Fig. 6c). All waves experience the same process. However, due to the influence of the refractory regions in the wake of the former waves, later ones get retarded and cannot proceed as far from the CR as did the former waves. Therefore, while the arms of the first SP to emerge from the CR reach the walls and are annihilated, the next SP's, spiral closer and closer to the CR and succeed in injecting parts of their arms back into the latter. But these parts are too far apart from each other and eventually disappear (Figs. 6e, 6f). Note that at the time of Fig. 6e all six waves have emerged from the CR. The arms of the 5th SP, although joining together inside of the CR, are not curved enough to create a tip that can become a new source. The arms of the 6th wave, how-

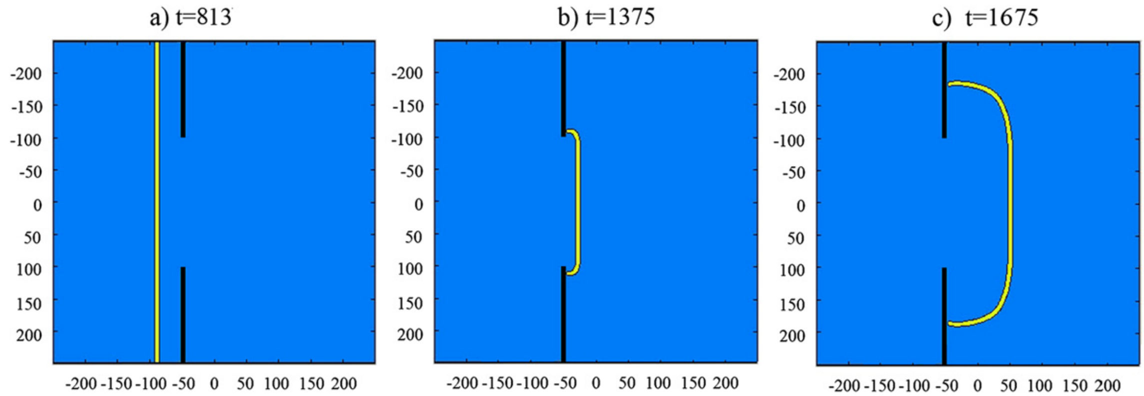

Fig. 8. A hole in the wall (HWD) with no lens for $a = 0.12$. No spiral pair is created (compare with Fig. 6).

ever, penetrate the CR, begin to rotate inside and join together just outside the CR perimeter $(50, 0)$. At the point of contact, they annihilate each other leaving only the tip of the wave (Figs. 6f, 6g). The latter *does become* an independent source for generating new waves (Fig. 6h). Thus, only the 6th wave, in this case, was able to create a tip, "strong" enough [28] to become a spiral pair source. This is the reason for the need, in this case, of several waves to create the source. A similar source, obtained by a different method, was discussed in Ref. [28].

Since the conditions for obtaining this effect are rather stringent in a 7-dimensional parameter space, it is quite possible that it is present in very small, isolated regions, making the mapping hardly conceivable.

5. Discussion

Besides its theoretical interest, study of the GS effects can be of importance for several applications. The possibility of a GS of such a specific form is small. Nonetheless, its appearance in the cardiac system can cause malfunctions severe enough to warrant its analysis. Such an appearance can be due either to an elongated blocking scar with a middle section of diminished excitability or to a string-fibrosis tissue with a middle section of diffuse fibrosis [21] which can lower excitability thus diminishing wave velocity there.

It should be noted that the two mechanisms for inducing spiral pair sources presented here are unlike any previous ones. In particular, we wish to show that they are hardly comparable to the mechanism of "hole in the wall" detachment (HWD) method [11–13]. The basic difference between the present model and the HWD is that detachment from the walls is achieved in the present model by the lens effect and it is therefore unnecessary to obtain it through lowering the excitability as in the HWD case.

This is intuitively seen by considering the converging effect of the lens which brings the tips of the segment away from the wall edges, and can be quantitatively gleaned from Figs. 7 and 8. Fig. 7 shows the R_{cr}/W ratio of Cabo et al. [11,12]. Here R_{cr} is the minimal 1D hole radius (half of its aperture) through which a plane wave can pass and W is the width of such a wave. Detachment in the HWD case occurs [11,12], when this ratio has approximately the value of 1 or higher, a situation which occurs when excitability becomes low. Fig. 7 depicts the variation of R_{cr}/W with the value of a_1 of the FitzHugh–Nagumo system and the ranges where HWD, flip–flop and the convergent lens mechanisms occur. It is clear that both latter cases do not fall within the HWD range.

Furthermore, compare Fig. 8, depicting a situation in which there exists a hole in the wall but no lens, with Fig. 6, both of which figures are calculated for the same $a_1 (= 0.12)$. In the former case no detachment is possible and no source is created while a source *is* created in the case of Fig. 6 since a wave segment appears there by lens convergence.

