Ion heating in oblique low-Mach number shocks

M. Gedalin

Department of Physics, Ben-Gurion University, Beer-Sheva, Israel[1]

1. Introduction

Ion heating at low-Mach number shocks has been the subject of intensive studies for quite a while. The downstream ion temperature typically substantially exceeds what could be expected from the adiabatic magnetic compression even for subcritical shocks [*Thomsen et al.*, 1985]. In contrast with the supercritical shocks, where the strong ion heating is due to the ion reflection [see *Sckopke et al.*, 1983; *Thomsen et al.*, 1985; *Burgess et al.*, 1989; *Sckopke et al.*, 1990; *Wilkinson and Schwartz*, 1990, and others], the role of reflected ions in the low-Mach number shocks is weak [*Thomsen et al.*, 1985; *Sckopke et al.*, 1990; *Wilkinson*, 1991; *Gedalin*, 1996; *Wilkinson*, 1997], since the fraction of reflected ions rarely exceeds 2-3%. *Lee et al.* [1986, 1987] proposed that the heating may be due to the nondeflection of ions at the ramp. In the de Hoffman-Teller frame the incident ions flow along the magnetic field and retain their velocity after crossing the ramp, while the magnetic fields rotates. As a result, ions begin to gyrate and contribute to the downstream heating as any other gyrophase-bunched distribution would. Using 1D hybrid simulations, *Wilkinson* [1991] showed that the non-deflection assumption is not satisfied at the shock front, although the downstream ion distribution certainly gyrates. *Gedalin* [1996] analytically studied the ion dynamics within a finite width shock transition layer and described the evolution of the ion distribution and downstream heating in the nearly-perpendicular shocks. The gyration of the downstream distribution was explained as the result of the insufficient deceleration of ions by the cross-shock potential and consequent mismatch between the ion velocity and the required downstream drift velocity. *Balikhin and Wilkinson* [1996] argued that the ion heating may be due to the ion demagnetization inside the ramp. However, it was shown analytically and numerically [*Gedalin*, 1996] that most of the downstream heating is due to the ion gyration and only a small part results from the heating at the ramp. Recent simulations at quasi-perpendicular shocks [*Wilkinson*, 1997] have confirmed the importance of the ion gyration and have shown only weak sensitivity of the heating to the ramp width. In most previous works only perpendicular or nearly-perpendicular geometry was considered. In this case, in the absence of time-varying fluctuations (as is accepted in theory and test particle analysis) the downstream hydrodynamical variables (such as density, pressure, etc.) exhibit strictly periodic dependence on the coordinate along the shock normal [cf. *Gedalin*, 1996], which does not allow direct comparison with simulations or observations. In the oblique case collisionless gyrophase mixing can be expected since each ion has its own drift velocity along the shock normal. Hence, some averaging is appropriate. In the present paper we generalize the analysis of *Gedalin* [1996] onto the case of the oblique low-Mach number shock. We analyze the behavior of ions and derive the ion pressure tensor within the ramp. We also estimate the maximum average downstream temperature after the gyrophase mixing occurs.

2. Ion distribution within the ramp

Ion heating in the shock front is primarily due to the interaction of the ions with the spatially varying but time-stationary electric and magnetic fields [*Sckopke et al.*, 1983; *Thomsen et al.*, 1985; *Sckopke et al.*, 1990; *Burgess et al.*, 1989; *McKean et al.*, 1995], while turbulent effects and deviations from one-dimensionality are significant at scales typically much larger than the shock width [cf. *Jokipii et al.*, 1993; *Giacalone et al.*, 1994]. In what follows we assume that the shock is stationary and everything depends only on the coordinate x along the shock normal. In such stationary one-dimensional shock front the downstream distribution is completely determined by the ion distribution after the ramp, hence, we start with the analysis of the ion dynamics within the ramp. The cornerstone of the present approach is the narrow ramp approximation [see *Gedalin*, 1996, for a more detailed description]. Since the ramp width is of the order of c/ω_{pi} [*Russell et al.*1982, @; *Farris et al.*, 1993], or $\pi c\cos\theta/M\omega_{pi}$ [*Mellott and Greendstadt*, 1984], or even several c/ω_{pe} [*Newbury and Russell*, 1996] (where M is the Alfven Mach number, ω_{pi} and ω_{pe} are ion and electron plasma frequencies, respectively, and θ is the angle between the shock normal and upstream magnetic field), an ion spends only as little as $\lesssim 0.1$ part of its gyroperiod inside the ramp. Therefore, a perturbative approach is appropriate for the description of the ion motion there. We assume that the magnetic and electric fields vary only inside the ramp of width L, while outside it the fields take their asymptotically homogeneous values [cf. *Mellott and Greendstadt*, 1984; *Mellott and Livesey*, 1987; *Farris et al.*, 1993]. The upstream ($x < 0$) magnetic field is $\mathbf{B}_u = B_u(\cos\theta, 0, \sin\theta)$, and the upstream plasma velocity is $\mathbf{V}_u = V_u(1, 0, 0)$ (we are working in the normal incidence frame). The ion equations of motion (generalization of those used by *Gedalin* [1996] onto three-dimensional case) take the following form:

