2D Relativistic MHD simulations of the Kruskal–Schwarzschild instability in a relativistic striped wind

(Ramandeep Gill, Jonathan Granot, Yuri Lyubarsky, 2018, MNRAS, 474, 3535)

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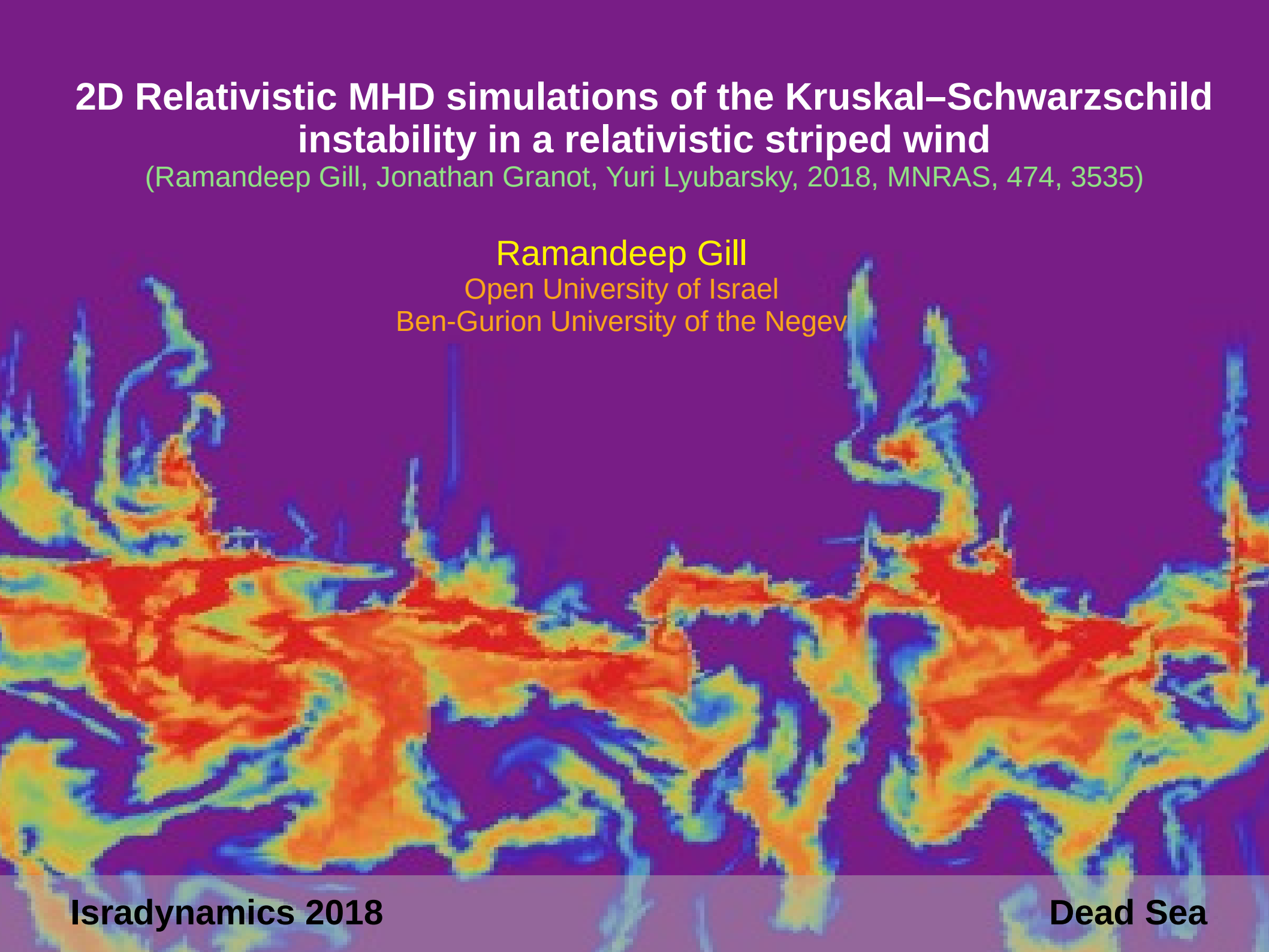
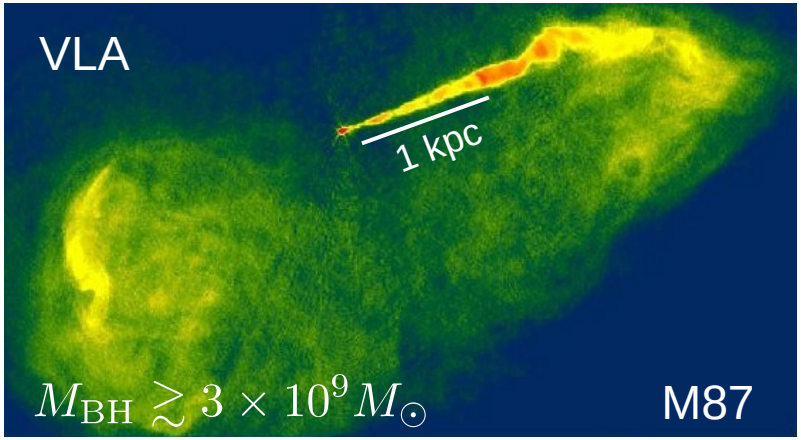


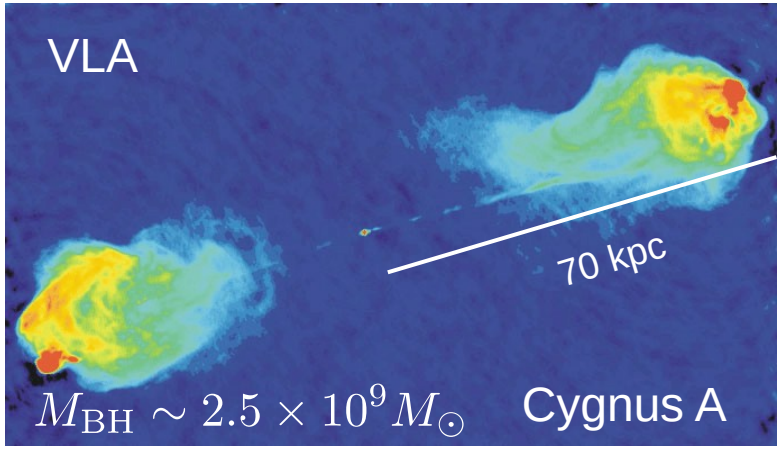
Isradynamics 2018 **Dead Sea**

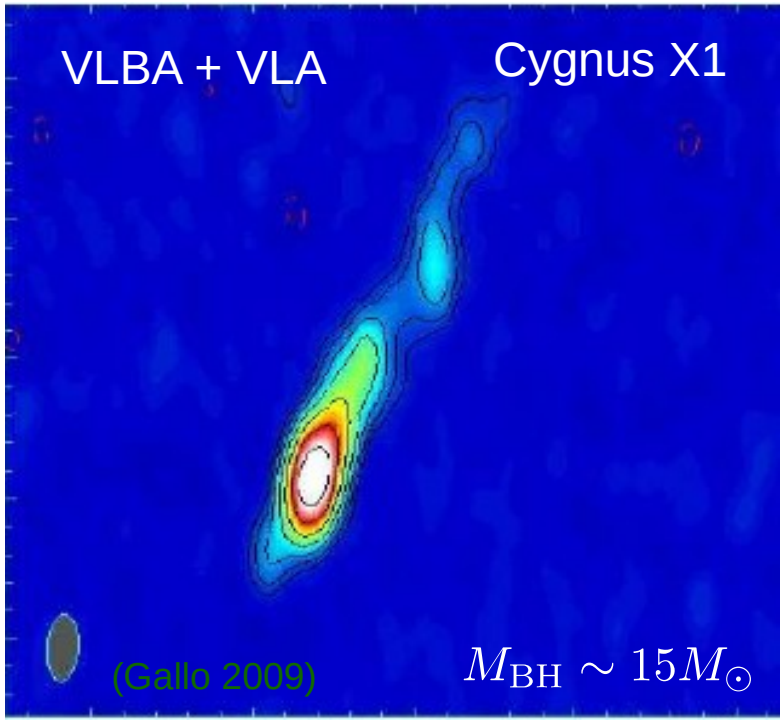
Outline of the Talk

- Relativistic jets in various sources (e.g. GRBs, AGNs, micro-quasars)
- Jet acceleration & energy dissipation mechanisms
- Kruskal-Schwarzschild Instability in a striped wind – analytic results
- Results from 2D Relativistic MHD simulations
- The role of buoyancy and the Kelvin-Helmholtz instability
- Conclusions

Jets Are Ubiquitous!







