Semilinear response for the heating rate of cold atoms in vibrating traps

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arXiv reference:

A. Stotland, D. Cohen and N. Davidson, arXiv (2008)

$DIP, $BSF

Diffusion and Energy absorption

Driven chaotic system with Hamiltonian $\mathcal{H}(X(t))$

$$X \quad = \quad \text{some control parameter}$$
$$\dot{X} \quad = \quad \text{rate of the (noisy) driving}$$

$\leadsto$ diffusion in energy space:

$$\boldsymbol{D} \quad = \quad \boldsymbol{G}_{\text{diffusion}} \ \overline{\dot{X}^2}$$

$\leadsto$ energy absorption:

$$\dot{\boldsymbol{E}} \quad = \quad \boldsymbol{G}_{\text{absorption}} \ \overline{\dot{X}^2}$$

[Ott, Brown, Grebogi, Wilkinson, Jarzynski, D.C.]

There is a dissipation-diffusion relation.

In the canonical case $\dot{E} = D/T$.

Below we use for G scaled units.

Linear response theory

$$\mathcal{H} = \{E_n\} - X(t)\{V_{nm}\}$$

$$\boldsymbol{G} \;=\; \pi\varrho_{\mathrm{E}}\ \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{algebraic}}$$

applies if

"strong quantum chaos"
(driven transitions $\ll$ relaxation)

otherwise

connected sequences of transitions are essential.