The function of the walls is two-fold. On the one hand they cut off the parts of the plane waves or any other wave parts impinging on them and on the other hand they retard the tips of the remaining segment so that they start to turn backwards, leading to rotation and eventually spiraling. It was also checked that the higher is the excitability constant a_3 of the walls, the more will the tips be retarded, reaching saturation at about $a_3 = 0.3$.

6. Conclusion

The use of a geometric structure consisting of a convergent lens surrounded by two "opaque" walls was shown to elicit two different sources of spiral pairs. Such a structure may arise in the heart tissue due to a scar having two 'opaque' parts with a gap in between of a lower excitability than the normal tissue or in fibrocystic heart tissue of an appropriate shape.

Acknowledgement

The authors would like to thank Professor V.S. Zykov, Dr. R. Finaly and Professor L. Prigozin for helpful discussions.

References

[1] See e.g. A.T. Winfree, Geometry of Biological Time, 2nd ed., Springer-Verlag, 2001.
[2] L. Glass, P. Hunter, A. Mc Culloch (Eds.), Theory of the Heart-Biomechanics, Biophysics and Nonlinear Dynamics of Cardiac Function, Springer-Verlag, NY, 1991.
[3] M.I. Rabinovich, P. Varona, A.I. Selverston, H.D.I. Abarbanel, Rev. Mod. Phys. 78 (2006) 1213.
[4] M. Voslar, I. Schreiber, Phys. Rev. E 69 (2004) 026210.
[5] S. Sawai, P.A. Thomason, E.C. Cox, Nature 433 (2005) 325.
[6] A. Lemarchand, B. Nowakowski, J. Phys. Condens. Matter 19 (2007) 065130.
[7] A. Provata, I.M. Sokolov, B. Spagnolo, Eur. Phys. J. B 65 (2008) 307.
[8] M.P. Mortell, R.E. OMalley, A. Pokrovskii, V.A. Sobolev, J. Phys. Conf. Ser. 55 (2006) 1.
[9] See e.g. B. Wohlfart, G. Ohlen, Clinical Physiol. 19 (1999) 11.
[10] C.F. Starmer, A.A. Lastra, W. Nesterenko, A.O. Grant, Circulation 84 (1991) 1364.
[11] C. Cabo, A.M. Pertsov, J.M. Davidenko, W.T. Baxter, R.A. Gray, J. Jalife, Biophys. J. 70 (1996) 1105.
[12] C. Cabo, A.M. Pertsov, J.M. Davidenko, J. Jalife, Chaos 8 (1998) 116.
[13] See e.g. M. Wellner, O. Berenfeld, Theory of reentry, in: Saunders, Cardiac Electrophysiology: From Cell to Bedside, 4th ed., 2004, p. 317, especially notice Fig. 35-5 there.
[14] A. Rabinovitch, M. Gutman, Y. Biton, I. Aviram, D.S. Rosenbaum, Phys. Rev. E 74 (2006) 061904.
[15] D. Nekhamkina, M. Sheintuch, Phys. Rev. E 75 (2007) 056210;
See also V.M. Eguiluz, E. Hernandez-Garsia, O. Piro, Physica A 283 (2000) 48.
[16] T.R. Chigwada, P. Parmanada, K. Showalter, Phys. Rev. Lett. 96 (2006) 244101.
[17] R.A. Gray, J. Jalife, A. Panfilov, W.T. Baxter, C. Cabo, J.M. Davidenko, A.M. Petrov, Circulation 91 (1995) 2454.

[18] I. Banville, R.A. Gray, R.E. Idekar, W.M. Smith, Circulation Res. 85 (1999) 742.
[19] S. Iravanian, Y. Nabutovsky, C.R. Kong, S. Saha, N. Bursac, L. Tung, Am. J. Physiol. Heart Circ. Physiol. 285 (2003) H449.
[20] E.J. Ciaccio, J. Coromilas, C.A. Costeas, A.L. Wit, J. Cardiovas. Elect. 15 (2004) 1293.
[21] J.M. De Bakker, M. Stein, H.M. Van Rijen, Heart Rhythm 2 (2005) 777;
Also, J.M. De Bakker, H.M. Van Rijen, J. Cardiovas. Electrophys. 17 (2006) 567.
[22] H.W.J. Kirsten, T. Tusschner, A.V. Panfilov, Europace 9 (2007) vi38.
[23] See e.g. E.M. Izhikevich, Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting, MIT Press, Cambridge, 2007.
[24] K. Kaly-Kullai, L. Roszol, A. Volford, Chem. Phys. Lett. 414 (2005) 326.
[25] Ref. [14] and M. Fialkovski, A. Bitner, B.A. Grzybowski, Phys. Rev. Lett. 94 (2005) 018303.
[26] T. Tsujikawa, T. Nagai, M. Mimura, R. Kobayashi, H. Ikeda, Jpn. J. Appl. Math. 6 (1989) 341.
[27] A.S. Mikhailov, V.A. Davydov, V.S. Zykov, Physica D 70 (1994) 1.
[28] A. Rabinovitch, Y. Biton, M. Gutman, I. Aviram, Comput. Biol. Medicine, in press.