Role of overshoots in the formation of the downstream distribution of adiabatic electrons

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Abstract. We analyze the influence of the magnetic overshoot on the downstream electron distributions formed as a result of the collisionless adiabatic motion of the electrons in the quasi-stationary electric field in shock front. We show that a substantial overshoot can result in a significant distortion of the downstream distribution due to cutting out the electrons with high perpendicular velocities. We calculate numerically the electron distribution in the $v_\parallel - v_\perp$ plane, as well as expected cuts through the distribution depending on the angle with respect to the magnetic field direction, also taking into account the averaging procedure of the ISEE type apparatus. These distorted distributions and cuts should be clearly observed in the downstream region just behind the shock transition layer. Absence of such observations would indicate that the above picture of adiabatic electron dynamics in the static field is significantly incomplete, and estimates based on these assumptions should be considered with caution.

1. Introduction

The prevailing view on the electron heating in shocks is that it is due to the quasi-static electric field in the shock front [*Feldman*, 1985; *Thomsen et al.*, 1987; *Schwartz et al.*, 1988]. The model suggested for not very thin shocks is based on the conservation of the magnetic moment in the weakly inhomogeneous (at the scale much larger than the electron gyroradius) magnetic field and energy conservation in the de Hoffman–Teller frame, where the cross–shock potential depends only on the coordinate along the shock normal [*Feldman et al.*, 1982; *Goodrich and Scudder*, 1984; *Feldman*, 1985; *Scudder et al.*, 1986c; *Schwartz et al.*, 1988; *Scudder*, 1995]. In this scenario, electrons are accelerated along the magnetic field when crossing the ramp. As a result, a gap (absence of low-energy electrons, see (7) in section 2) is formed in the downstream distribution, which is assumed to be filled owing to some preexisting downstream electron population [*Feldman*, 1985] or owing to whistler instabilities [*Veltri et al.*, 1990; *Veltri and Zimbardo*, 1993a, b]. In the recent study by *Hull et al.* [1998] the gap filling is simply taken as something given.

It was shown that if the ramp is very thin [*Balikhin et al.*, 1993; *Balikhin and Gedalin*, 1994; *Gedalin et al.*, 1995a, b] or if there is some small-scale structure present [*Gedalin and Balikhin*, 1997; *Gedalin et al.*, 1997], electrons become demagnetized in the ramp and accelerated along the shock normal. (The existence of these thin shocks has been demonstrated by *Newbury and Russell* [1996]). In this scenario, downstream distribution also possesses a large gap which has to be filled.

In both cases the collisionless electron dynamics in the quasi-static fields of the shock front determine only a part of the downstream distribution, not explaining the gap filling. The downstream electron distribution consists of electrons which came from upstream and those which are not connected with the upstream region at all and therefore, which do not carry "fingerprints" of the shock fields. Therefore only high-energy tail observations can be used to draw reliable conclusions about electron dynamics.

Recently [*Hull et al.*, 1998], downstream electron distributions formed owing to the adiabatic electron dynamics in the shock were analyzed. In this analysis it was assumed that the Liouville mapping depends only on the upstream and downstream magnetic fields and cross-shock potential and does not depend on the details of the field profiles in the shock front. In particular, overshoot is not taken into account.

Effects related to the nonmonotonic behavior of the magnetic field were studied earlier by *Veltri et al.* [1990, 1992] and *Veltri and Zimbardo* [1993a, b] using numerical Monte Carlo simulations and taking into account both reversible and diffusive effects. It was found that downstream distributions lack electrons with low parallel velocity and high perpendicular velocities when only reversible (collisionless) effects are taken into account. It was suggested [*Veltri and Zimbardo*, 1993a, b] that the gap in the distribution can be filled owing to electrostatic instabilities within the drift approximation and that the drift approximation should be violated (by the presence of high-frequency waves excited owing to the whistler instabilities) in order to explain the values of the observed downstream perpendicular temperature.

In the present brief report we perform an analytical study of the influence of the overshoot field on the downstream distributions formed in the adiabatic regime. In section 2 we present general analytical expressions for downstream distributions of adiabatic electrons as a function of the magnetic compression ratio and cross-shock potential. In section 3 we visualize the distributions and cuts at different angles with respect to the downstream magnetic field. We also take into account the averaging performed by the ISEE type apparatus.