$$\dot{u}_x = e + u_y b_z - u_z b_y, \tag{1}$$
$$\dot{u}_y = \sin\theta + u_z \cos\theta - u_x b_z, \tag{2}$$
$$\dot{u}_z = u_x b_y - u_y \cos\theta, \tag{3}$$

where we use dimensionless variables, normalizing them as follows: $\mathbf{v}/V_u = \mathbf{u}$, $\mathbf{B}/B_u = \mathbf{b}$, $b_x = \cos\theta$, $\Omega_u t = \tau$, $\Omega_u = eB_u/m_i c$, $\Omega_u x/V_u = X$, $2e\varphi/m_i V_u^2 = \phi$, $e = -(1/2)d\phi/dX$, and φ is the cross-shock electrostatic potential, $E_x = -(\partial\varphi/\partial x)$. Dot means derivative with respect to τ and $u_x = \dot{X}$. The dimensionless upstream ion thermal velocity is $v_T = \sqrt{\beta_i/2M^2}$, where $\beta_i = 8\pi n_u T_u/B_u^2$.

Throughout the paper $\mathbf{u}$ stands for varying ion velocity (function of X or τ), and $\mathbf{w} = \mathbf{u}_{|X=0}$ and $\mathbf{W} = \mathbf{u}_{|X=L}$. We will assume that the influence of reflected or quasi-reflected [*Gedalin et al.*, 1996] ions is negligible and consider only directly transmitted ions. Taking into account that $u_x > 0$ across the ramp, we substitute $d/d\tau = u_x(d/dX)$ and find

$$u_x^2 = w_x^2 - \phi + 2\int_0^X dX(u_y b_z - u_z b_y), \tag{4}$$
$$u_y = w_y + \int_0^X \frac{dX}{u_x}(\sin\theta + u_z\cos\theta - u_x b_z), \tag{5}$$
$$u_z = w_z + \int_0^X \frac{dX}{u_x}(u_x b_y - u_y\cos\theta). \tag{6}$$

Since there are no collisions the distribution function $f(\mathbf{u}, X) = f_0(\mathbf{w})$, and averaging over the distribution is done as follows [*Gedalin and Zilbersher*, 1995; *Gedalin*, 1996]:

$$\begin{aligned}\langle g(\mathbf{u})\rangle &\equiv \int g(\mathbf{u}) f(\mathbf{u}, X) d^3\mathbf{u} \\ &= \int \tilde{g}(\mathbf{w}) f_0(\mathbf{w}) J d^3\mathbf{w} \equiv \langle J\tilde{g}\rangle_0,\end{aligned} \tag{7}$$

where $g(\mathbf{u}) = \tilde{g}(\mathbf{w})$, and J is the Jacobian of the transformation $\mathbf{w} \to \mathbf{u}$, which in the stationary one-dimensional case takes the following simple form: $J = |w_x/u_x|$. We will assume f_0 to be Maxwellian and $v_T \ll 1$. In the low β plasma ($|w_x - 1|, |w_y|, |w_z| \sim v_T \ll 1$) and thin ramp ($L \ll 1$) case one may expand (4) up to the first order in L and second order in v_T as follows:

$$\begin{aligned}u_x = \sqrt{1-\phi}[1 + (w_x - 1 + A_y w_y \\ + A_z w_z)/(1-\phi) - \phi(w_x - 1)^2/2(1-\phi)^2],\end{aligned} \tag{8}$$

where $A_y = \int_0^X b_z dX$ and $A_z = -\int_0^X b_y dX$. Now, using $N = \langle J\rangle_0$ (where $N = n/n_u$ is the normalized density) and taking into account that the only nonzero averages are $\langle (w_x - 1)^2\rangle_0 = \langle w_y^2\rangle_0 = \langle w_z^2\rangle_0 = v_T^2$, one immediately finds

$$N = (1-\phi)^{-1/2}\left[1 + \frac{3\phi}{2(1-\phi)^2}\frac{\beta_i}{2M^2}\right]. \tag{9}$$