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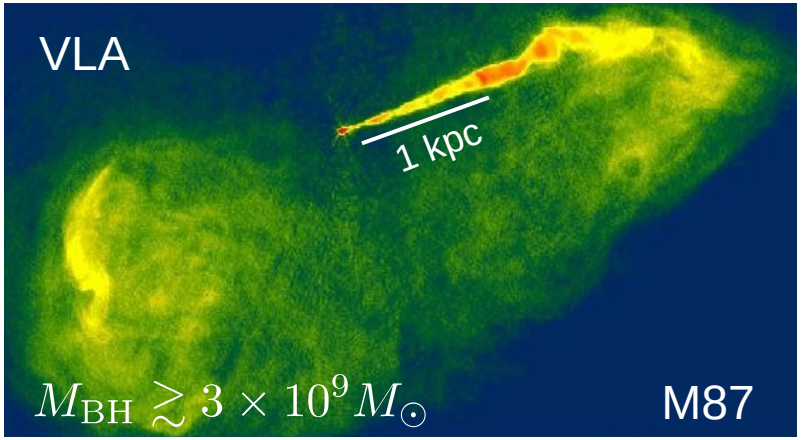
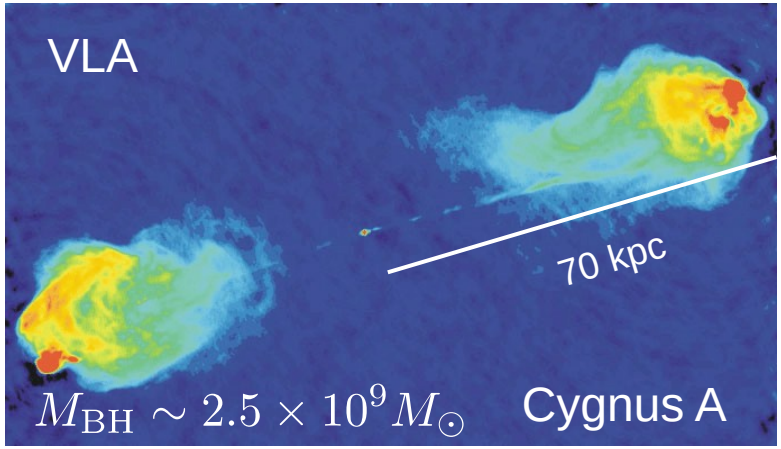
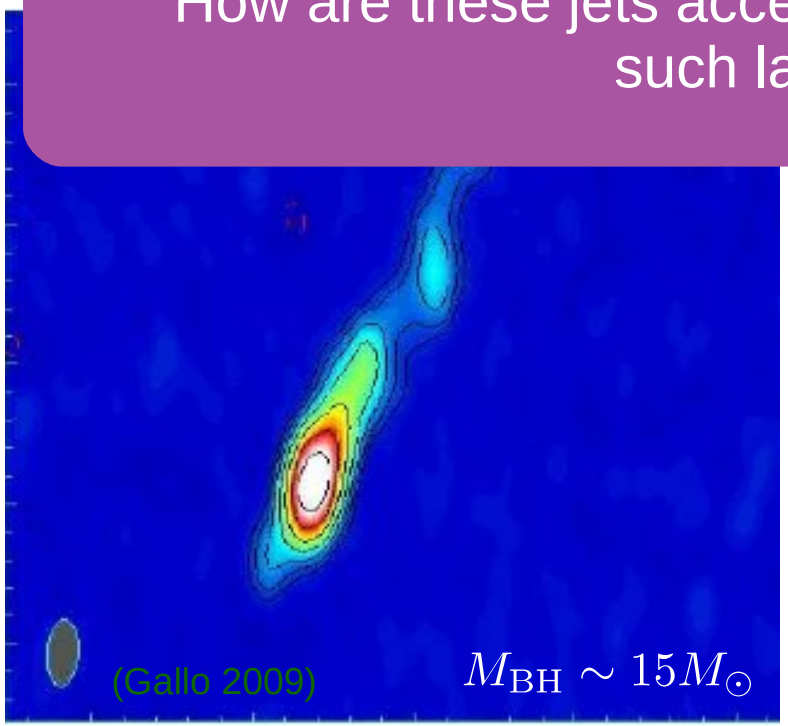




How are these jets accelerated to relativistic speeds and such large distances?



Jet Acceleration (in GRBs)

- There are two main options: Thermal acceleration and magnetic acceleration depending on the magnetization of the jet:

$$\sigma \equiv \frac{w_B}{w_m} = \frac{B'^2}{4\pi(\rho' c^2 + 4p')}$$

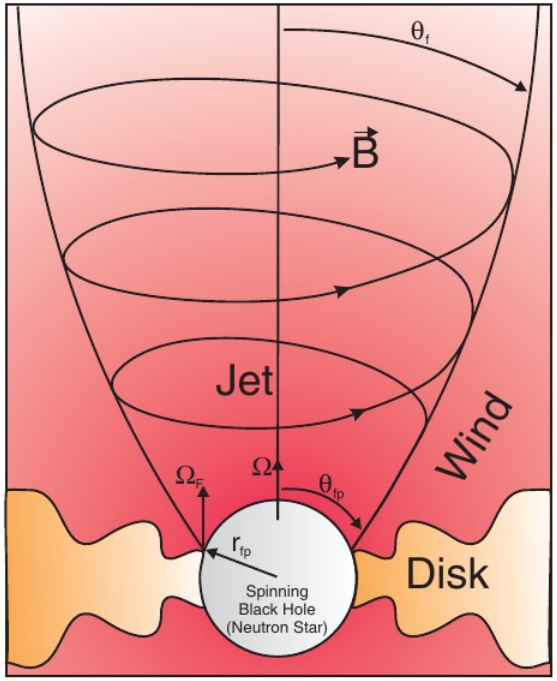
- Thermal acceleration $(\sigma < 1)$, due to neutrino anti-neutrino annihilation, might play some role in the acceleration of GRB jets, where the flow is optically thick $(\tau_T \gg 1)$ near the central engine. It cannot work for AGN jets.

- It is widely believed that magnetic acceleration $(\sigma > 1)$ might be the dominant mechanism, in which case a Poynting-flux dominated jet is launched.