2. Downstream Distribution of Connected Electrons

Let us denote the upstream electron distribution as $f_u = f_u(v_\perp, v_\parallel)$, where $\perp$ and $\parallel$ refer to the direction of the local (upstream in this case) magnetic field. We are working in the de Hoffman-Teller frame where the only electric field component is along the shock normal (x for convenience), that is, the cross-shock potential $\phi = \phi(x)$. If the electron upstream velocity is $(v_{\perp,u}, v_{\parallel,u})$, then the magnetic moment and energy conservation give

$$v_\perp = v_{\perp,u}\sqrt{B/B_u}, \quad (1)$$

$$v_\perp^2 + v_\parallel^2 = v_{\perp,u}^2 + v_{\parallel,u}^2 + \frac{2e\phi}{m_e}, \quad (2)$$

where B and ϕ are the magnetic field and potential, respectively, in the point of interest.

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Hence the electron distribution in the point B, ϕ is given as $f(v_\perp, v_\parallel) = f_u(v_\perp, v_\parallel)$, where

$$v_{\perp,u} = v_\perp \sqrt{B_u/B}, \tag{3}$$

$$v_{\parallel,u} = \sqrt{Q}\,\mathrm{sign}(v_\parallel), \tag{4}$$

$$Q = v_\perp^2(1 - B_u/B) + v_\parallel^2 - \frac{2e\phi}{m_e}, \tag{5}$$

if $Q \geq 0$, that is, if the point $(v_\perp, v_\parallel)$ in the velocity space is accessible for upstream electrons. Negative sign in (4) corresponds to downstream electrons which leak into the upstream region.

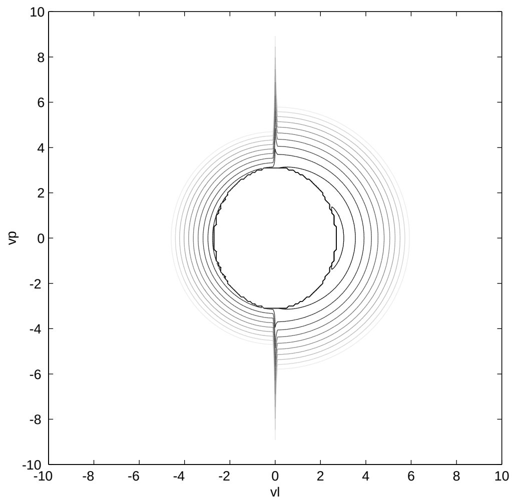

Figure 1. Downstream distribution (contour plot) formed without overshoot after crossing the shock with the following parameters: Alfvenic Mach number $M = 7.7$, angle between the shock normal and upstream magnetic field $\theta = 76°$, downstream-to-upstream magnetic field ratio $B_d/B_u = 3$, and electron $\beta_e = 1.6$.

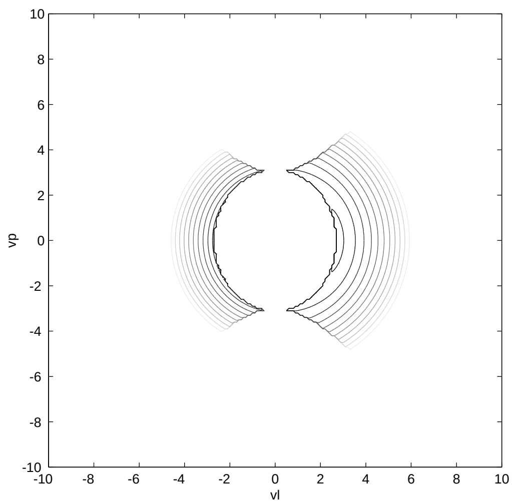

Figure 2. Downstream distribution (contour plot) formed with the overshoot taken into account, for the same shock parameters as those in Figure 1 and with the overshoot height $B_m/B_u \approx 7$.

