Two-stream instability of electrons in the shock front

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1. Introduction

It is widely believed (see *Scudder* [1995] and references therein) that electron heating in the shock front is mainly due to the electron interaction with the quasi-static electric and magnetic fields in the essentially one-dimensional stationary shock profile. During this interaction each electron, crossing the shock from upstream to downstream, gets the same amount of energy, namely $e\varphi_0^{\mathrm{HT}}$, where φ_0^{HT} is the cross-shock potential in the de Hoffman-Teller frame. Adiabatic mechanism [*Feldman et al.*, 1982; *Goodrich and Scudder*, 1984; *Feldman*, 1985; *Thomsen et al.*, 1987; *Schwartz et al.*, 1988; *Hull et al.*, 1998], which works in most shock profiles, implies that the electron magnetic moment is conserved across the shock, that is, $v_\perp^2/B = \text{const}$ everywhere (index $\perp$ refers to the local magnetic field direction). In this case the electrons are efficiently accelerated along the magnetic field. Some electrons are reflected because of the magnetic barrier. In very thin shocks, where the ramp width is of the order of the electron inertial length [*Newbury and Russell*, 1996], electrons may be demagnetized in a part of the ramp and accelerated across the magnetic field [*Balikhin et al.*, 1993; *Gedalin et al.*, 1995; *Gedalin and Balikhin*, 1998; *Gedalin et al.*, 1997]. This results in a different distribution of the acquired energy among the parallel and perpendicular degrees of freedom. Independently of what is the particular mechanism of the electron energization, the upstream electrons, whose parallel velocities are directed upstream and which never enter the shock, must have come from the downstream region. A substantial part of the phase space is not accessible to electrons, so that the electron distribution inside the ramp and just behind it has a hole in the distribution [*Hull et al.*, 1998], which should be filled somehow to produce the observed distributions. It was suggested that this hole may be filled with some preexisting electron population [*Feldman et al.*, 1982; *Feldman*, 1985], in which case the electron distribution forms nonlocally. Only the high energy tail is produced at the shock while the rest of the distribution is formed at another reflecting boundary, which requires very good coordination between the shock and this boundary. This scenario was never analyzed in detail. Another suggestion is the electrostatic and whistler instabilities in the drift (adiabatic) regime [*Veltri et al.*, 1990, 1992 ; *Veltri and Zimbardo*, 1993a, b]. *Hull et al.* [1998] adopted the phenomenological approach where the hole is filled homogeneously up to the density required by the Rankine-Hugoniot relations, which does not provide physical explanation why such filling should occur.

In this paper we consider the consequences of the fact that the collisionless electron distribution inside the ramp essentially consists of two counterstreaming beams, which is the classical two-stream instability case. We find the growth rate of the instability and show that it is capable of smoothing the electron distribution inside the ramp. In what follows we consider the adiabatic case, since in the demagnetized regime the electron distribution cannot be determined analytically. However, the results should not differ substantially, especially for strong shocks with large de Hoffman-Teller cross-shock potential.

2. Electron distribution inside the ramp

For simplicity we assume that the upstream distribution is Maxwellian

$$f_u(v_{u,\|}, v_{u,\perp}) = n_u(2\pi v_T^2)^{-3/2} \cdot \exp[-\frac{(v_{u,\|} - V_{\mathrm{sh}})^2 + v_{u,\perp}^2}{2v_T^2}], \tag{1}$$

where $V_{\mathrm{sh}} = V_u/\cos\theta$ is the bulk upstream plasma velocity along the magnetic field in the de Hoffman-Teller frame (V_u is the plasma velocity in the normal incidence frame, and θ is the angle between the shock normal and upstream magnetic field). The distribution function is normalized so that $2\pi \int f_u dv_{u,\|} vu, \perp dv_{u,\perp} = n_u$.