Rotational energy of the compact source is tapped by magnetic field lines

⬇

Poynting-flux dominated flow



(McKinney & Uzdensky 2012)

Magnetic Energy Dissipation

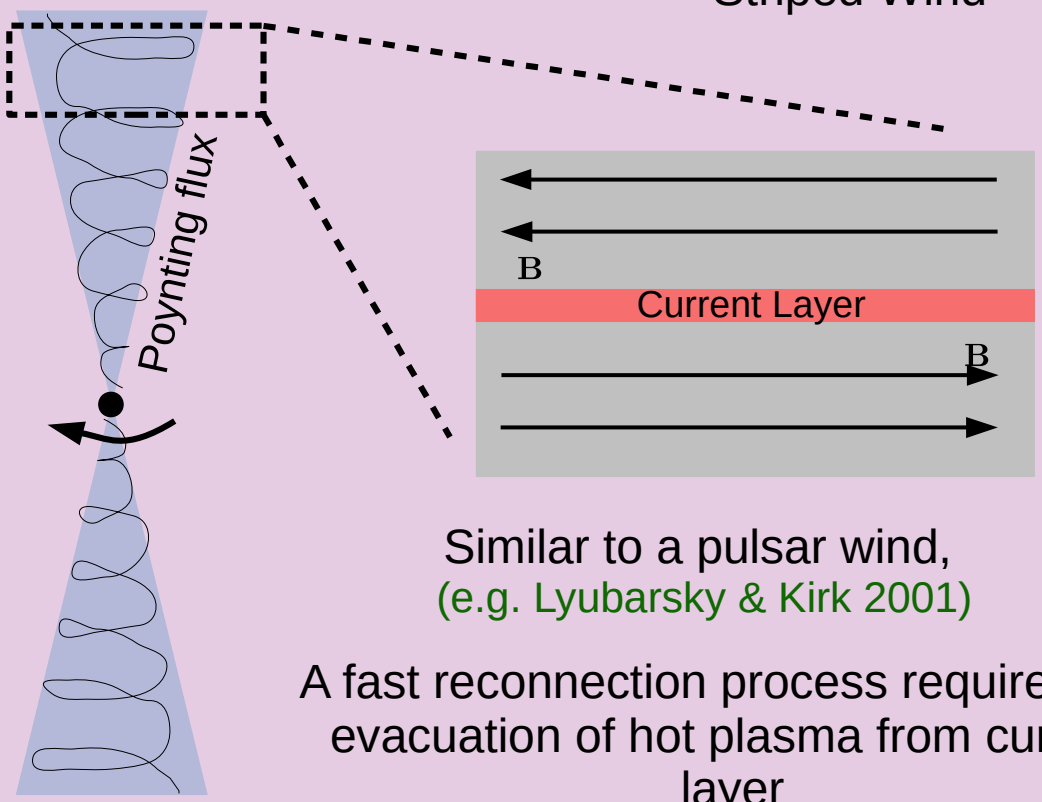
Magnetic dissipation in Poynting-flux dominated flows

→ Acceleration of the flow

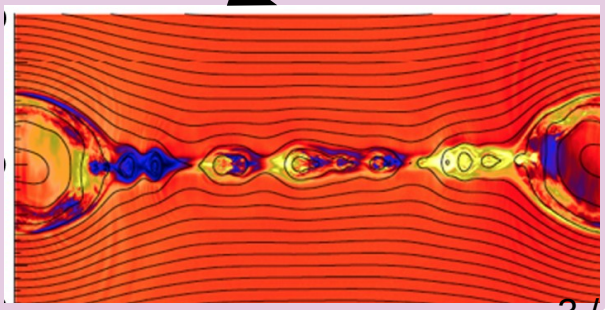
→ Non-thermal emission
(e.g. prompt GRB emission)

e.g. Romanova & Lovelace 1992, Lyubarsky 1992, Thompson 1994, Usov 1994, Lyubarsky & Kirk 2001, Lyutikov & Blackman 2001, Lyutikov 2006, Thompson 2006, Petri & Lyubarsky 2007,…,

Striped Wind



Reconnection!

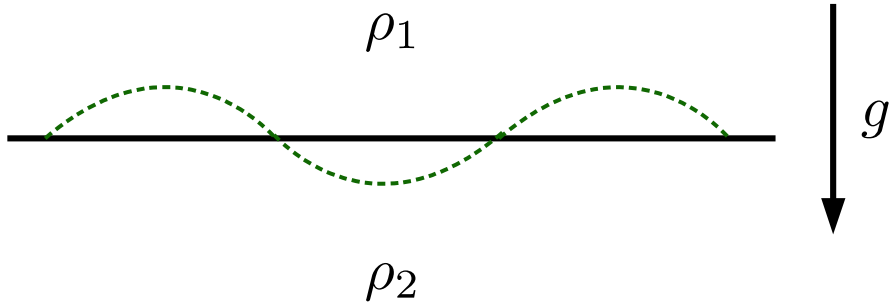


Similar to a pulsar wind,
(e.g. Lyubarsky & Kirk 2001)

A fast reconnection process requires fast evacuation of hot plasma from current layer

Digression: Rayleigh-Taylor Instability (RTI)

- RTI occurs at the interface of two fluids with different densities, where the heavier fluid rests on top of the lighter fluid: $\rho_1 > \rho_2$



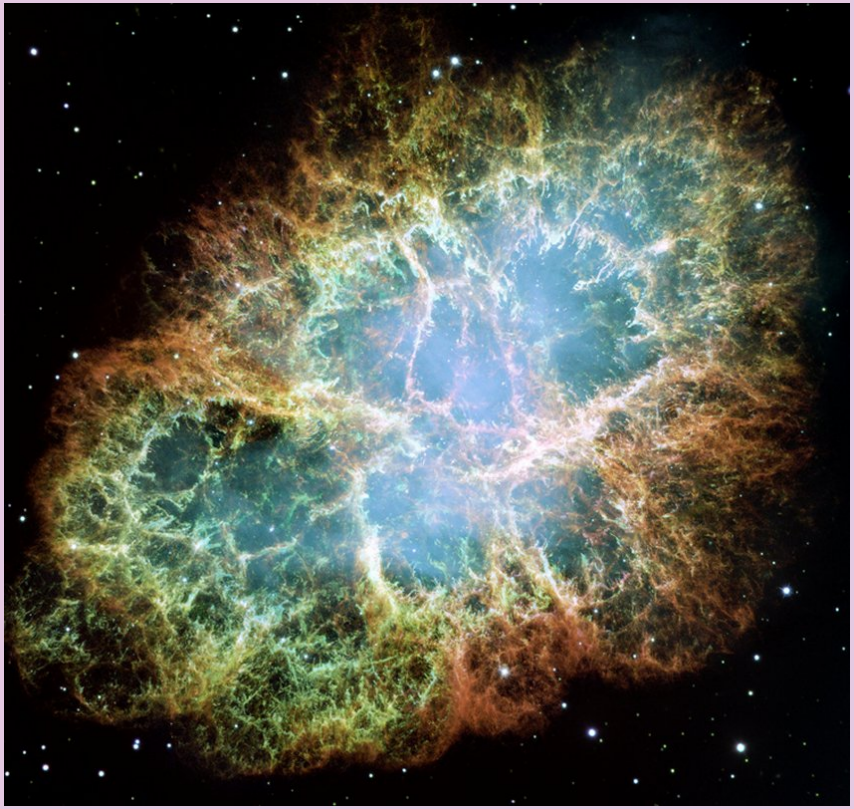
- This setup is always unstable to perturbations, which grow over time.

