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Non-diagonal ion pressure in nearly-perpendicular collisionless shocks

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1. Introduction

It is well known that the ion distribution upstream of the Earth's bow shock [*Gurgiolo et al.*, 1981; *Sckopke et al.*, 1983] and well downstream in the magnetosheath [*Sckopke et al.*, 1990] is nongyrotropic and contains gyrophase-bunched ions. Consequences of non-gyrotropy for wave features and micro-stability has been studied extensively [*Brinca et al.*, 1993]. Temperature anisotropy caused by gyrophase-bunched ions also was studied both observationally [*Sckopke et al.*, 1990] and in simulations [*Burgess et al.*, 1989; *Wilkinson*, 1991; *McKean et al.*, 1995]. In these studies only the diagonal components of the temperature were considered, either in the shock coordinates or relative to the magnetic field direction. The complete pressure structure, however, has not been considered, and in the studies of the shock structure (and magnetosheath plasma as well) it is still widely accepted to assume pressure isotropy or at least gyrotropy [*Lyu and Kan*, 1986; *Hau et al.*, 1993; *Chao et al.*, 1995]. In the last case the pressure tensor is assumed to be axisymmetric with a symmetry axis along the magnetic field. Less attention has been devoted to the off-diagonal components of the ion pressure tensor (in the shock coordinates, where x is along the shock normal, y is the noncoplanarity direction, and z is along the main component of the magnetic field, these off-diagonal components are p_{xy}, p_{xz} and p_{yz}). Recently, *Li et al.* [1995] found that the ion pressure tensor in the magnetosheath, far from the bowshock, is only approximately axisymmetric, and its symmetry axis is not directed along the magnetic field, that is, there are nonzero off-diagonal components which *Li et al.* [1995] relate to the strong local obliquity of the magnetic field. These off-diagonal components indicate presence of gyrophase bunched ions. Since there is a strong gyrophase-bunched component of the ion distribution in the foot of the high Mach number supercritical shocks, due to the large fraction of reflected ions ($> 20\%$ for $M \sim 8$ [*Sckopke et al.*, 1983]), one can expect that substantial nonzero off-diagonal components of the pressure tensor should be observable there even in the case of the perpendicular geometry.

The hydrodynamical structure of collisionless shocks depends crucially on the ion pressure behavior in the shock front. While neglecting the off-diagonal components of the ion pressure may be a satisfactory approximation at scales $\gtrsim V_u/\Omega_u$ (where V_u is the plasma upstream velocity and $\Omega_u = eB_u/m_i$ is the upstream ion gyrofrequency), these components should be taken into account when analyzing the fine structure of the shock. They are related to the noncoplanar magnetic field enhancement and cross-shock electric field, which have been described earlier by introducing unknown ion current velocity [*Jones and Ellison*, 1991] and/or generalized Ohm's law with phenomenological resistivity of collisionless plasma [*Scudder*, 1995].

The objective of the present paper is to investigate the structure of the ion pressure in the foot of the perpendicular collisionless shock, where there is a mixture of incident and reflected-gyrating ions, and to analyze some implications for the shock physics.

2. Pressure Tensor Structure

Let us consider a perpendicular shock geometry (shock normal and magnetic field along x and z, respectively). Equations of the two-fluid hydrodynamics for the one-dimensional, stationary, and quasineutral shock read

$$nm_e v_x \frac{dv_x}{dx} = -neE_x - nev_{e,y}B_z - \frac{dp_{e,xx}}{dx}, \tag{1}$$

$$nm_i v_x \frac{dv_x}{dx} = neE_x + nev_{i,y}yB_z - \frac{dp_{i,xx}}{dx}, \tag{2}$$

$$nm_e v_x \frac{dv_{e,y}}{dx} = -en(E_y - v_x B_z) - \frac{dp_{e,xy}}{dx}, \tag{3}$$

$$nm_i v_x \frac{dv_{i,y}}{dx} = en(E_y - v_x B_z) - \frac{dp_{i,xy}}{dx}, \tag{4}$$

$$\frac{dB_z}{dx} = -\mu_0 ne(v_{i,y} - v_{e,y}), \tag{5}$$

where $n_e = n_i = n$, $v_{e,x} = v_{i,x} = v_x$, $nv_x = n_u V_u =$ const, and $E_y = V_u B_u =$ const.