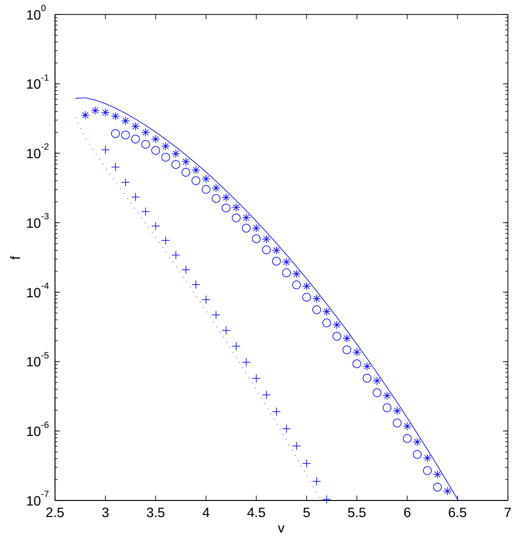

Figure 3. Cuts of the downstream distribution (no overshoot) at the angle φ with respect to the downstream magnetic field: $\varphi = 0°$ (along the magnetic field, solid line), $\varphi = 40°$ (stars), $\varphi = 80°$ (circles), $\varphi = 120°$ (crosses), and $\varphi = 160°$ (dotted line).

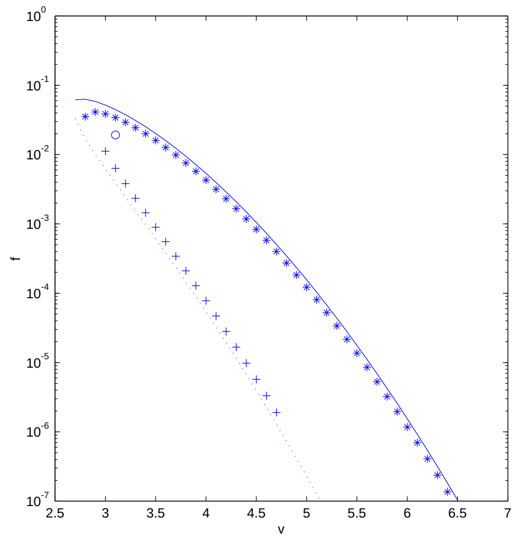

Figure 4. Cuts of the downstream distribution (overshoot present) at the angle φ with respect to the downstream magnetic field: $\varphi = 0°$ (solid line), $\varphi = 40°$ (stars), $\varphi = 80°$ (circles), $\varphi = 120°$ (crosses), and $\varphi = 160°$ (dotted line).

For definiteness, in what follows we restrict ourselves with the Maxwellian upstream distribution

$$f_u(v_{\perp,u}, v_{\parallel,u}) = (2\pi)^{-3/2} v_T^{-3} \cdot \exp\{-([(v_{\parallel,u} - V_u/\cos\theta)^2 + v_{\perp,u}^2]/2v_T^2\}, \tag{6}$$

where $V_u/\cos\theta$ is the bulk upstream electron flow velocity along the magnetic field in the de Hoffman-Teller frame, V_u being the incident plasma velocity in the normal incident frame and θ being the angle between the shock normal and upstream magnetic field.

In general, ϕ and B/B_u depend on the coordinate along the shock normal, so that the condition (4) becomes x dependent. The distribution of connected electrons is determined by the condition $Q(x) \geq 0$ in all points throughout the shock front. It is impossible to describe this distribution in the general case of arbitrary dependence $B(x)$ and $\phi(x)$. For more definiteness, in the present paper we shall adopt the assumption by *Hull et al.* [1998] that $\phi(x) = \phi_d(B(x)/B_u - 1)/(B_d/B_u - 1)$, where d refers to downstream. Let $B_m = \max(B)$ and $\phi_m = \phi_d(B_m/B_u - 1)/(B_d/B_u - 1)$. It is easy to show that in this case the accessibility criterion $Q(x) \geq 0$ can be rewritten as the accessibility criteria in the upstream region and overshoot, as follows:

$$Q = v_\perp^2(1 - B_u/B_d) + v_\parallel^2 - \frac{2e\phi_d}{m_e} \geq 0, \tag{7}$$

$$Q_m = v_\perp^2(1 - B_m/B_d) + v_\parallel^2 - \frac{2e\phi_d}{m_e}\left(\frac{B_d - B_m}{B_d - B_u}\right) \geq 0. \tag{8}$$

In weak shocks the overshoot is negligible and $B_m = B_d$, so that (8) is satisfied automatically. However, any noticeable overshoot would alter the downstream distribution relative to what is expected only from (7).

