The Noise Spectrum of an Interacting Quantum Dot

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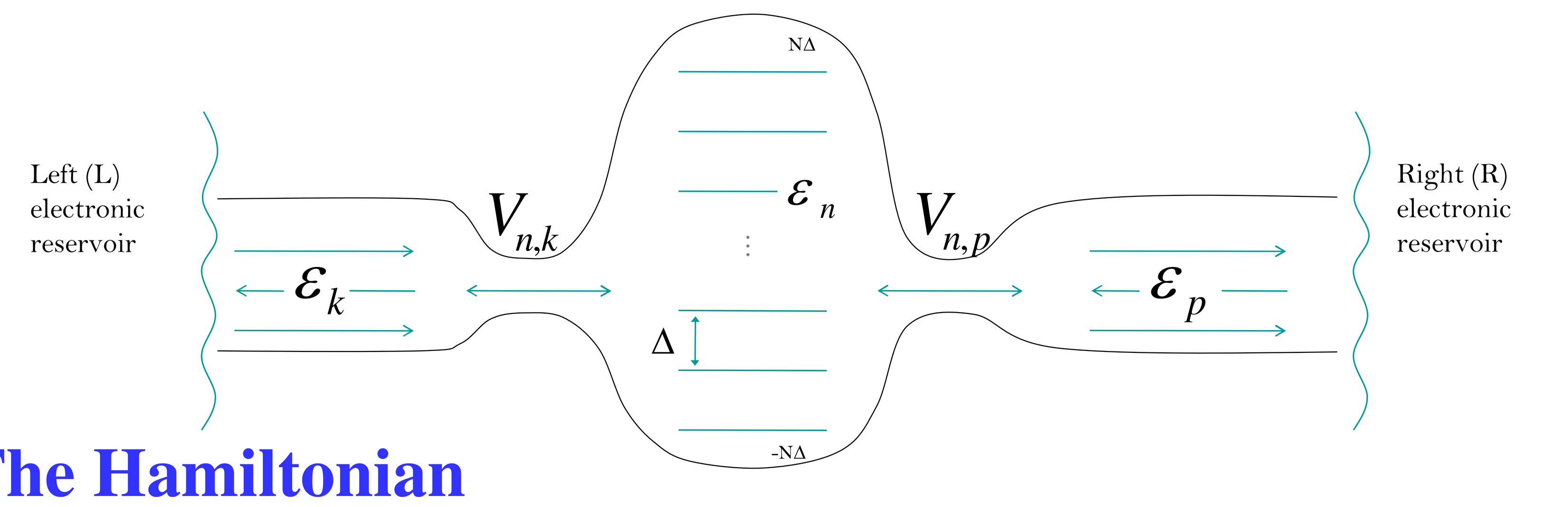
We calculate the noise spectrum of a quantum dot connected to two leads for zero temperature and bias voltage.
The interaction was treated within the Hartree and Hartree-Fock approximations.
The results generalize our earlier work on a non interacting two-level quantum dot [1] and a single-level quantum dot [2].

The Model

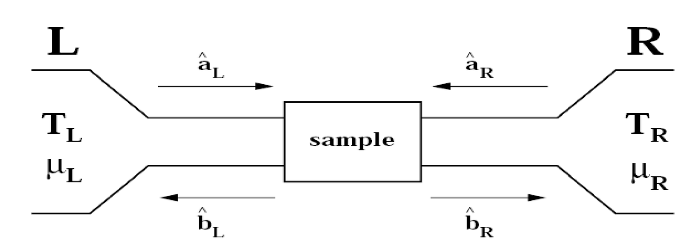


The Hamiltonian

$$\hat H = \sum \varepsilon_k \hat c_k^\dagger \hat c_k + \sum V_{n,k}(\hat c_k^\dagger \hat d_n + h.c.) + \sum_{n=-N}^{N} \varepsilon_n \hat d_n^\dagger \hat d_n - \frac{1}{2}\sum_{n\neq m} J_{nm}(\hat d_n^\dagger \hat d_m + \hat d_m^\dagger \hat d_n) + \frac{U}{2}\left(\sum_{n=-N}^{N} (\hat d_n^\dagger \hat d_n - N)\right)^2$$
$$+ \sum \varepsilon_p \hat c_p^\dagger \hat c_p + \sum V_{n,p}(\hat c_p^\dagger \hat d_n + h.c.)$$