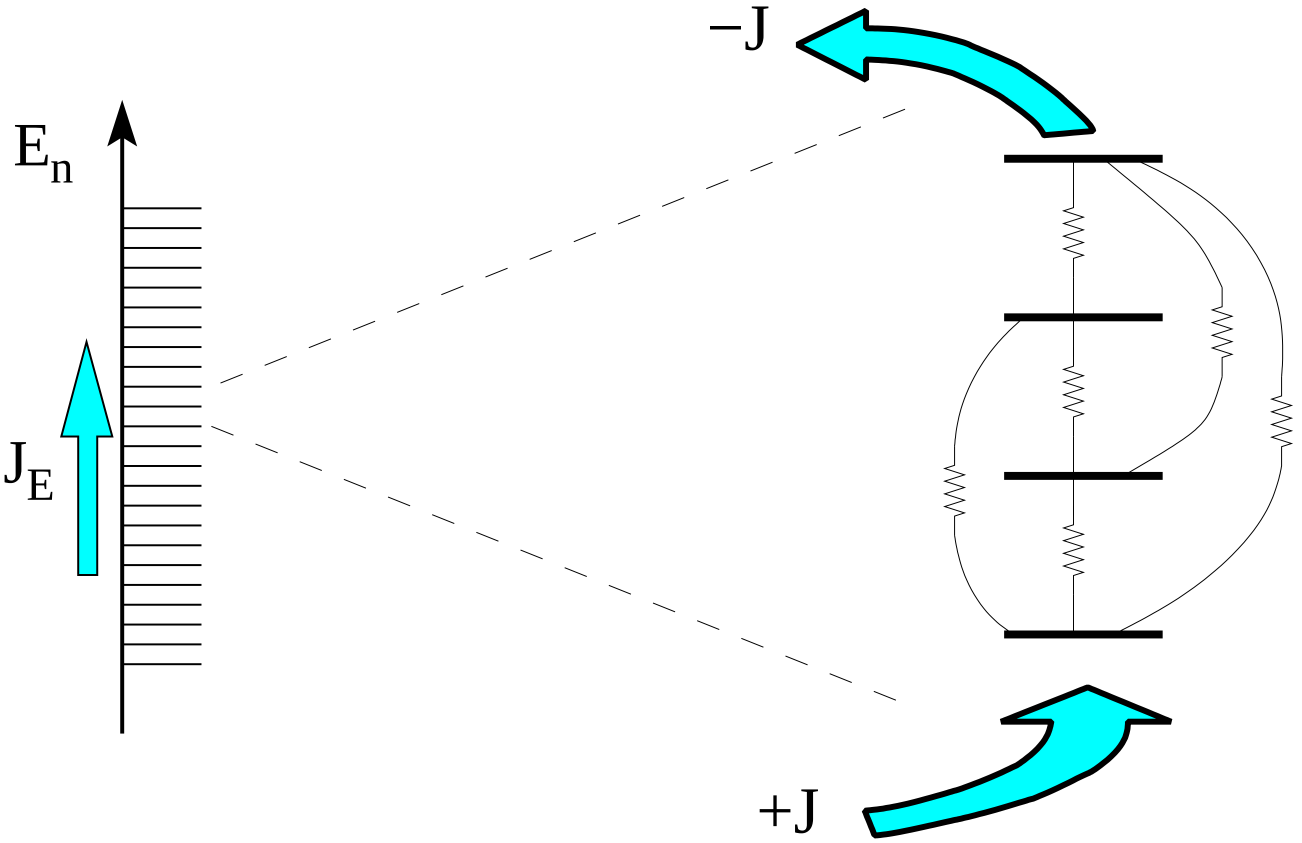
leading to

Semi Linear Response Theory (SLRT)

Semi Linear Response Theory

$$\mathcal{H} = \{E_n\} - X(t)\{V_{nm}\}$$

$$\boldsymbol{G} \;=\; \pi\varrho_{\mathrm{E}}\; \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{SLRT}}$$



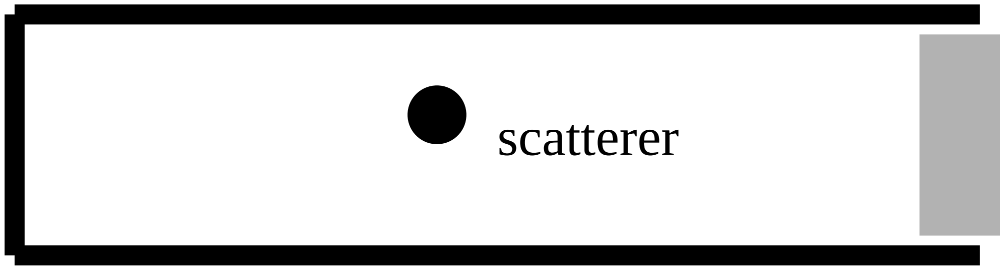
$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3}\;\frac{|V_{nm}|^2}{(E_n{-}E_m)^2}\;\tilde{F}(E_n{-}E_m)$$

$\langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{SLRT}} \equiv$ inverse resistivity of the network

$$\langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{harmonic}} \ll \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{SLRT}} \ll \langle\langle|V_{mn}|^2\rangle\rangle_{\mathrm{algebraic}}$$

The model

A particle in a 2-D box with a vibrating wall.



Deforming potential: smooth Gaussian / s-scatterer

The Hamiltonian in the $\boldsymbol{n} = (n_x, n_y)$ basis:

$$\mathcal{H} = \mathrm{diag}\{E_{\boldsymbol{n}}\} + u\{U_{\boldsymbol{nm}}\} + f(t)\{V_{\boldsymbol{nm}}\}$$

The matrix elements for the wall displacement:

$$V_{\boldsymbol{nm}} = -\delta_{n_y,m_y} \times \frac{\pi^2}{\mathsf{m} L_x^3} n_x m_x$$

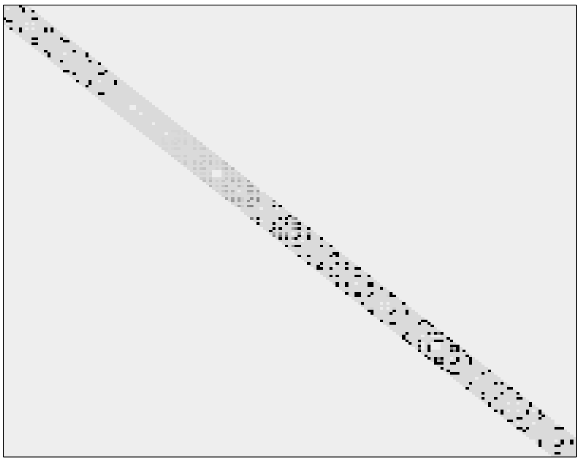
The Hamiltonian in the E_n basis:

$$\mathcal{H} = \mathrm{diag}\{E_n\} + f(t)\{V_{nm}\}$$

$$\langle\langle |V_{nm}|^2 \rangle\rangle_a \approx \frac{\mathsf{m} v_{\mathrm{E}}^3}{2\pi L_x^2 L_y}$$

$$\langle\langle |V_{nm}|^2 \rangle\rangle_g \approx \left(\frac{\mathsf{m}^2 v_{\mathrm{E}}^2}{2\pi L_x}\right)^2 \exp\left[-2\mathsf{m}^2 v_{\mathrm{E}}^2 (\sigma_x^2 + \sigma_y^2)\right] \times u^2$$