3. Visualization

We visualize the expressions derived in section 2 on the example of a high Mach number shock described by *Scudder et al.* [1986a, b, c] with the following parameters: Alfvenic Mach number $M = 7.7$, angle between the shock normal and upstream magnetic field $\theta = 76°$, downstream-to-upstream magnetic field ratio $B_d/B_u = 3$, and electron $\beta_e = 1.6$. The overshoot height (maximum compression ratio) $B_m/B_u \approx 7$. The normalized de Hoffman-Teller cross-shock potential is estimated as $e\phi_d/T \approx 3.5$. As we have seen in section 2 the distribution of the potential in the shock front is crucial for the determination of the downstream distribution. Here we shall follow the suggestion of *Hull et al.* [1998] that $\phi \propto (B/B_u - 1)$ throughout the shock, which means that in the overshoot $\phi_m = \phi_d(B_m - B_u)/(B_d - B_u)$.

Figure 1 shows the downstream electron distribution which would be formed in this shock if the overshoot were absent (upstream distribution assumed Maxwellian). It should be compared with Figure 2b by *Hull et al.* [1998], where the discontinuity where $v_\parallel = 0$ is smoothed. Note that we show only those parts of the downstream distribution which can be Liouville mapped to the upstream electron distribution. The inner part of the distribution shown in Figure 2b by *Hull et al.* is filled rather arbitrarily, and parameters of those electrons are not directly related to the shock parameters nor to collisionless electron dynamics in the shock front.

In Figure 2 the overshoot presence is taken into account. The distortion of the distribution for large $v_\perp$ in the vicinity of $v_\parallel = 0$ is clearly seen. It appears because the overshoot is a more efficient magnetic mirror than the downstream field is, and it prevents more electrons from crossing the shock than that which would cross the shock in the absence of overshoot. A similar phenomenon (lack of electrons with low $v_\parallel$ and high $v_\perp$ in the downstream distributions) has been observed by *Veltri et al.* [1990] in numerical Monte Carlo simulations.

Corresponding cuts of the downstream distribution for different $\varphi = \arctan(v_\perp/v_\parallel)$ are shown in Figures 3 and 4 withoutand with the overshoot present. As expected, the most drastic changes are seen for the cuts nearly perpendicular to the magnetic field. Figure 4 shows that there are no high-energy electrons for the cut with $\varphi = 80°$, while in Figure 3 this cut is quite smooth and extends to high $v_\perp$ (compare with Figure 4b by *Hull et al.* [1998]).

Finally, Figure 5 shows the cuts which are measured by an ISEE type apparatus. The ISEE type detector has a shape of a fan (few degrees wide in azimuth) with an opening angle of 110° (±55° in elevation angle above and below the spacecraft equatorial plane). The detector covers the energy range from 50 eV to 20 keV per charge in 16 contiguous energy bins (corresponding to the relative width of $\Delta v/v \approx 18\%$) and provides 16 azimuth angle measurements during one satellite rotation (3 s). For the present model the spacecraft was assumed to be moving slowly in the shock front (its velocity neglected) along the shock normal and measuring electrons in the normal incidence frame. We chose the spacecraft rotation axis also along the shock normal direction (shock normal is along x, and the upstream magnetic field is in the $x - z$ plane). The angle $\varphi = 0°$ corresponds to measuring electrons in the $x - z$ plane with the velocities directed from upstream to downstream. The angle $\varphi = 90°$ corresponds to measuring electrons in the $x - y$ plane, which is almost perpendicular to the downstream magnetic field. In Figure 5 we take into account the averaging over energy and polar angle but not averaging over azimuthal angle. The azimuthal averaging was considered by *Hull et al.* [1998] and found to provide some smoothing which is not significant here (see Figure 2).

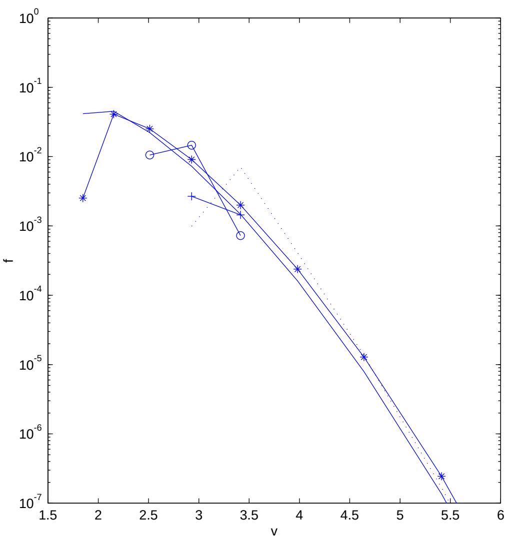

Figure 5. Cuts measured by an ISEE type apparatus. Markers are as those in Figure 4. The apparatus measurements are shown at a discrete number of energy levels (16; see description in the text) and connected with curves for better visualization, as is usual.