Let $\varphi(x)$ be the de Hoffman-Teller potential and $B(x)$ the total magnetic field throughout the shock. Conservation of energy and magnetic momentum read

$$v_\|^2 + v_\perp^2 = v_{u,\|}^2 + v_{u,\perp}^2 + \frac{2e\varphi}{m_e}, \tag{2}$$

$$v_\perp^2 = v_{u,\perp}^2 (B/B_u). \tag{3}$$

Collisionless dynamics implies that throughout the shock $f(v_\parallel, v_\perp) = f_u(v_{u,\parallel}, v_{u,\perp})$, where the functional form of $f_u(v_{u,\parallel}, v_{u,\perp})$ is given by (1), while the corresponding $v_{u,\parallel}$ and $v_{u,\perp}$ are found by solving (2)–(3):

$$v_{u,\perp}^2 = v_\perp^2(B_u/B), \quad v_{u,\parallel} = \sqrt{Q}\,\mathrm{sign}(v_\parallel), \tag{4}$$

$$Q = v_\parallel^2 + v_\perp^2(1 - B_u/B) - \frac{2e\varphi}{m_e}. \tag{5}$$

Obviously, the condition $Q < 0$ defines the region in the velocity space which is not accessible for electrons.

In what follows we need the one-dimensional distribution function $F(v_\parallel) = 2\pi \int f(v_\parallel, v_\perp) v_\perp dv_\perp$. Figure 1 shows

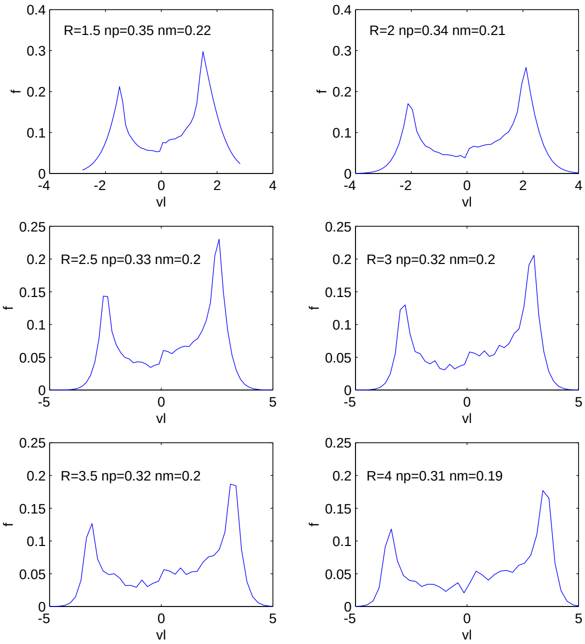

Figure 1. Evolution of the distribution function $F(v_\parallel)$ through the ramp. The parameters are as follows: de Hoffman-Teller cross-shock potential $e\varphi_0^{\mathrm{HT}} = 12T_{eu}$, $V_u/v_{Te}\cos\theta = 0.2$. In the figure $R = B/B_u$, n_f is the density of transmitted electrons, and n_b is the density of the backstreaming electrons normalized on the upstream density. R monotonically increases from $R = 1$ (upstream) to $R = 4$ (downstream).

the evolution of the distribution function $F(v_\parallel)$ across the shock front. The de Hoffman-Teller cross-shock potential is estimated with the widely used polytropic relation [*Schwartz et al.*, 1988]:

$$e\varphi_0^{\mathrm{HT}} = \frac{\gamma}{\gamma - 1}(T_{ed} - T_{eu}), \tag{6}$$

where we chose $\gamma = 2$ and $T_d/T_u = B_d/B_u$ (“adiabatic” case), and $B_d/B_u = 4$. The parameter $V_{\mathrm{sh}}/v_{Te} = 0.2$, that is, upstream electrons are subsonic in the de Hoffman-Teller frame. The distribution function depends on the local magnetic compression ratio $R = B/B_u$, and we assume that the potential follows the magnetic field profile [*Hull et al.*, 1998; *Gedalin et al.*, 1998]: $\varphi = \varphi_0(B - B_u)/(B_d - B_u)$. The transmitted and backstreaming electron densities (normalized on the upstream density) are denoted in the figure as n_f and n_b, respectively.

3. Two-stream instability

We consider the high-frequency electrostatic oscillations with the wavevector $\mathbf{k} \parallel \mathbf{B}$. The corresponding dispersion relation is $\epsilon_\parallel = 0$, where

$$\epsilon_\parallel = 1 - \frac{\omega_{pe}^2}{k} \int \frac{dv_\parallel}{\omega - kv_\parallel} F(v_\parallel), \tag{7}$$