$\{|V_{nm}|^2\}$ **as a random matrix** $\{X\}$

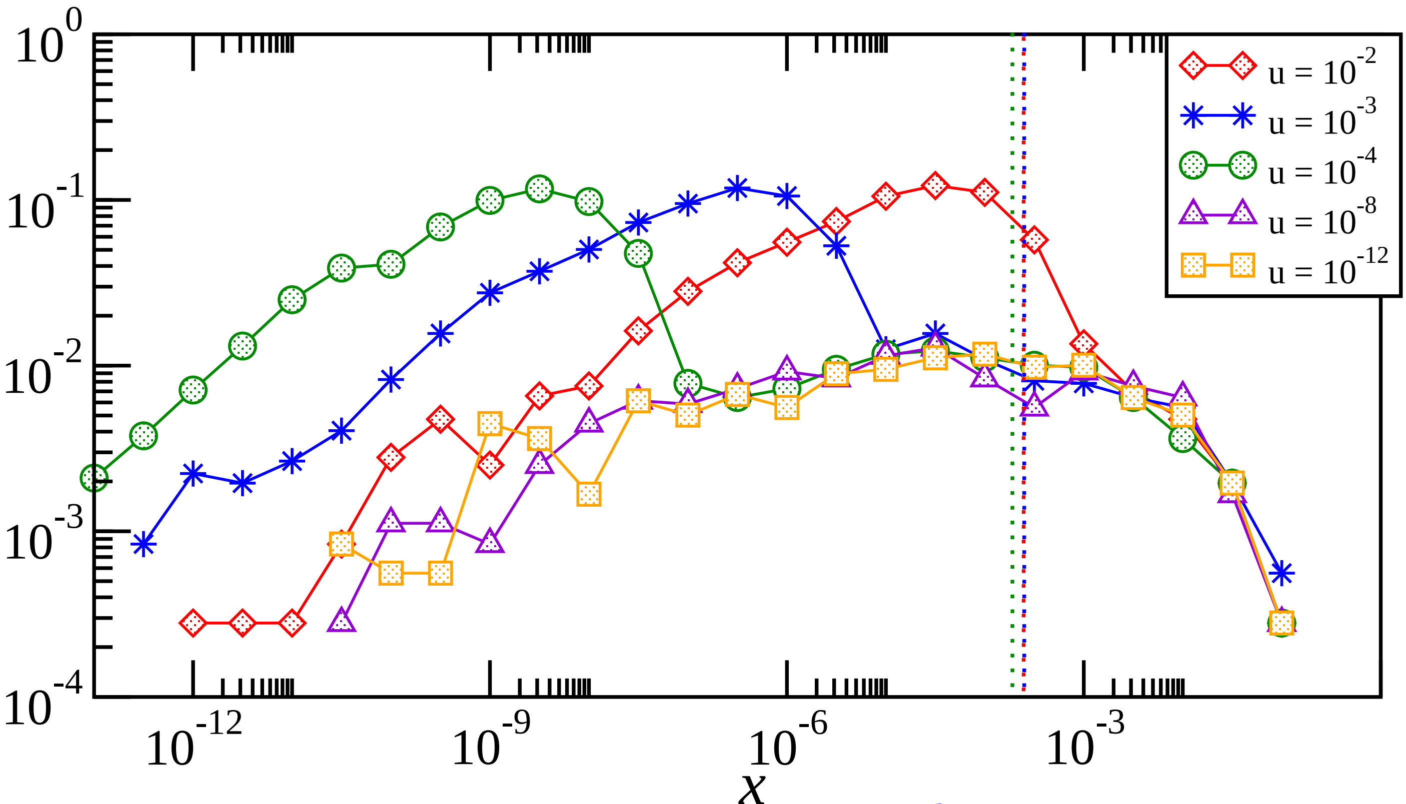


- sparsity:

$$q = \frac{\langle\langle x\rangle\rangle_{\mathrm{g}}}{\langle\langle x\rangle\rangle_{\mathrm{a}}}$$

- texture

Histogram of X :