Momentum states in R/L reservoirs | Coupling R/L reservoir-QD | Energy levels in the QD | Coupling between the energy levels | Charging energy

Büttiker's Scattering Formalism [3]



$$\hat I_\alpha(t) = \frac{e}{2\pi}\int dE dE' \left[\hat a_\alpha^\dagger(E')\hat a_\alpha(E) - \hat b_\alpha^\dagger(E')\hat b_\alpha(E)\right] e^{i(E'-E)t}$$
$$\hat b_\alpha = \sum S_{\alpha\beta}\hat a_\beta$$
$$\hat I_\alpha(t) = \frac{e}{2\pi}\int dE dE' \sum_{\beta,\gamma} \hat a_\beta^\dagger(E') A_{\beta\gamma}(\alpha,E',E)\hat a_\gamma(E) e^{i(E'-E)t}$$
$$A_{\beta\gamma}(\alpha,E',E) = \delta_{\alpha\beta}\delta_{\alpha\gamma} - S_{\alpha\beta}^\dagger(E')S_{\alpha\gamma}(E)$$

Quantum statistical average:

$$\langle \hat a_\beta^\dagger(E')\hat a_\gamma(E)\rangle = \delta_{\beta\gamma}\delta(E'-E)f_\beta(E) \qquad f_\alpha(E) = [e^{(E-\mu_\alpha)/k_BT}+1]^{-1}$$

The noise definition:

$$C(\omega) = \int dt e^{i\omega t}\left\langle \delta\hat I(t)\delta\hat I(0)\right\rangle \qquad \begin{cases}\delta\hat I \equiv \hat I - \langle \hat I\rangle \\ \hat I \equiv (\hat I_L - \hat I_R)/2\end{cases}$$

The Hartree Fock Approximation

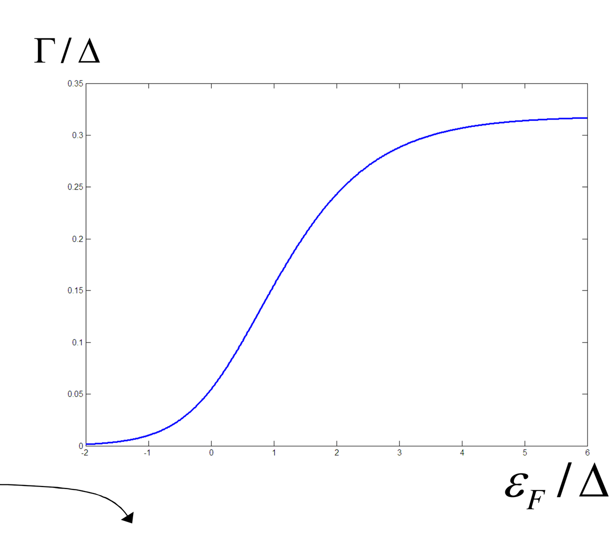
$$\hat d_n^\dagger \hat d_n \hat d_m^\dagger \hat d_m \to \langle \hat d_n^\dagger \hat d_n\rangle \hat d_m^\dagger \hat d_m + \langle \hat d_m^\dagger \hat d_m\rangle \hat d_n^\dagger \hat d_n - \langle \hat d_n^\dagger \hat d_m\rangle \hat d_m^\dagger \hat d_n - \langle \hat d_m^\dagger \hat d_n\rangle \hat d_n^\dagger \hat d_m$$

