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The fluctuations of the current within a mesoscopic ring

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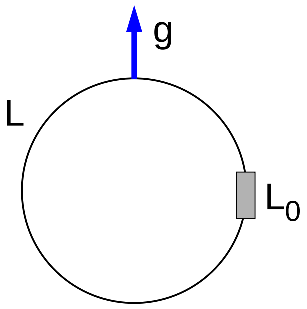
References:

A.Stotland and D.Cohen, in preparation

$DIP

Definition of the problem



$$\mathcal{H}_0 = \frac{p^2}{2\mathsf{m}} + u\delta(x)$$

$$g(E) = \left[1 + \left(\frac{\mathsf{m}}{\hbar^2 k_E} u\right)^2\right]^{-1}$$

Current operator:

$$I = \frac{e}{2\mathsf{m}} \left(p\ \delta_{L_0}(x - x_0) + \delta_{L_0}(x - x_0)\ p\right)$$

$$\mathbf{I} = \sum_{mn} I_{mn} a_m^\dagger a_n$$

What are the fluctuations of the current?

$$S(\omega) = \mathrm{FT}\left[\, \langle \mathbf{I}(t)\mathbf{I}(0)\rangle \,\right]$$

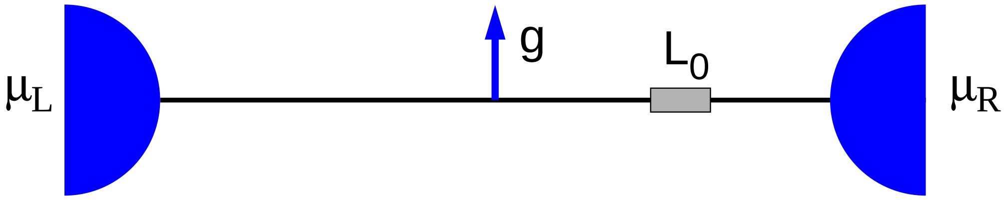
given

$$\langle I \rangle \sim I_0$$

(non-equilibrium state)

Known results

Open geometry:



$$\mu \equiv \mu_R - \mu_L$$

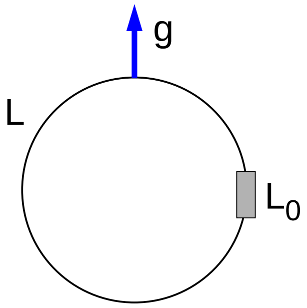
$$S(\omega; \mu, \beta) = \frac{e^2}{2\pi}\Big[2g^2 F(\omega) + g(1-g)(F(\omega+\mu) + F(\omega-\mu))\Big]$$

$$F(x) \equiv \frac{x}{1 - e^{-\beta x}}$$

References:

- Y. Imry, Introduction to Mesoscopic Physics, 2nd ed.
- Ya. M. Blanter, M. Büttiker, Phys. Rep. 336, 1 (2000)
- M. J. M. de Jong, C. W. J. Beenakker, cond-mat/9611140 (1996)
- U. Gavish, Y. Imry, Y. Levinson, cond-mat/0211681 (2002)
- U. Gavish, Y. Levinson and Y. Imry, Phys. Rev. Lett. 87, 216807 (2001)