$X \sim$ LogNormal

Algebraic average: $\langle\langle x\rangle\rangle_a = \langle x\rangle$

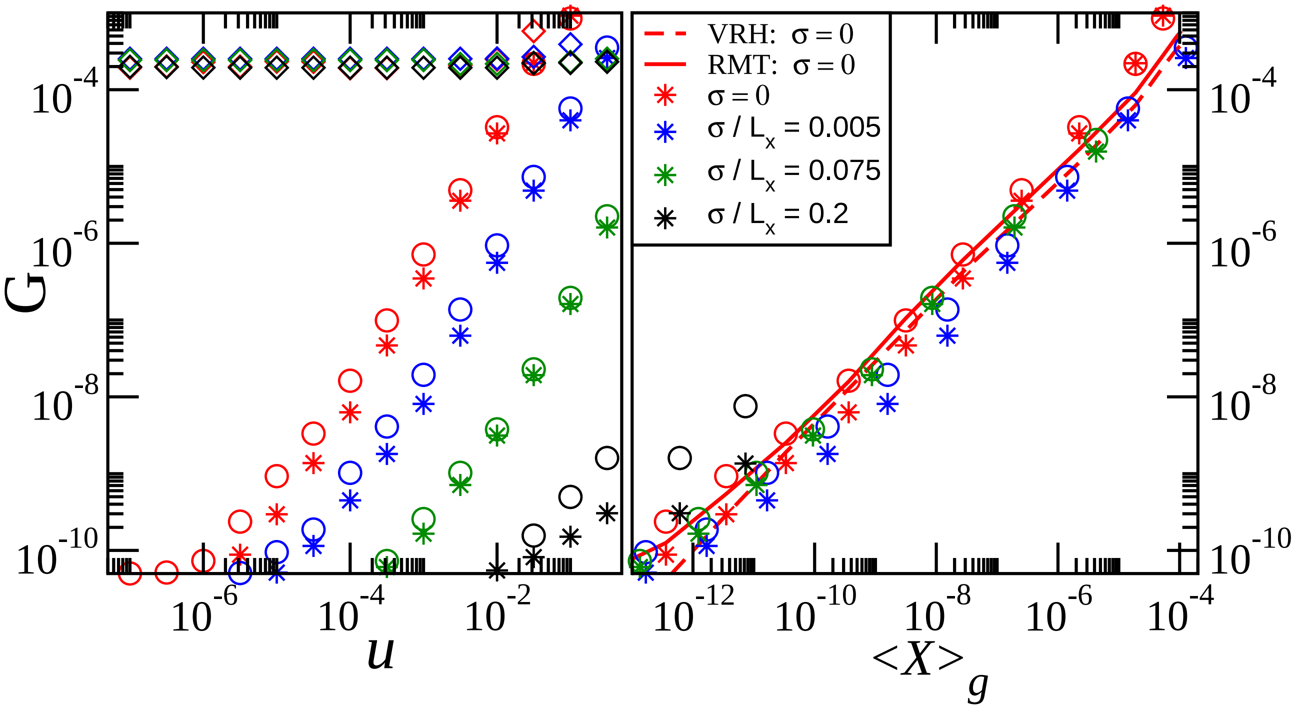
Harmonic average: $\langle\langle x\rangle\rangle_h = [\langle 1/x\rangle]^{-1}$

Geometric average: $\langle\langle x\rangle\rangle_g = \exp[\langle \log x\rangle]$