- Elementary linear stability analysis yields the dispersion relation (e.g. Chandrasekhar 1961):

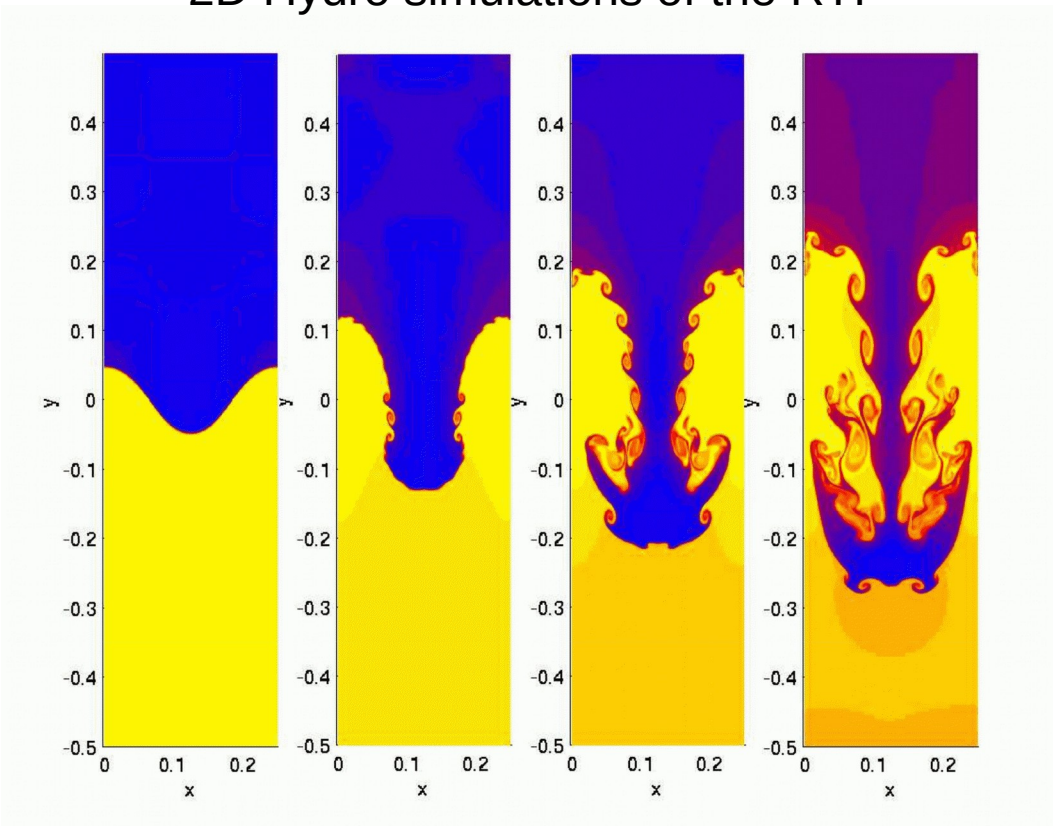
$$\omega^2 = \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) gk < 0 \qquad \text{for} \qquad \rho_1 > \rho_2$$

- g can also be the effective gravity that an accelerated system feels in the comoving frame.

Digression: Rayleigh-Taylor Instability (RTI)



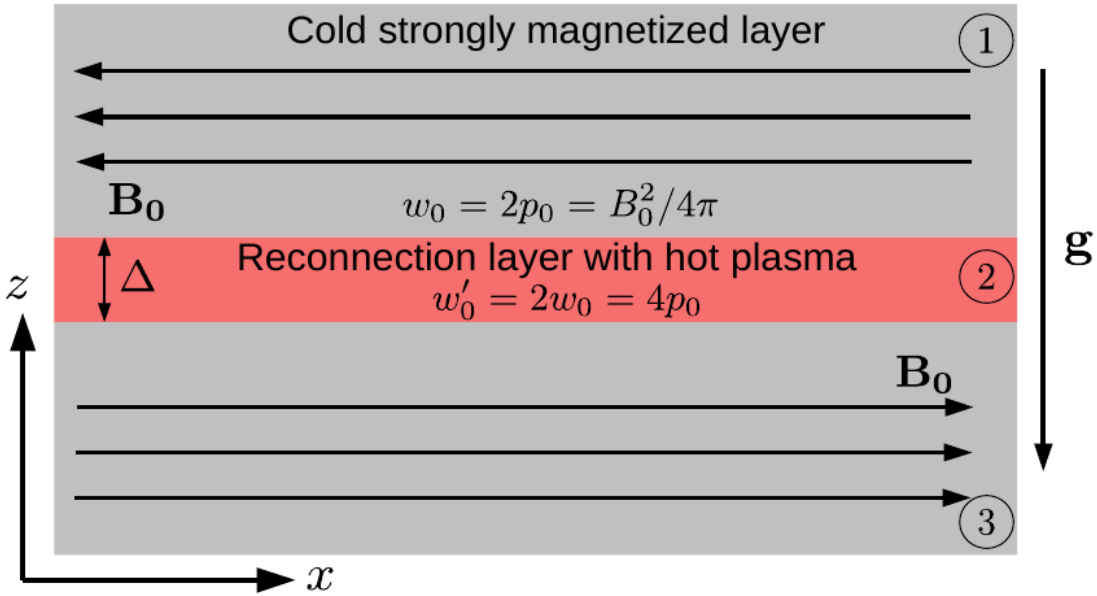
2D Hydro simulations of the RTI



(Li & Li, LANL)

- A bubble of relativistically hot plasma – a pulsar wind nebula – drives a shock wave, as it expands, into the much denser SN ejecta.
- The contact discontinuity between the two fluids is RTI unstable.

Kruskal-Schwarzschild Instability (KSI) in a Relativistic Fluid



- Consider two relativistic fluids with different enthalpy densities, where again the heavier fluid rests on top of the lighter fluid. The enthalpy density is given by

$$w = \rho c^2 + \frac{\hat{\gamma}}{\hat{\gamma} - 1} p$$

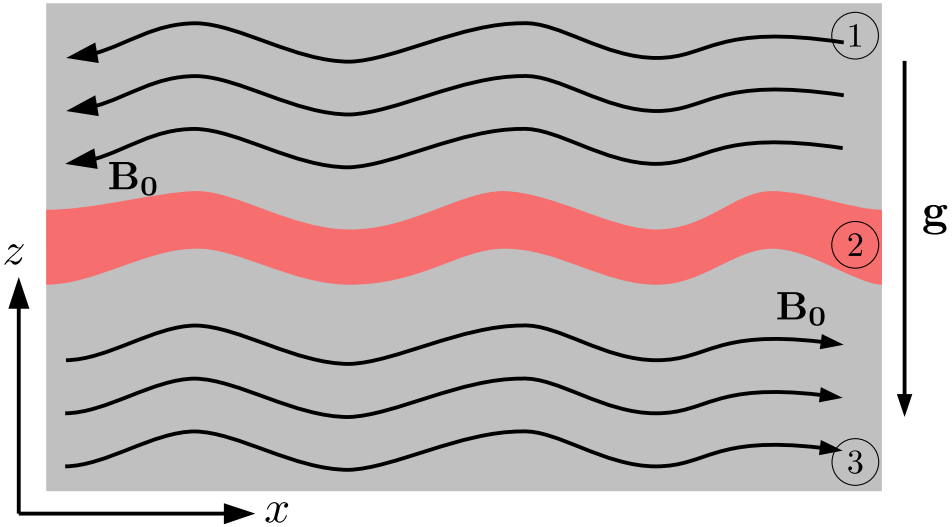
- In equilibrium, the fluid pressure is vertically stratified to counter-balance the force of gravity, such that

$$\partial_z p_0 = -\frac{w_0 g}{c^2} \qquad p_0(z) = p_0(0)\left[1 + \mathcal{O}\left(\frac{gz}{c^2}\right)\right]$$