4. Discussion

In the present paper we critically reconsidered the formation of the downstream electron distribution within the stationary one-dimensional shock model. In doing so we followed the prevailing [*Feldman et al.*, 1982; *Goodrich and Scudder*, 1984; *Feldman*, 1985; *Thomsen et al.*, 1987; *Schwartz et al.*, 1988] view that the electron motion is adiabatic; that is, magnetic moment is conserved. Previous treatment of this system [*Hull et al.*, 1998] used direct Liouville mapping depending only on the upstream and downstream magnetic fields and overall cross-shock potential in the de Hoffman-Teller frame.

We have seen in section 3 that upstream-to-downstream Liouville mapping is not so straightforward and depends, in general, on the field distribution within the shock front. Presence of a substantial overshoot may result in a significant distortion of the downstream distribution relative to that which could be expected without overshoot (in the assumption that the electron motion is collisionless and adiabatic). In particular, electrons with high $v_\perp$ and low $v_\parallel$ are cut out from the distribution. Lack of these electrons in the downstream distribution was shown earlier by direct numerical Monte Carlo simulation of *Veltri et al.* [1990, 1992]. Such distributions are not observed, which means that either at least one of the assumptions about electron dynamics is incorrect or the potential distribution in the shock front is self adjusted so that (8) does not place any constraint in addition to (7). The latter possibility is doubtful, since it is difficult to reconcile hyperbolic condition of (8) with the elliptic condition (7). Moreover, such reconciliation would require very large de Hoffman-Teller potential drop at the ramp, which could well result in the breakdown of adiabaticity [*Gedalin et al.*, 1995a].

It was argued [*Scudder*, 1996] that only strictly stationary fields should be taken into account when Liouville mapping upstream and downstream distributions, while seemingly nonstationary fields (waves) should be removed from the consideration before the mapping is performed. Respectively, it was suggested [*Scudder et al.*, 1986b] that average (low resolution) magnetic field should be used in the electron dynamics analysis, as has been done by *Veltri et al.* [1992] and *Veltri and Zimbardo* [1993a, b]. This averaging removes high overshoots (magnetic compression of $B/B_u \approx 7$ in the high-resolution profile against $B/B_u \approx 4$ in the low-resolution profile) and any small-scale structure, which is supposed to be nonstationary. This usage of average low-resolution field does not seem to be correct for the following reasons. If the timescale of the magnetic field and electric field variations is substantially larger than the electron transit time, these fields act essentially as quasi-stationary fields. If, on the other hand, these fields vary quickly on the typical electron transit time, the electron motion can hardly be expected to be adiabatic. Thus, when the electron motion is assumed adiabatic, one must consider the whole shock profile as quasi-stationary; otherwise, adiabaticity is not well grounded.

To conclude, our present analysis has shown that the inhomogeneous structure of the shock front (in particular, overshoot) results in very peculiar downstream distributions of the electrons, collisionlessly and adiabatically crossing the shock. We have found the features of the downstream distribution as a function of the downstream and overshoot magnetic fields and the cross-shock potential. We have also calculated the cuts through downstream distribution as they should be measured by the ISEE type apparatus. Such distributions should be clearly seen just behind the shock transition layer, both in observations and in simulations, provided the other factors, possibly affecting the distribution formation, are weak. Absence of observations of the above described features in a number of cases, together with the finding that simple Liouville mapping is inconsistent with the observed suprathermal electron fluxes downstream of the Earth's bow shock [*Gosling et al.*, 1989], probably means that the electron collisionless dynamics in the shock front should be reconsidered, taking into account strong inhomogeneity of the field and possible nonstationarity (and deviations from one-dimensionality), which could result in altering the relation between de Hoffman-Teller and normal incidence frames [*Gedalin et al.*, 1997]. One such scenario (whistler instability and pitch angle diffusion) has been suggested by *Veltri and Zimbardo* [1993a, b]. Yet another possibility is weak nonstationarity of the shock front, owing to which the downstream electron distribution could be a superposition of distributions formed locally with different cross-shock potentials. Respectively, estimates of the cross-shock potential from the electron heating should be accepted with caution, since their precision is unclear.

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