Strategy

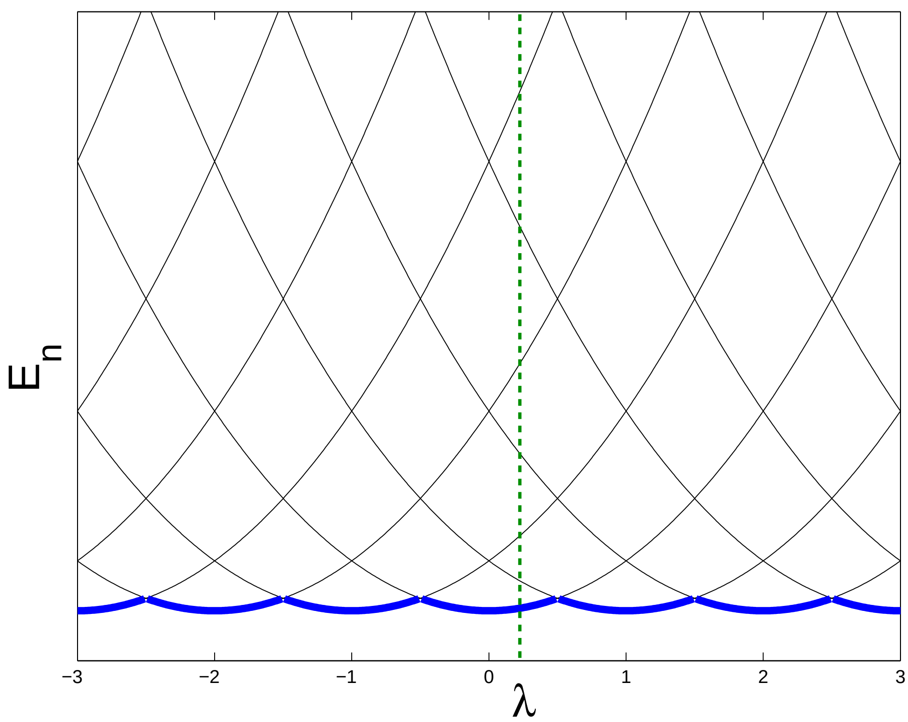


$$\mathcal{H}_0 = \frac{p^2}{2\mathrm{m}} + u\delta(x)$$

Many body parameters: μ, β

$$\begin{cases} \langle\Psi|\mathcal{H}_0|\Psi\rangle = \text{minimum} \\ \langle\mathbf{I}\rangle = I_0 \end{cases}$$

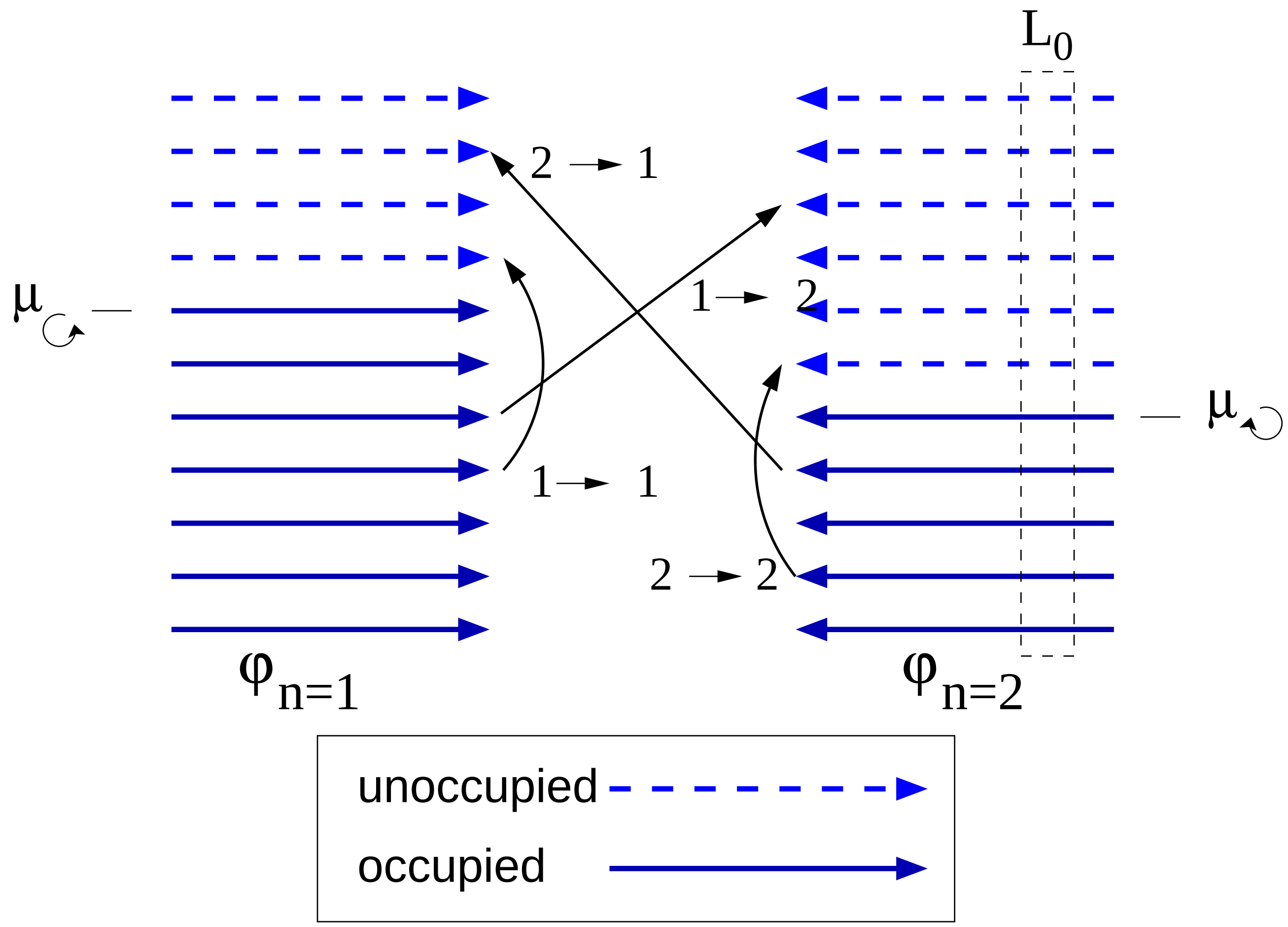
$$\mathcal{H} = \mathcal{H}_0 + \lambda\mathbf{I}$$