Other moments are calculated similarly, using the definitions $nV_i = \langle Ju_i\rangle_0$ and $P_{ij} = \langle Ju_i u_j\rangle_0 - nV_iV_j$ (where $V_i = v_i/V_u$ is the normalized hydrodynamical velocity and $P_{ij} = p_{ij}/n_u m_i V_u^2$ is the normalized pressure tensor). Omitting straightforward but lengthy algebra, we presents the results in the following form (only leading terms are retained):

$$V_y = \lambda_0 - A_y, \quad V_z = -A_z \tag{10}$$
$$P_{xx} = (1-\phi)^{-3/2}P_0, \quad P_0 = \beta_i/2M^2, \tag{11}$$
$$P_{yy} = P_{zz} = (1-\phi)^{-1/2}P_0, \tag{12}$$
$$P_{xy} = (A_y - \lambda_1\sin\theta)(1-\phi)^{-1}P_0, \tag{13}$$
$$P_{xz} = A_z(1-\phi)^{-1}P_0, \quad P_{yz} = 0, \tag{14}$$
$$\lambda_0 = \int_0^X \frac{dX}{(1-\phi)^{1/2}}, \quad \lambda_1 = \int_0^X \frac{dX}{(1-\phi)^{3/2}}. \tag{15}$$

These expressions generalize the results of *Gedalin* [1996], obtained for perpendicular geometry, and correct their Eq. (22). In the leading order, neglecting $\beta \ll 1$, one has $N = (1-\phi)^{-1/2}$ and $P_{xx} = N^3P_0$, $P_{yy} = P_{zz} = NP_0$ which means that the ions are completely demagnetized so that the pressure P_{xx} along the shock normal obeys the one-dimensional adiabatic law, while the perpendicular temperature does not change.

3. Downstream heating

Having determined the ion velocity change within the ramp we know now the ion distribution at the downstream edge of the ramp. From this point on ions proceed in the homogeneous downstream magnetic field, which we write in the following dimensionless form: $\mathbf{B}_d = (\cos\theta, 0, R\sin\theta)$, $R = B_{zd}/B_{zu}$. It is convenient to switch to the de Hoffman-Teller frame (HT), where the upstream plasma velocity is along the magnetic field and the motional electric field is absent. The velocity transformation reads:

$$\bar{u}_{x,y} = u_{x,y}, \qquad \bar{u}_z = u_z + \tan\theta \tag{16}$$

(barred variables refer to the de Hoffman-Teller frame), while the pressure and temperature are invariant under this transformation. The HT equations of motion read:

$$v_\parallel = \text{const}, \quad \dot{v}_{\perp,1} = B_d v_{\perp,2}, \quad \dot{v}_{\perp,2} = -B_d v_{\perp,1}, \tag{17}$$

where we introduced the notation $v_\parallel = (\bar{u}_z R\sin\theta + \bar{u}_x\cos\theta)/B_d$, $v_{\perp,1} = \bar{u}_y$, $v_{\perp,2} = (\bar{u}_x R\sin\theta - \bar{u}_z\cos\theta)/B_d$, and $B_d = (R^2\sin^2\theta + \cos^2\theta)^{1/2}$. The solution of (17) with the initial condition $X_{|\tau=0} = L$ and $\bar{\mathbf{u}}_{|\tau=0} = \bar{\mathbf{W}}$ has the following form:

$$v_{\perp,1} = V_\perp\cos(B_d\tau + \psi), \tag{18}$$

$$v_{\perp,2} = -V_\perp\sin(B_d\tau + \psi), \tag{19}$$

$$X = L + \frac{v_\parallel\cos\theta}{B_d}\tau + \frac{R\sin\theta V_\perp}{B_d^2}[\sin(B_d\tau + \psi) - \sin\psi], \tag{20}$$

where the constant parameters $V_\perp$, ψ, and $v_\parallel$ are

$$V_\perp = \left[\bar{W}_y^2 + \frac{(\bar{W}_x R\sin\theta - \bar{W}_z\cos\theta)^2}{B_d^2}\right]^{1/2}, \tag{21}$$

$$\psi = \tan^{-1}\left[\frac{\bar{W}_x R\sin\theta - \bar{W}_z\cos\theta}{B_d\bar{W}_y}\right], \tag{22}$$

$$v_\parallel = \left[\bar{W}_x\cos\theta + \bar{W}_z R\sin\theta\right]/B_d. \tag{23}$$