where $\omega_{pe}^2 = 4\pi n_u e^2/m_e$ is the upstream plasma frequency, and the distribution function $F(v_\parallel)$ is qualitatively shown in Figure 1. Qualitative analysis of the stability of this two-humped distribution can be done without solving the dispersion relation. The distribution should be unstable with respect to excitation of Langmuir waves, propagating in both directions. Analysis of the exact distribution function is difficult. Moreover, in thin shocks or with weak nonstationarity the distribution function can be expected to be distorted from (1). We shall, therefore, perform a semi-quantitative analysis of the model distribution consisting of a waterbag distribution $f_0 = n_0\theta(V_0^2 - v_\|^2)$ and two beams at the end of it $f_b = n_b[\delta(v_\| - V_0) + \delta(v_\| + V_0)]$. The distribution $F = f_0 + f_b$ resembles the numerically found distributions in Figure 1. The beams are taken of equal densities to simplify the analysis (this does not change qualitative conclusions). It can be seen from Figure 1 that the beam and waterbag densities are comparable. The corresponding dispersion relation reads:

$$1 - \frac{\omega_0^2}{\omega^2 - k^2V_0^2} - \frac{2\omega_b^2(\omega^2 + k^2V_0^2)}{(\omega^2 - k^2V_0^2)^2} = 0. \tag{8}$$

Hydrodynamic instability occurs when $k^2V_0^2 < 2\omega_b^2 - \omega_0^2$. Figure 2 shows the temporal growth rate of the two-stream

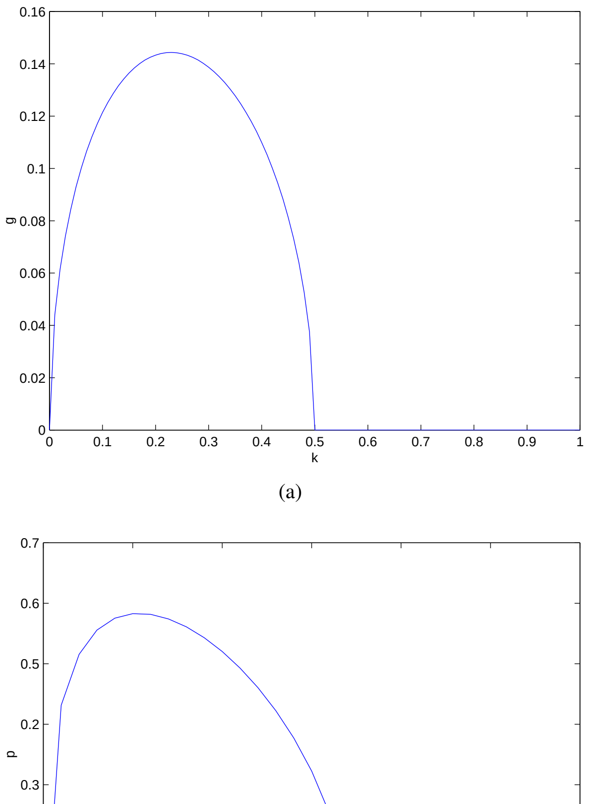

Figure 2. (a) Growth rate Γ and (b) spatial growth rate κ of the two-stream instability for $n_b/n_0 = 0.75$.

instability for $n_b/n_0 = 0.75$ as a function of k. The maximum growth rate Γ is $\approx 0.14\omega_0$ and occurs at $k \approx 0.25\omega_0/V_0$. Taking $V_0 \approx v_{Te}$ and $\omega_0 \approx 0.5\omega_{pe}$, one finds the growth rate of $\Gamma \sim 0.1\omega_{pe}$ and $kr_{De} \sim 0.1$. Spatial development of the instability is described by the spatial growth rate κ (spatial dependence is $\propto \exp(\kappa x)$) as a function of frequency. It is seen that the maximum spatial growth rate occurs at about $\omega \approx 0.7\omega_0$ and the spatial scale $1/\kappa \sim V_0/\omega_0$ is much less than the electron inertial length.

If $n_b < 0.5n_0$ the model distribution is not suitable for the analysis and the instability should be considered kinetically. In the kinetic regime the instability is not aperiodic and Langmuir waves are excited. Analytical consideration does not seem possible for general distribution of the form (1) but some qualitative conclusions can be made. The total density is $\approx 0.5n_u$ so that the excited wave frequency is $\omega_r \lesssim 0.7\omega_{pe}$. The expected resonant wavenumbers lie in the range$(0.2 - 0.5)\omega_r/v_{Te}$