The single electron effective Hamiltonian

$$\hat H_{dot}^{HF} = \sum_n \tilde\varepsilon_n \hat d_n^\dagger d_n - \frac{1}{2}\sum_{n\neq m}\tilde J_{nm}(\hat d_n^\dagger \hat d_m + \hat d_m^\dagger \hat d_n)$$

The effective parameters

$$\tilde\varepsilon_n = \varepsilon_n - \frac{NU}{2} + U\underbrace{\sum_{n\neq m}\langle \hat d_m^\dagger \hat d_m\rangle}_{Hartree} \qquad \tilde J_{nm} = J_{nm} + U\underbrace{\langle \hat d_n^\dagger \hat d_m\rangle}_{Fock}$$



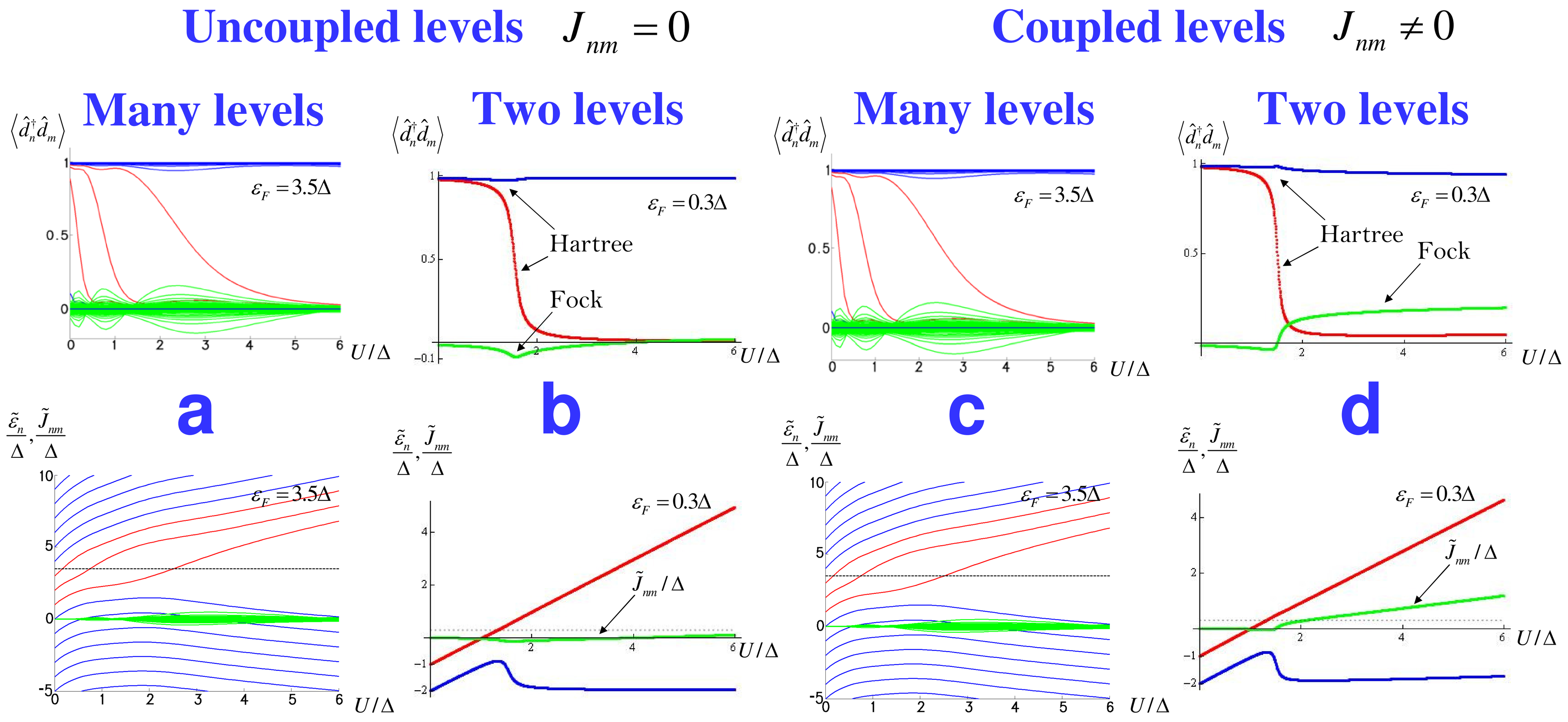
The self consistent calculation

$$\langle d_n^\dagger d_m\rangle = -\int_{-\infty}^{\varepsilon_F}\frac{dE}{\pi}\,\mathrm{Im}[E - H_{dot}^{HF} + i\Gamma]_{nm}^{-1}$$

$$\begin{cases}\Gamma = \Gamma_L + \Gamma_R & \text{level width}\\ \Gamma_L = \alpha\Gamma & \text{asymmetry parameter}\end{cases}$$