- Pressure is homogeneous on length scales: $z \ll \frac{c^2}{g}$

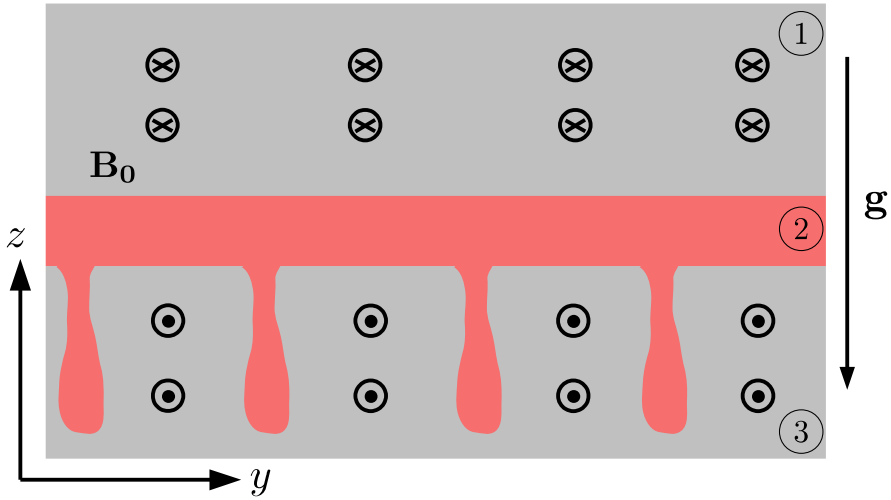
KSI in a Relativistic Fluid

- The perturbation can only grow orthogonal to the magnetic field lines.
- Due to tension of magnetic field lines, the growth of modes in the direction parallel to field lines is suppressed.



Mode parallel to the field lines

$$\hat{k} \parallel \hat{B}$$



Mode normal to the field lines

$$\hat{k} \perp \hat{B}$$

KSI in a Relativistic Fluid – Linear Stability Analysis

- The dynamical equations, in the ideal MHD limit, follow from conservations laws:

$$\partial_\mu(\rho u^\mu) = 0 \qquad\qquad \partial_\mu T^{\mu\nu} = 0$$

- The stress-energy tensor is a sum of the plasma and electromagnetic components:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{plasma}} + T^{\mu\nu}_{\text{em}} = \frac{w u^\mu u^\nu}{c^2} + p\eta^{\mu\nu} - b^\mu b^\nu$$

- In ideal MHD, the flux-freezing condition applies: $\dfrac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$

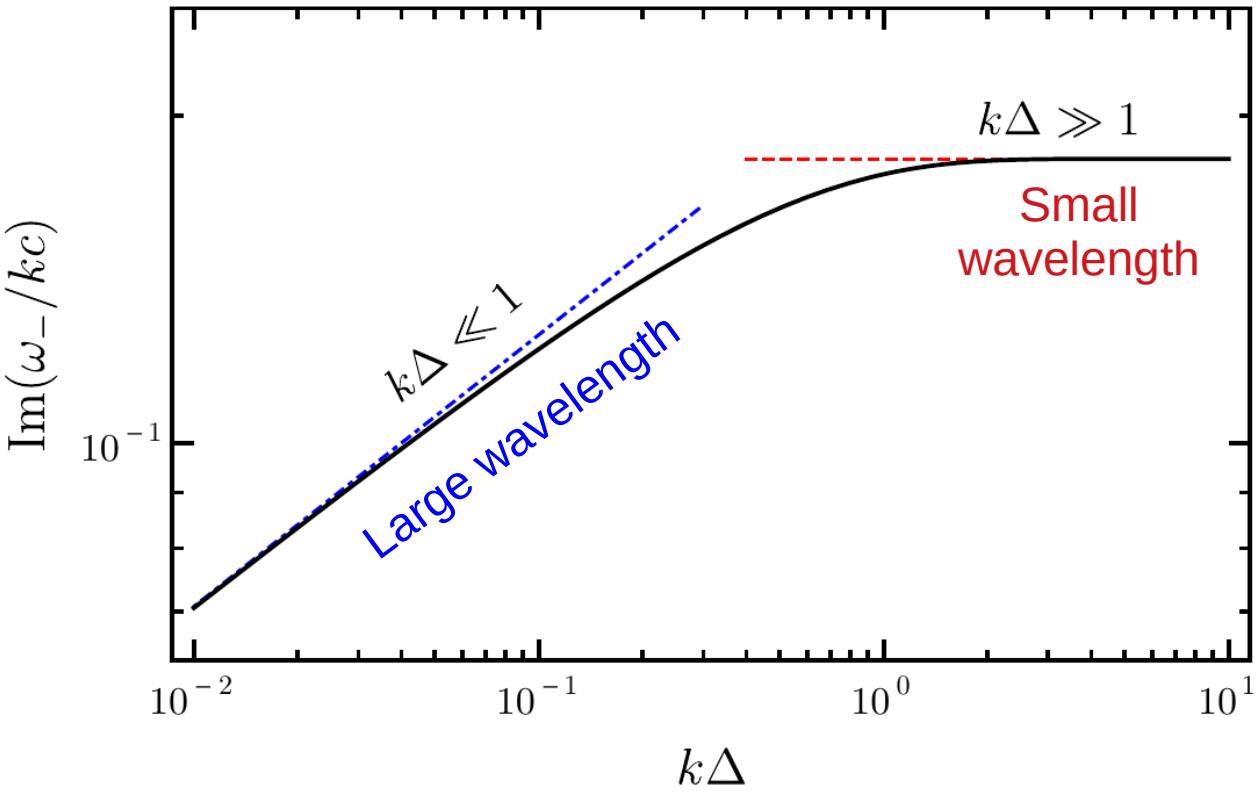
- Pressure and density are related to each other by the adiabatic condition:

$$\frac{d}{dt}\left(\frac{p}{\rho^{\hat{\gamma}}}\right) = 0$$

- By assuming harmonic perturbations, we find the dispersion relation (Lyubarsky 2010)

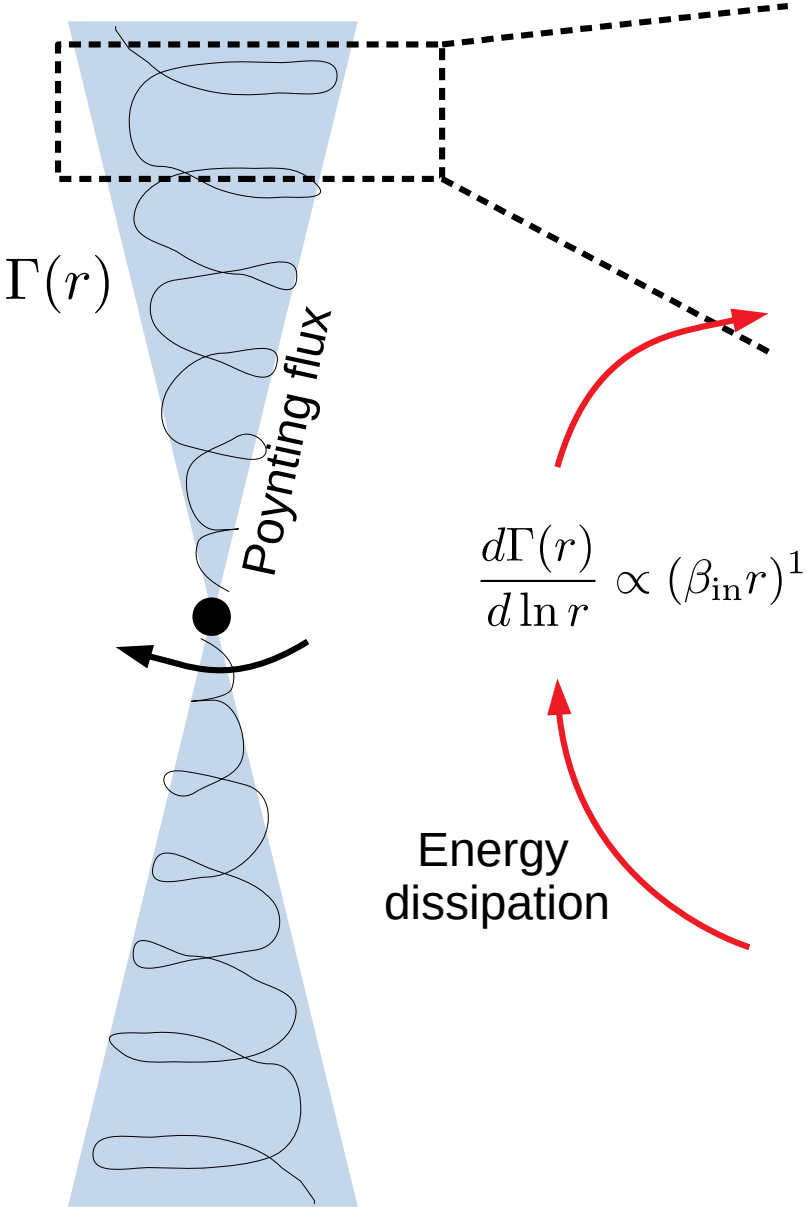
$$\omega_\pm^2 = \pm gk\left(5 + \frac{4}{\tanh(k\Delta)}\right)^{-1/2}$$