so that $kr_{De} \lesssim 1$. The growth rate of the instability can be also expected to be $\Gamma \lesssim 0.1\omega_r$. The corresponding spatial scale is $\sim V_u/\Gamma \ll c/\omega_{pe}$, and therefore is much smaller than the typical scale of the magnetic field variation. The instability should be very efficient in relaxation of the distribution function, this relaxation taking place at the time scale substantially less than the ramp crossing time. The relaxation starts at the edges of the ramp and results in the filling of the gap and lowering of the peaks. If we consider only the parallel instability ($\mathbf{k} \parallel \mathbf{B}$) the distribution relaxation occurs at constant $v_\perp$ and in the final state a plateau $\partial f/\partial v_\parallel = 0$ is formed in the distribution (see, for example, *Swansson* [1989]), which could be responsible for the observed flattops. The edges of the plateau correspond to larger $|v_\parallel|$ than $|v_\parallel|$ corresponding to the maximum of $F(v_\parallel)$, which means that the Liouville mapping of the maximum in the upstream distribution to the edge of the observed downstream distribution [*Schwartz et al.*, 1988] would result in overestimates of the de Hoffman-Teller cross-shock potential. This is in agreement with the *Schwartz et al.* found discrepancy between this method and the method of the polytropic estimate based on (6): it was found that the edge mapping gives systematically higher estimates of the cross-shock potential than the polytropic approximation.

In reality not only waves with $\mathbf{k} \parallel \mathbf{B}$ are excited, and the relaxation should involve the perpendicular velocity $v_\perp$ as well. Respectively, instead of plateau a quasi-plateau is formed [*Swansson*, 1989], which, nevertheless, does not change the basic conclusion that the two-stream instability should result in the fast gap filling and flattopped distribution formation.

4. Conclusions

As a result of the collisionless electron dynamics in the shock front a two-stream unstable distribution is formed inside the ramp. Typical temporal and spatial scales of the developing instability are much smaller than the ramp crossing time and magnetic field inhomogeneity scale, respectively. Therefore, the instability may provide fast filling of the gap in the collisionless electron distribution. During the relaxation the distribution tends to acquire a flattop-like shape. The high energy end of the distribution is not involved in the relaxation and may be Liouville mapped onto the upstream distribution. Because of the relaxation the edge of the downstream electron distribution does not correspond to the maximum in the upstream distribution but to incident electrons with higher energies. Therefore, the standard Liouville mapping of the edge, used by *Schwartz et al.* [1988] for the determination of the de Hoffman-Teller cross-shock potential, should overestimate the potential.

Obviously, comprehensive quantitative analysis of the high frequency instabilities requires, in general, consideration of oblique waves too. The above analysis exploited the properties of the distribution formed due to the acceleration of transmitted electrons and deceleration of electrons leaking from the downstream region, that is, a particular mechanism of the distribution formation inside the ramp. Nevertheless, the growth of beams should be the common feature of the all collisionlessly formed distributions. We expect that these distributions are two-stream (bump-on-tail) unstable. Since these electrostatic instabilities are fast, they may result in effective bringing of the distribution to the marginally stable state, with low or no beams at all. Since the quasilinear diffusion can be considered as turbulent collisions, we argue that the formation of the inner part of the electron distribution is collisional, while the high energy tail is formed collisionlessly. If the above electrostatic instabilities occur they would be observed as appearance of the electrostatic noise inside the ramp at frequencies below the upstream plasma frequency. Such electrostatic noise at frequencies between the ion and electron plasma frequencies has been observed at the shock front [*Rodriguez and Gurnett*, 1975] and later tentatively identified by *Onsager et al.* [1989] as electron beam mode waves. It has been shown recently [*Matsumoto et al.*, 1997; *Bale et al.*, 1998] that this noise exists in the form of the short nonlinear structures with the typical spatial scale of several Debye lengths, in agreement with our estimated of the typical scale of the most unstable perturbations. *Omura et al.* [1994] have shown that "electron holes" can be formed as a result of nonlinear evolution of the two-stream instability with large beam to background density ratio. These holes can nonlinearly decay into smaller holes [*Saeki and Genma*, 1998] and probably result in large amplitude electric field spikes. This inhomogeneous turbulence should scatter electrons producing effective electron resistivity. *Bale et al.* [1998] suggested that the "electron holes" may scatter ions too. More reliable quantitative conclusions require detailed analysis of the behavior of the electron distribution inside the ramp (which in turn requires better knowledge of the shock structure). However, it seems that the two-stream instability of the collisionlessly formed electron distribution may trigger the electrostatic noise generation in the observed frequency band.

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