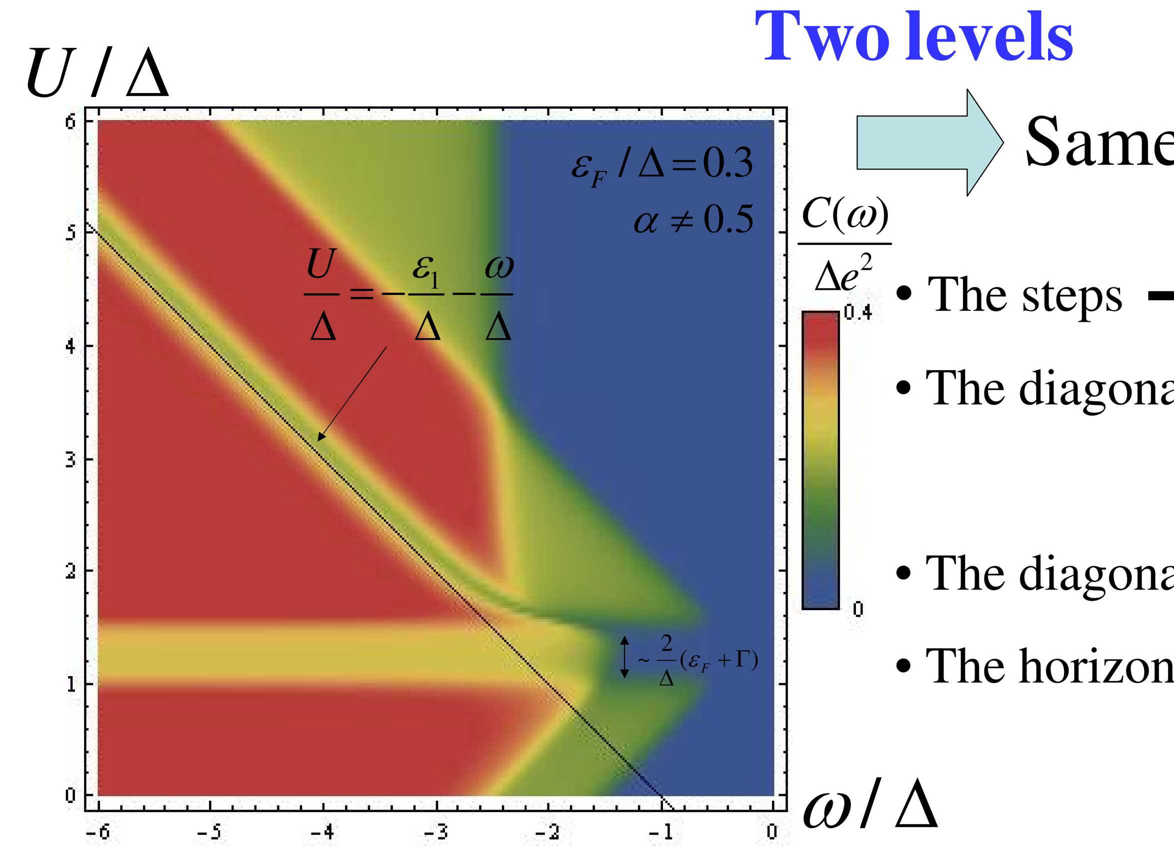
Numerical results

Uncoupled levels $J_{nm} = 0$ | Coupled levels $J_{nm} \neq 0$



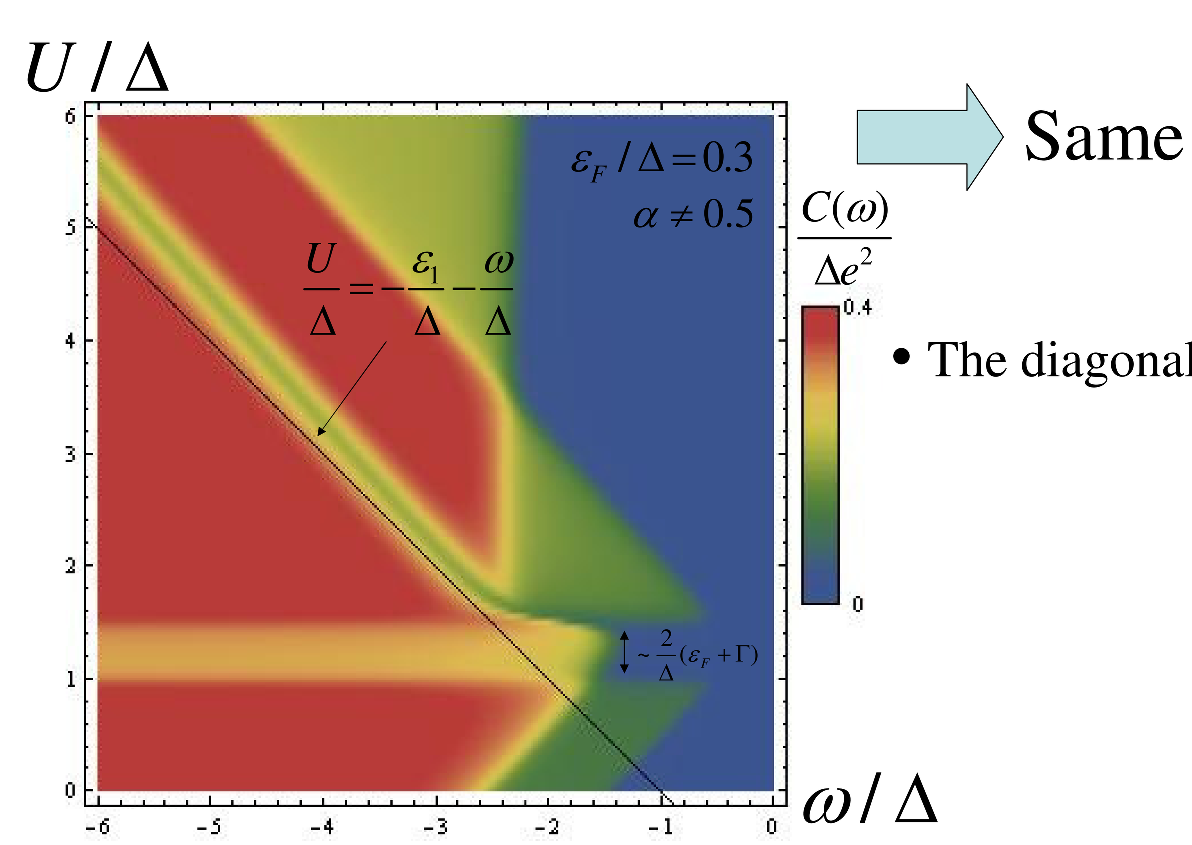
The Noise Spectrum

Two levels



Same scenario as in **b**

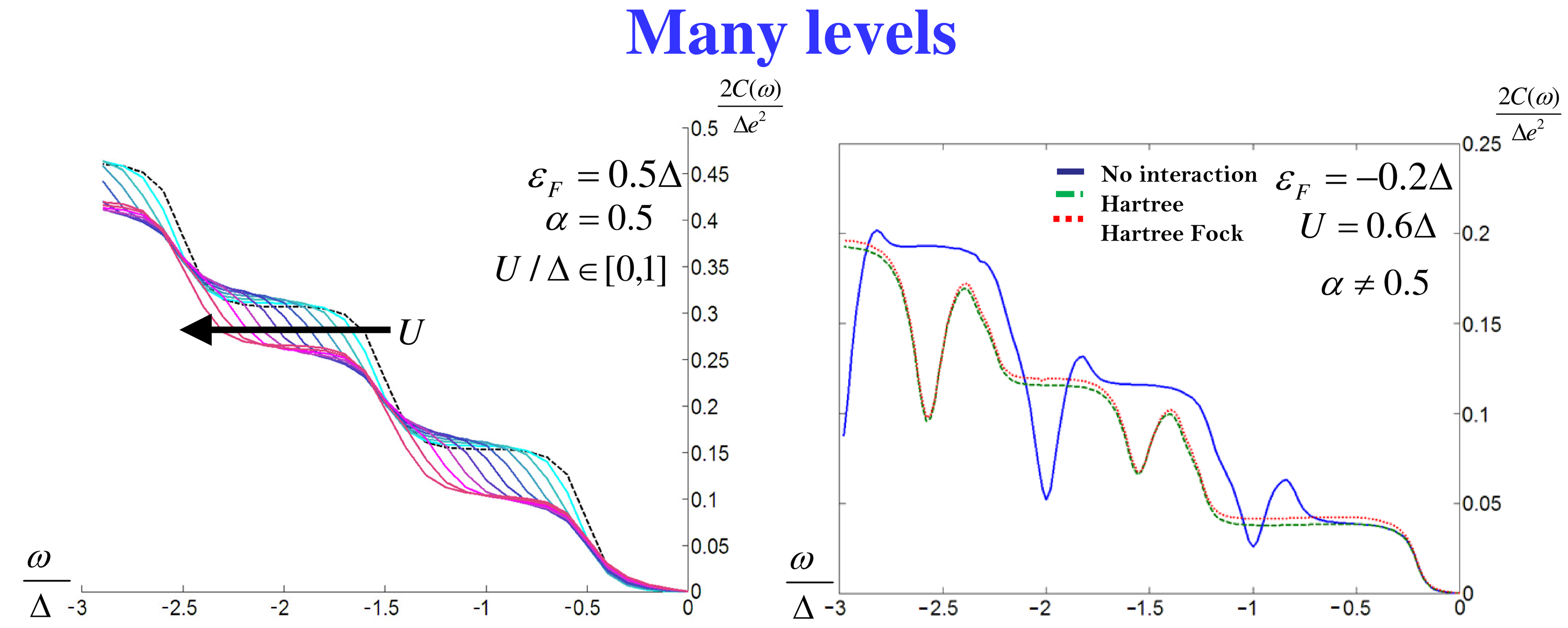
- The steps ⟶ QD resonances
- The diagonal dip ⟶ resonances energy difference
- The diagonal dip is linear in ω
- The horizontal dip ⟶ when $|\tilde\varepsilon_1| \le |\varepsilon_F|$



Same scenario as in **d**

- The diagonal dip is not linear in ω

Many levels



- Same behavior as the noise for a 2 level QD.
- No qualitative difference between Hartree and Hartree Fock

[1] O. Entin-Wohlman, Y. Imry, S. A. Gurvitz, and A. Aharony, Phys. Rev. B **75**, 193308 (2007).
[2] E. A. Rothstein, O. Entin-Wohlman, and A. Aharony, Phys. Rev. B **79**, 075307 (2009).
[3] M. Büttiker, Phys. Rev. B **46**, 12485 (1992).