Maximum current $\Leftrightarrow \lambda = \pi/2$

λ is like flux.

Transitions between one particle energy levels



$$\mu \equiv \mu_{\circlearrowleft} - \mu_{\circlearrowright}$$

Closed geometry

$$|I_{nm}^{\rightleftarrows}|^2 \sim 1-g$$

$$|I_{nm}^{\updownarrow}|^2 \sim g$$

Open geometry

$$|I_{nm}^{\rightleftarrows}|^2 \sim g(1-g)$$

$$|I_{nm}^{\updownarrow}|^2 \sim g^2$$

Fluctuations in equilibrium

One particle:

$$\begin{aligned} S^{[1]}(\omega;E) &= \mathrm{FT}\Big[\,\langle I(t)I(0)\rangle\,\Big] \\ &= \sum_{nm} p_n |I_{mn}|^2\, 2\pi\delta(\omega - (E_m - E_n)) \end{aligned}$$

p_n are the microcanonical weights ($E_n \sim E$).

$\varrho_{\mathrm{F}} = \mathrm{DOS}$

Many particles:

$$\begin{aligned} S(\omega) &= \mathrm{FT}\Big[\,\langle \mathbf{I}(t)\mathbf{I}(0)\rangle\,\Big] \\ &= \sum_{nm} |I_{mn}|^2 \left\langle a_n^\dagger a_m a_m^\dagger a_n \right\rangle 2\pi\delta(\omega - (E_m - E_n)) \\ &= \sum_{nm} (1 - f(E_m)) f(E_n) |I_{mn}|^2\ 2\pi\delta(\omega - (E_m - E_n)) \\ &= \int \varrho_{\mathrm{F}} dE (1 - f(E+\omega)) f(E)\ S^{[1]}(\omega;E) \\ &= \frac{\varrho_{\mathrm{F}}\omega}{1 - \mathrm{e}^{-\omega/T}}\ S^{[1]}(\omega) \equiv \varrho_{\mathrm{F}}\, F(\omega)\, S^{[1]}(\omega) \end{aligned}$$

Fluctuations in non-equilibrium

General formula:

$$
\begin{aligned}
S(\omega;\mu,\beta) \quad &= \quad \varrho_{\mathrm{F}}\, S^{[1]}_{\uparrow}(\omega)\, F(\omega) \\
&+ \quad \varrho_{\mathrm{F}}\, S^{[1]}_{\downarrow}(\omega)\, F(\omega) \\
&+ \quad \varrho_{\mathrm{F}}\, S^{[1]}_{\leftarrow}(\omega)\, F(\omega{+}\mu) \\
&+ \quad \varrho_{\mathrm{F}}\, S^{[1]}_{\rightarrow}(\omega)\, F(\omega{-}\mu)
\end{aligned}
$$

For our model system:

$$
S(\omega;\mu,\beta) = \frac{e^2}{2\pi}\Big[2gF(\omega) + (1{-}g)(F(\omega{+}\mu) + F(\omega{-}\mu))\Big]
$$

For open geometry - extra g factor.

For the optimal occupation:

$$
I_0 = \frac{e}{2\pi}\sqrt{g}\,(\mu_{\circlearrowleft} - \mu_{\circlearrowright})
$$

For a steady state of a driven system the $I_0(\mu)$ relation is not optimal!

What next?

- Quantum-classical correspondence
- Multimode (chaotic) case
- The effect of the environment
- Path integrals method