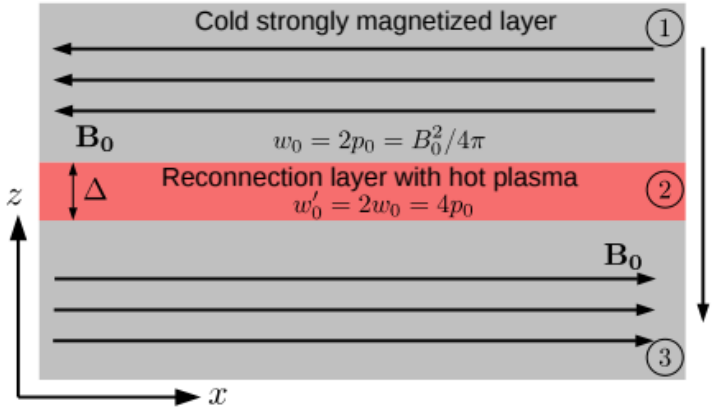
KSI in a Relativistic Fluid – Linear Stability Analysis



$$\eta = \begin{cases} \sqrt{\dfrac{gk}{3}}\,, & k\Delta \gg 1 \\ \left(\dfrac{g}{2}\right)^{1/2} k^{3/4}\Delta^{1/4}\,, & k\Delta \ll 1 \end{cases}$$

KSI in a Relativistic Striped Wind – Key Concept



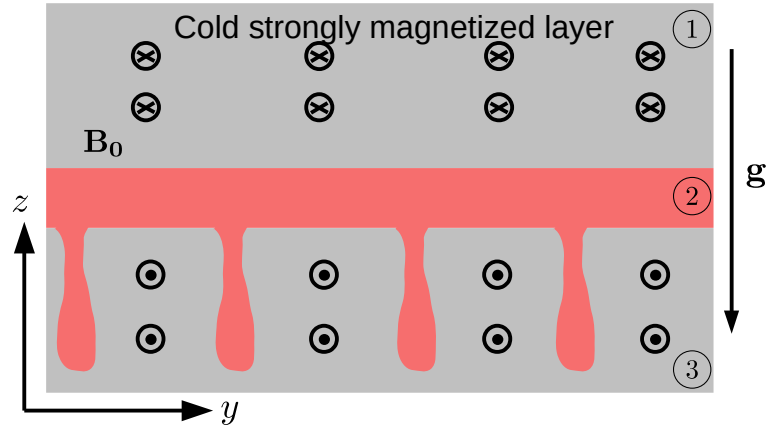


$$\mathbf{g} = -c^2 \frac{d\Gamma(r)}{dr}\hat{z}$$

$$\frac{d\Gamma(r)}{d\ln r} \propto (\beta_{\rm in} r)^{1/3}$$

KSI sets in and the hot plasma starts to drip out of the current layer

Energy dissipation

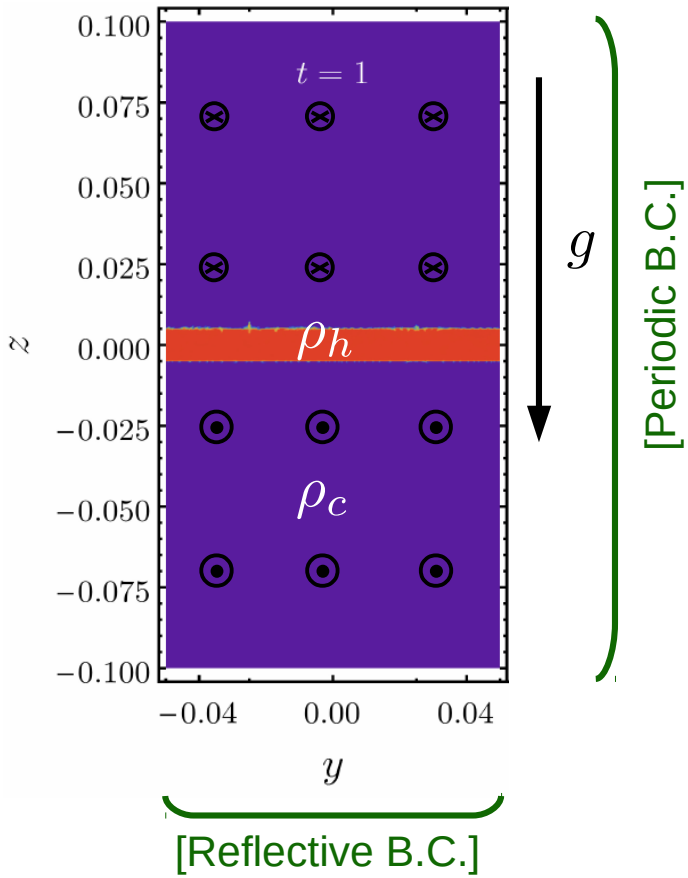


Reconnection rate:

$$\beta_{\rm in} = \frac{v_{\rm in}}{c}$$

Linear & Non-Linear Analysis with 2D Ideal-MHD Simulations in Athena

Large (Single) Wavelength Mode - Setup

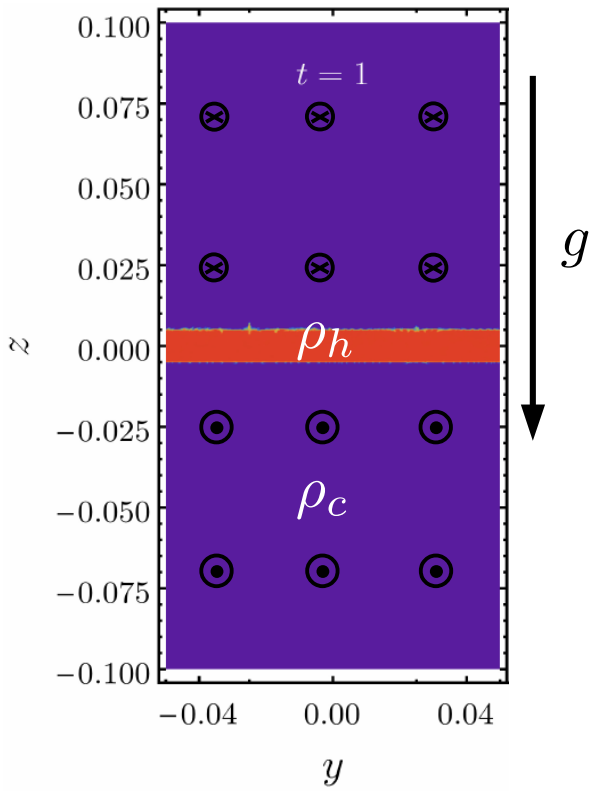


- Initial density contrast: $\rho_h = \psi\rho_c = \psi$
- Magnetization: $\sigma_0 \equiv \dfrac{w_{B,0}}{w_{m,0}} = \dfrac{b_0^2}{\rho_{0,c} + 4p_{0,c}} \xrightarrow[\text{cold}]{} \dfrac{b_0^2}{\rho_{0,c}}$