Assuming gyrotropy of the electron pressure $p_{e,xy} = 0$ due to small electron gyroradius and high electron gyrofrequency and using the widely accepted massless electron approximation $m_e = 0$ one finds

$$v_{i,y} = -\frac{p_{i,xy}}{n_u m_i V_u}. \tag{6}$$

If $p_{i,xy} = 0$ the ion current velocity $v_{i,y} = (m_e/m_i)v_{e,y}$ is negligible, in contrast with observations on high Mach number shocks. Using (6) one has, consequently,

$$eE_x = -\frac{1}{n}\frac{dp_{e,xx}}{dx} - \frac{1}{2\mu_0 n}\frac{dB_z^2}{dx} - ev_{i,y}B_z, \tag{7}$$

which is well known [*Scudder*, 1995], and

$$eE_x = -\frac{1}{n}\frac{dp_{e,xx}}{dx} - \frac{1}{2\mu_0 n}\frac{dB_z^2}{dx} + \frac{ep_{i,xy}}{n_u m_i V_u}B_z. \tag{8}$$

The last term in (8) directly relates the potential electric field to the off-diagonal elements of the ion pressure tensor. To evaluate this potential we use the approximations $n \propto B$ [*Scudder et al.*, 1986] and $p_{e,xx} \propto n^{\gamma_e - 1}$ [*Schwartz et al.*, 1988] to obtain

$$\begin{aligned} e\varphi &= \frac{\gamma_e}{\gamma_e - 1}(T_{e,xx} - T_{e,u}) + \frac{B_u}{\mu_0 n_u}(B_z - B_u) \\ &+ \frac{e}{n_u m_i V_u}\int p_{i,xy}B_z\,dx. \end{aligned} \tag{9}$$

While the first two terms in (9) depend only on the initial and final values of the electron temperature and magnetic field, respectively, the last term is significantly nonlocal. It may contribute considerably to the overall cross-shock potential at the shock front.

In the present paper we analyze the ion pressure in the foot of a supercritical shock within the model of specular ion reflection. In the spirit of *Schwartz et al.* [1983] we assume that $\mathbf{B} = (0, 0, B_u) = \text{const}$ and $\mathbf{E} = (0, V_u B_u, 0) = \text{const}$ in the foot $-D < x < 0$. Ions are reflected off the ramp $x = 0$ specularly, that is, in the reflection point v_x changes its sign, while v_y does not change. The effect of the gradual increase of the magnetic field in the foot can be approximately taken into account by equating B_u to the mean magnetic field in the foot [*Sckopke et al.*, 1983]. It results also in the weak bulk acceleration of the incident ions in y-direction [cf., for example, *Burgess et al.*, 1989], which, probably, can be estimated perturbatively. In the present letter we restrict ourselves to the simplest model of a constant magnetic field in the foot, assuming also that the cross-shock potential has already done its job by specularly reflecting certain fraction of incident ions, and do not take into account the weak potential in the foot [*Wilkinson and Schwartz*, 1990]. In this way we rely on the observations of *Sckopke et al.* [1983], who found that the model of specular reflection provides a satisfactory description of the reflected ion distribution in the foot, within the limits of observational errors.

An ion trajectory in the foot can be represented as follows

$$v_x = V_u + w_y \sin\psi + (w_x - V_u)\cos\psi, \tag{10}$$

$$v_y = w_y \cos\psi - (w_x - V_u)\sin\psi, \tag{11}$$

$$\Omega_u x = (V_u\psi + (w_x - V_u)\sin\psi + w_y(1 - \cos\psi)), \tag{12}$$

where $\psi > 0$ is the ion gyrophase, and the initial conditions read $x = 0$, $v_x = w_x$, and $v_y = w_y$ at $\psi = 0$. We define the two single-valued reflection functions $\psi_1(\xi)$ and $\psi_2(\xi)$, where $\xi = \Omega_u x/V_u$, which are the solutions of (12), such that $0 < \psi_1 < \psi_0 - \alpha < \psi_2 < 2\pi - \psi_0 - \alpha$, where $\alpha = \arcsin(w_y/v_\perp)$, $\psi_0 = \arccos(V_u/v_\perp)$, and $v_\perp^2 = (V_u - w_x)^2 + w_y^2$. Obviously, ψ_1 and ψ_2 are the ion gyrophases in the point x before and after the turnaround, respectively.