Eq. (20) shows that the drift velocity in the direction of the shock normal $V_d = v_\parallel\cos\theta/B_d$ depends on the peculiar ion velocity, so that gradual collisionless gyrophase mixing should be expected in the oblique case [*Gedalin et al.*, 1996]. Finding $\tau = \tau(X, v_\parallel, V_\perp, \psi)$ from (20) (there will be multiple solutions if $R\sin\theta V_\perp/B_d > v_\parallel\cos\theta$), substituting it in (18)-(19), and using (7), one can obtain in principle the moments of the downstream ion distribution. Unfortunately, (20) cannot be solved analytically, in general, and the resulting expressions are not very useful [cf. *Gedalin and Zilbersher*, 1995; *Gedalin*, 1996]. Instead, we shall estimate the upper limit on the downstream temperature in the low-β case, assuming that the gyrophase mixing results in a gyrotropized ring-like distribution far downstream. Alternatively, this can be considered as the spatial averaging of the distribution or pressure (but not temperature !) over $\Delta X \gtrsim R\sin\theta V_\perp/B_d^2$. In this approach we lose the gyrophase information and treat the ions in the guiding center approximation, collecting ions at the spatial length which greatly exceeds their gyroradii (this is the usual approximation for electrons whose gyroradii are too small to be resolved observationally). This approach is similar to what has been used to estimate the contribution of reflected ions in the downstream heating [*Sckopke et al.*, 1983]. This immediately gives the estimate $T_{\text{max}}/m_iV_u^2 \approx \frac{1}{2}V_\perp^2$. For this estimate it is sufficient to restrict ourselves to the cold ion approximation and ignore corrections related to the finite ramp width. Then from (4)-(6) one has $W_x = \sqrt{1-\phi_0}$, $W_y = W_z = 0$, where $\phi_0 = \phi_{|X=L}$. Substituting into (16) we find $\bar{W}_X = \sqrt{1-\phi_0}$, $\bar{W}_y = 0$, $\bar{W}_z = \tan\theta$, and finally $V_\perp = |R\sqrt{1-\phi_0} - 1|\sin\theta/B_d$. Now the estimate of the maximum downstream temperature looks as follows:

$$\frac{T_{\text{max}}}{m_iV_u^2/2} = \tfrac{1}{2}V_\perp^2 = \frac{(R\sqrt{1-\phi_0} - 1)^2\sin^2\theta}{2B_d^2}. \tag{24}$$

Because of the nonzero velocity spread of the initial distribution, the above considered ring will be filled due to the gyrophase mixing, so that the eventual heating will be somewhat lower. Although it is difficult to estimate the exact deviation without detailed knowledge of the distribution, in the low β limit the effect should not be strong. The estimated temperature is the perpendicular temperature, while in this approximation the parallel heating is weak, if any. The above findings are in qualitative agreement with the results of the previous numerical analysis of the perpendicular shock [*Gedalin*, 1996], which shows that the non-gyrotropic pressure, averaged over large spatial length, is negligible, and the average distribution is nearly gyrotropic. Writing the average pressure tensor in the gyrotropic form $\langle P_{ij}\rangle = P_\perp\delta_{ij} + (P_\parallel - P_\perp)b_ib_j$ one finds $\langle P_{yy}\rangle = P_\perp$, which provides easy comparison with simulations or observational data.

Thus, the maximum normalized downstream temperature depends only on the magnetic compression ratio and NIF cross-shock potential. Further estimates can be made using the pressure balance equation and the estimate of the cross-shock potential in the form [cf. *Gedalin*, 1996]:

$$\frac{2M^2}{N} + P_{e,xx} + P_{i,xx} + b_y^2 + b_z^2$$
$$= 2M^2 + \beta_e + \beta_i + \sin^2\theta, \tag{25}$$
$$\phi_0 = \frac{1}{M^2}\int_0^L \frac{1}{N}\frac{d}{dX}(\mathbf{b}^2 + P_{e,xx})dX. \tag{26}$$

In the low $\beta_{e,i} \ll 1$ limit, approximating $P \propto N^2$ and $N/|\mathbf{b}| \approx$ const, one finds

$$B_d = \tfrac{1}{2}[-1 + \sqrt{8M^2/(1+\beta_e+\beta_i)+1}], \tag{27}$$
$$\phi_0 = \frac{2(B_d-1)(1+\beta_e)}{M^2}. \tag{28}$$

Combination of (24) with (27) and (28) provides a rough estimate of the maximum ion heating at the quasi-perpendicular shock as a function of the Mach number only and can be a basis for the search of the correlation between the Mach number and ion heating.

4. Conclusions

We have derived the ion pressure tensor within the ramp of the low-Mach number oblique shock. We have found also the upper limit on the downstream heating which would be achieved after complete gyrophase mixing and gyrotropization far downstream (such smoothing would be faster in reality due to turbulence present in the shock front [*McKean et al.*, 1995; *Wilkinson*, 1997]). Additional factors, which may provide faster smoothing, may be the weak nonstationarity of the shock front and rippling of the shock surface, due to which ions, moving in slightly different field patterns, meet at the same place and time downstream. The maximum downstream temperature depends only on the magnetic compression and total NIF cross-shock potential. This estimate provides also the upper limit on the heating that would be observed averaging the ion downstream over a spatial region, large compared to the downstream ion gyroradius, and thus may be verified both observationally and in numerical simulations. For the low-β quasi-perpendicular shocks both magnetic compression and cross-shock potential may be approximately expressed in terms of the Mach number. As a result, the estimated downstream temperature becomes a function of the Mach number and upstream plasma parameters only, which may provide a basis for the observational studies of correlations between the shock Mach number and ion heating.

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