[In Lorentz-Heaviside units]
$c = 4\pi = 1$

- Homogeneous pressure: $p_0 = \sigma_0/2$ for $\rho_{0,c} = 1$

Large (Single) Wavelength Mode - Setup



- Characteristic velocity: $v_A = \sqrt{\dfrac{\sigma_0}{1+\sigma_0}}$
- Velocity perturbation:

$$v_{1z}(y,z) = \frac{\kappa v_A}{2}\left[\sin\left(\frac{2\pi m_0 y}{L_y}\right)\right]\left[1+\cos\left(\frac{2\pi z}{L_z}\right)\right]$$

Mode number

$$m_0 = \frac{L_y}{\lambda_0}$$

- Initial density contrast: $\rho_h = \psi\rho_c = \psi$
- Magnetization: $\sigma_0 \equiv \dfrac{w_{B,0}}{w_{m,0}} = \dfrac{b_0^2}{\rho_{0,c} + 4p_{0,c}} \xrightarrow[\text{cold}]{} \dfrac{b_0^2}{\rho_{0,c}}$

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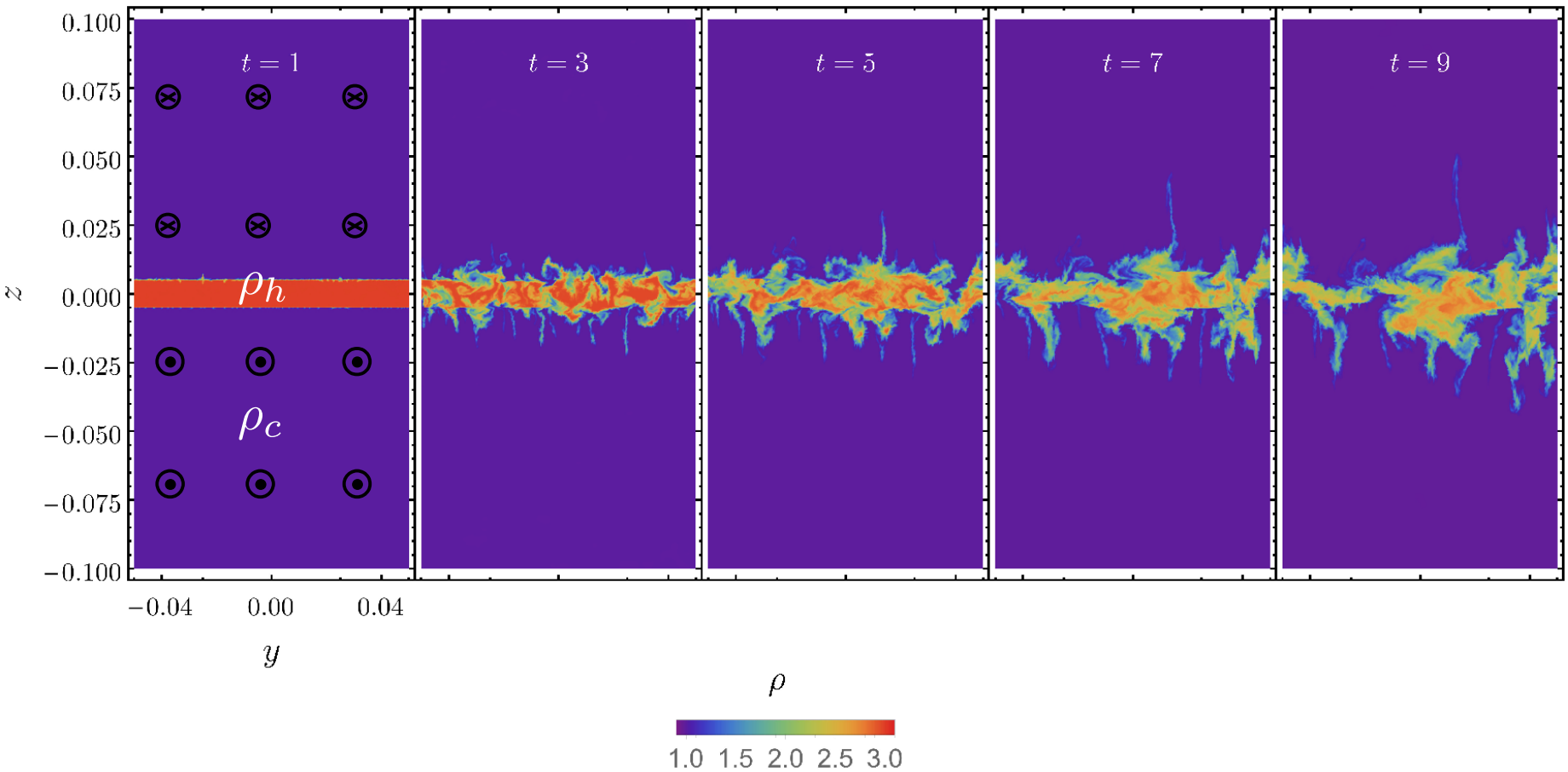
Large (Single) Wavelength Mode: $k\Delta \ll 1\ ,\ m_0 = 1$