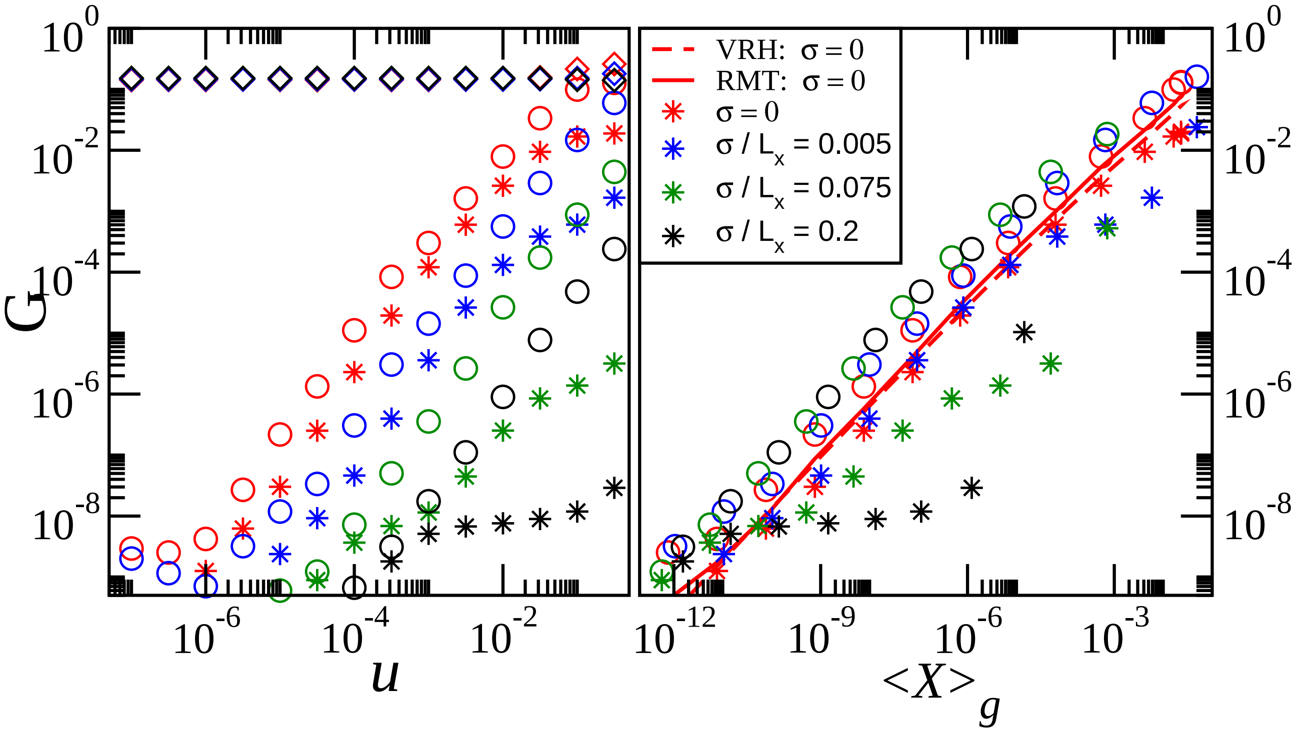
$$\langle\langle x\rangle\rangle_h \ll \langle\langle x\rangle\rangle_g \ll \langle\langle x\rangle\rangle_a$$

Numerical Results

$AS = 1$



$AS = 20$



$$G_{\mathrm{LRT}} = \pi \varrho_{\mathrm{E}} \, \langle\langle |V_{nm}|^2 \rangle\rangle_{\mathrm{algebraic}}$$

$$G_{\mathrm{SLRT}} = q \; \exp\left[2\sqrt{-\ln q}\right] \times G_{\mathrm{LRT}}$$

The RMT modeling and VRH

Log-normal distribution:

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)$$

$$\mu = \ln \langle\langle x \rangle\rangle_{\mathrm{g}}$$

$$\sigma^2 = 2\ln \frac{\langle\langle x \rangle\rangle_{\mathrm{a}}}{\langle\langle x \rangle\rangle_{\mathrm{g}}}$$

A typical matrix element for connected transitions:

$$\left(\frac{\omega}{\Delta}\right) \mathrm{Prob}\big(x > x_\omega\big) \sim 1$$

$$x_\omega \approx \langle\langle x \rangle\rangle_{\mathrm{g}} \exp\left[2\sqrt{-\ln q}\right]$$

Generalized VRH estimate:

$$G_{\mathrm{SLRT}} = q \; \exp\left[2\sqrt{-\ln q}\right] \times G_{\mathrm{LRT}}$$

Conclusions

(*) Wigner ($\sim$ 1955):
The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.

Not always...

1. No “strong quantum chaos“ $\Longrightarrow$ log-normal distribution.
2. The heating process $\sim$ a percolation problem.
3. Resistors network calculation to get $G_{\rm SLRT}$.
4. Generalization of the VRH estimate
5. SLRT is essential whenever the distribution of matrix elements is wide (“sparsity”) or if the matrix has “texture”.

[1] D. Cohen, T. Kottos and H. Schanz, JPA (2006)

[2] S. Bandopadhyay, Y. Etzioni and D. Cohen, EPL (2006)

[3] M. Wilkinson, B. Mehlig and D. Cohen, EPL (2006)

[4] D. Cohen, PRB (2007)

[5] A. Stotland, R. Budoyo, T. Peer, T. Kottos and D. Cohen, JPA(FTC) (2008)