Given the distribution of reflected ions $f_r(w_x, w_y)$ at $x = 0$, the distribution in an arbitrary point $x < 0$ is $f(v_x, v_y, x) = f_r(w_x(v_x, v_y, x), w_y(v_x, v_y, x))$. Any function $g(v_x, v_y)$ is averaged as follows

$$\begin{aligned} \langle g(v_x, v_y)\rangle &= \int g(v_x, v_y)f(v_x, v_y, x)\,dv_x dv_y \\ &= \int g(v_x, v_y)f_r(w_x, w_y)|w_x|/|v_x|\,dw_x dw_y \end{aligned} \tag{13}$$

where $v_x = v_x(w_x, w_y, x)$, $v_y = v_y(w_x, w_y, x)$ are obtained by substituting ψ_1 and ψ_2 into (10) and (11), and $J = |w_x/v_x|$ is the Jacobian of the transformation $\mathbf{w} \to \mathbf{v}$ for $x = \text{const}$. Approximating the incident ion distribution by the Maxwellian

$f(v_x, v_y) = (n_0/2\pi v_T^2)\exp(-((v_x - V_u)^2 + v_y^2)/2v_T^2)$, and taking moments one obtains

$$n = n_0 + \sum_s \langle\langle J^{(s)} \rangle\rangle, \tag{14}$$

$$nV_x = n_0 V_u + \sum_s \langle\langle v_x^{(s)} J^{(s)} \rangle\rangle = n_0 V_u, \tag{15}$$

$$nV_y = \sum_s \langle\langle v_y^{(s)} J^{(s)} \rangle\rangle, \tag{16}$$

$$\begin{aligned} p_{xx} = m_i(n_0(V_u^2 + v_T^2) \\ + \sum_s \langle\langle (v_x^{(s)})^2 J^{(s)} \rangle\rangle - nV_x^2), \end{aligned} \tag{17}$$

$$p_{yy} = m_i(n_0 v_T^2 + \sum_s \langle\langle (v_y^{(s)})^2 J^{(s)} \rangle\rangle - nV_y^2), \tag{18}$$

$$p_{xy} = m_i(\sum_s \langle\langle v_x^{(s)} v_y^{(s)} J^{(s)} \rangle\rangle - nV_x V_y), \tag{19}$$

where p_{ij} is the pressure tensor, $J^{(s)} = |w_x|/|v_x^{(s)}|$, the summation is over the roots of (12) ($s = 1, 2$ correspond to ψ_1 and ψ_2, respectively), $v_T = \sqrt{T/m_i} = V_u\sqrt{\beta_i/2}/M$, $\beta_i = 8\pi nT/B^2$, M is the Alfven Mach number, and we introduced the notation $\langle\langle \ldots \rangle\rangle = \int (\ldots) f_r(w_x, w_y) dw_x dw_y$ for brevity.

(14)-(19) represent the hydrodynamical variables in terms of the reflection functions ψ_1 and ψ_2, for arbitrary f_r. If the reflection is specular $f_r(w_x, w_y) = (n_r/2\pi v_T^2)\exp(-((w_x + V_u)^2 + w_y^2)/2v_T^2)$. In the high Mach number shocks $v_T^2/V_u^2 = \beta_i/2M^2 \ll 1$, unless $\beta_i \sim M^2 \sim 10^2$ for $M \gtrsim 6$. It is easy to see that Taylor expansion of the integrands in (14)-(19) goes on powers of $(v_T/v_x)^2$. Sufficiently far from the turnaround point $v_x \sim V_u$ and the terms $\sim (v_T/v_x)^2$ and higher can be neglected. Therefore, in the lowest order at $x = 0$ one can put $f_r = n_r\delta(w_x + V_u)\delta(w_y)$. Then $v_x = V_u(1 - 2\cos\psi)$, $v_y = 2V_u \sin\psi$, and

$$\Omega_u x/V_u = \psi - 2\sin\psi, \tag{20}$$

and the moments of the total distribution function are

$$n = n_0 + n_r(|K_1|^{-1} + |K_2|^{-1}), \tag{21}$$

$$nV_y = 2n_r V_u(\sin\psi_1 |K_1|^{-1} + \sin\psi_2 |K_2|^{-1}), \tag{22}$$

$$\begin{aligned} p_{xx} = m_i(n_0(V_u^2 + v_T^2) \\ + n_r V_u^2(K_2 - K_1) - nV_x^2), \end{aligned} \tag{23}$$

$$\begin{aligned} p_{yy} = m_i(n_0 v_T^2 + 4n_r V_u^2(\sin^2\psi_1 |K_1|^{-1} \\ + \sin^2\psi_2 |K_2|^{-1}) - nV_y^2), \end{aligned} \tag{24}$$

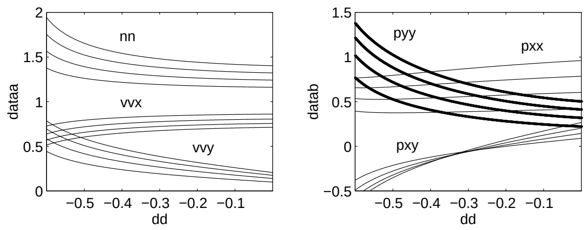
$$p_{xy} = m_i(2n_r V_u^2(\sin\psi_2 - \sin\psi_1) - nV_x V_y), \tag{25}$$

where $K_i = 1 - 2\cos\psi_i$, ψ_1 and ψ_2 are determined by (20), and $K_1 < 0 < K_2$. All ions turn around in the point $v_x = 0 \to \psi_t = \pi/3$, $x = V_u(\pi/3 - 2\sin(\pi/3))/\Omega_u \approx -0.68(V_u/\Omega_u)$ [*Woods*, 1971]. The thermal component $n_0 m_i v_T^2$ of the incident ion pressure cannot be neglected a priori since it might be that $n_0/n_r \gg 1$.

(21)-(25) describe the typical pressure tensor for the mixture of incident and reflected ions in the foot of a high Mach number shock sufficiently far from the turnaround point $x = -0.68(V_u/\Omega_u)$. The approximation $v_T/v_x \ll 1$ does not work near the turnaround point and this region requires separate consideration. This question is beyond the scope of the present letter.

The profiles of the total density and plasma velocities, predicted by (21)-(22), are shown in Figure 1a for $M = 6$, $\beta_i = 1$, and four different values of the reflected ion fraction $n_r/n_0 = 0.1, 0.15, 0.2, 0.25$. Both density and V_y increase with the increase of n_r/n_0, while V_x decreases. The corresponding pressure tensor components are shown in Figure 1b. One can see that $p_{xx}, p_{yy} \sim n_0 m_i V_u^2$. The contribution of the thermal incident ion pressure $p_{xx}^{(T)} = p_{yy}^{(T)} = n_0 m_i v_T^2$ is small $\delta p/p \sim v_T^2/V_u^2 \ll 1$, so that p_{ij} is rather insensitive to β_i. The absolute value of p_{xy} changes less than p_{xx} and p_{yy}, although relative variations are similar. The components p_{xx} and p_{xy} monotonically increase with the increase of x, while p_{yy} decreases. Both p_{xx} and p_{yy} increase monotonically with the increase of n_r/n_0. The component p_{xy} is correlated with n_r/n_0 near the ramp, but they anti-correlate near the turnaround point. This component changes its sign approximately in the middle of the foot.

If $B_z =$ const the electric field $E_x \propto p_{xy}$ (see (8)). The behavior of this E_x and its value at the ramp $\approx 0.25E_y$ are in agreement with observations [*Scudder et al.*, 1986]. The distribution of the potential in the foot is shown in Figure 2. The potential drop constitutes about 0.1 of the incident ion energy, in qualitative agreement with observations [*Scudder et al.*, 1986] and simulations [*Burgess et al.*, 1989]. It is rather insensitive to the reflected ion fraction because of the partial



(a) (b)

Figure 1. Ion (a) density and velocity components and (b) pressure tensor components in the foot for different $n_r/n_0 = 0.1, 0.15, 0.2, 0.25$.

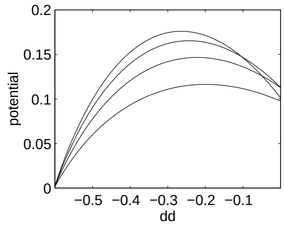
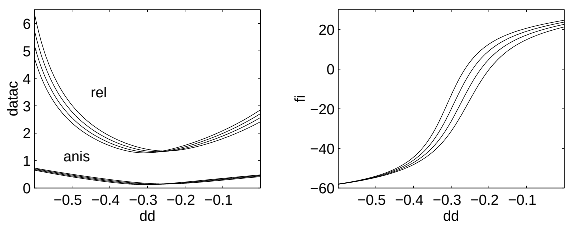


Figure 2. Electric potential distribution in the foot for different n_r/n_0.



(a) (b)

Figure 3. The pressure anisotropy parameters (a) $r = p_{\text{max}}/p_{\text{min}}$ and $A = (p_{\text{max}} - p_{\text{min}})/(p_{\text{max}} + p_{\text{min}})$ and (b) angle between the major axis of the pressure tensor ellipse and x-axis for different n_r/n_0.

cancellation in (9) due to the change of sign of p_{xy}. The potential maximum in the middle of the foot increases with the increase of n_r/n_0.

Due to $p_{xx} \neq p_{yy}$ and $p_{xy} \neq 0$ the pressure tensor is no more gyrotropic. The ratio $r = p_{\text{max}}/p_{\text{min}}$ and anisotropy $A = (p_{\text{max}} - p_{\text{min}})/(p_{\text{max}} + p_{\text{min}})$, where $p_{\text{max,min}} = (p_{xx} + p_{yy} \pm ((p_{xx} - p_{yy})^2 + 4p_{xy}^2)^{1/2})/2$, are shown in Figure 3a, while the angle between the major axis and the shock normal $\theta = \arctan{(p_{\text{max}} - p_{xx})/p_{xy}}$ is shown in Figure 3b. They depend weakly on n_r/n_0. The anisotropy drops to ~ 1 approximately in the middle of the foot and is large at the edges. The major axis of the pressure tensor ellipse rotates to almost $90\,\deg$ across the foot. The anisotropy is larger than the observed by *Li et al.* [1995] in the magnetosheath (in the plane perpendicular to the magnetic field). It is not surprising since the shock front is an efficient source of the reflected-gyrating ions in the foot. These ions may contribute also to the ions pressure features found by *Li et al.* [1995], if they can propagate far downstream without being completely isotropized.

The temperature is defined as $T_{ij} = p_{ij}/n$. While the total perpendicular temperature $T_\perp = (T_{xx}+T_{yy})/2$ is independent of p_{xy}, the anisotropy $A = ((T_{xx} - T_{yy})^2 + 4T_{xy}^2)^{1/2}/T_\perp$ can be substantially higher than the usually defined $A_0 = |T_{xx} - T_{yy}|/T_\perp$, due to $T_{xy} \neq 0$.

3. Conclusions

To conclude, we have shown that the gyrophase bunched reflected ions in the shock foot may contribute significantly to the off-diagonal elements of the ion pressure tensor. General expressions are derived for the pressure tensor of the mixture of incident and reflected ions in the widely accepted model of the foot. When the reflection is nearly specular the pressure tensor takes definite typical form which is rather insensitive to the incident ion temperature.

The potential electric field is directly related to the off-diagonal ion pressure components. The behavior of this electric field and the found potential drop $\sim 0.1(m_i V_u^2/2)$ across the foot are in a qualitative agreement with observations [*Scudder et al.*, 1986]. Presence of this electric field and increase of the magnetic field in the foot due to the reflected ion current would modify the ion motion in this region, which should be taken into account in a more sophisticated model. Observations [*Sckopke et al.*, 1983; *Scudder et al.*, 1986] and simulations [*Burgess et al.*, 1989] show that the bulk ion velocity changes by $\sim 0.1 V_u$, so that we expect that the corresponding corrections be $\lesssim 10\%$. Non-specular character of the ion reflection [*Sckopke et al.*, 1983] would probably result in a more significant modification of the quantitative results of our model. This question is beyond the scope of the present letter.

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