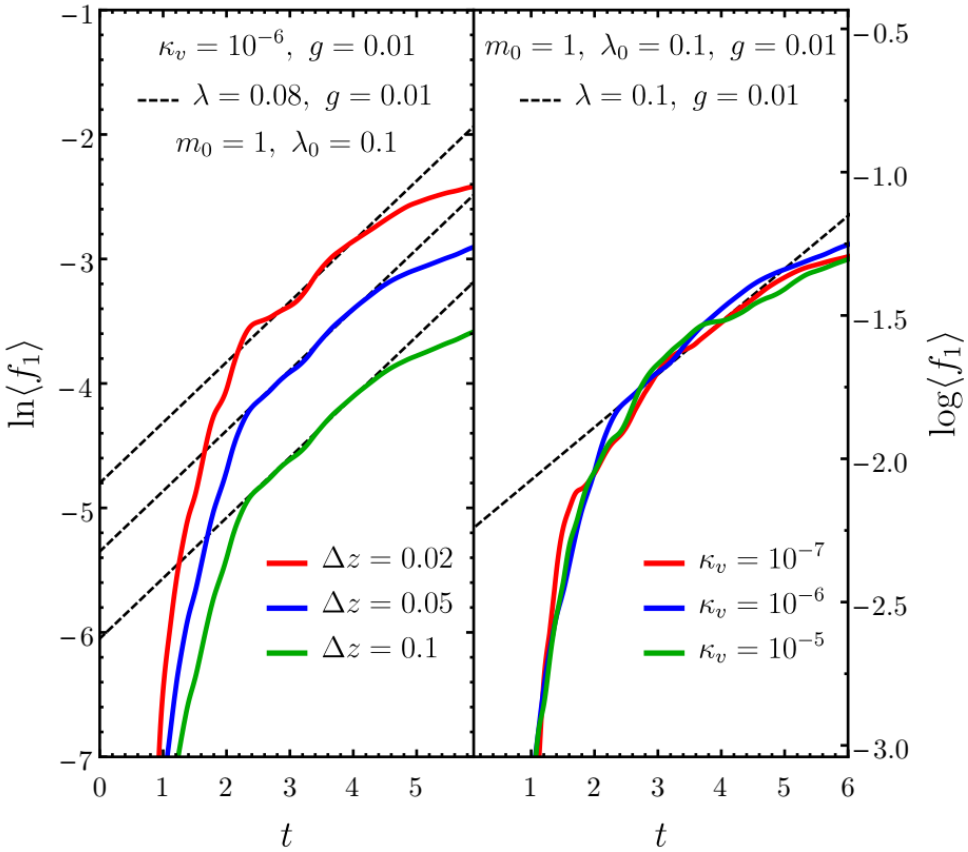
$$\kappa_v = 10^{-6}$$

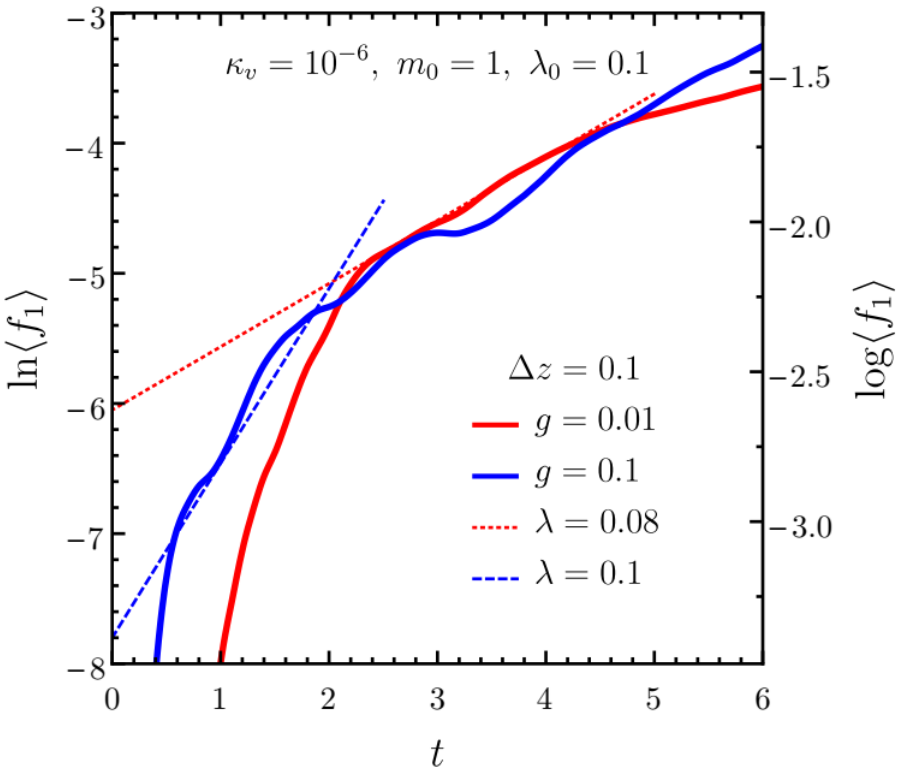
$$g = 10^{-2} \rightarrow z_{\text{crit}} = g^{-1} = 10^2$$

Large (Single) Wavelength Mode: $k\Delta \ll 1\ ,\ m_0 = 1$

Linear Growth Rate: Theory Vs Simulations

- There's good agreement between the linear theory and simulation in the linear stage.

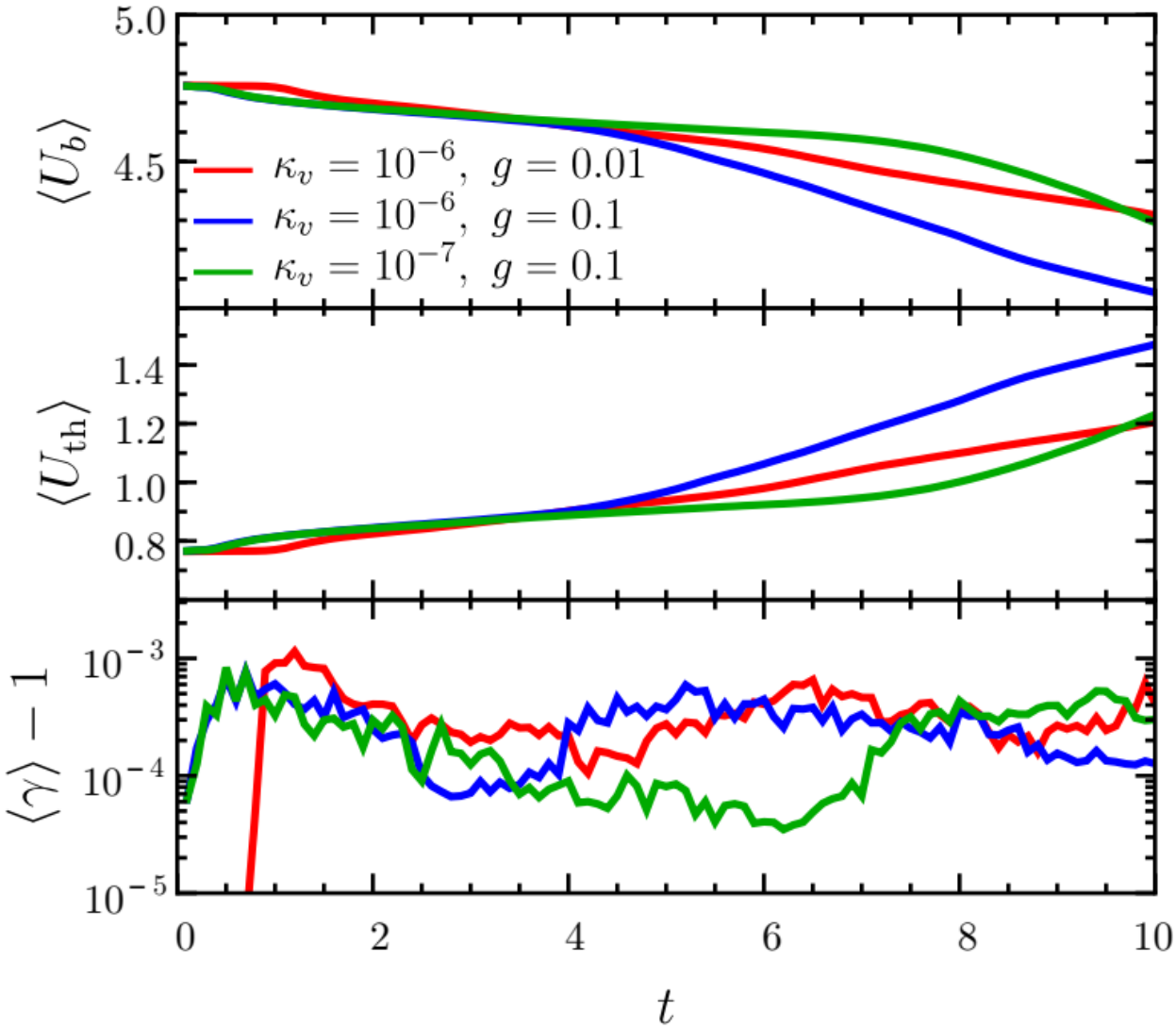




$\langle f_m \rangle$ = Spatially averaged Fourier amplitude of density perturbations for a given mode number m

Large (Single) Wavelength Mode: $k\Delta \ll 1\ ,\ m_0 = 1$

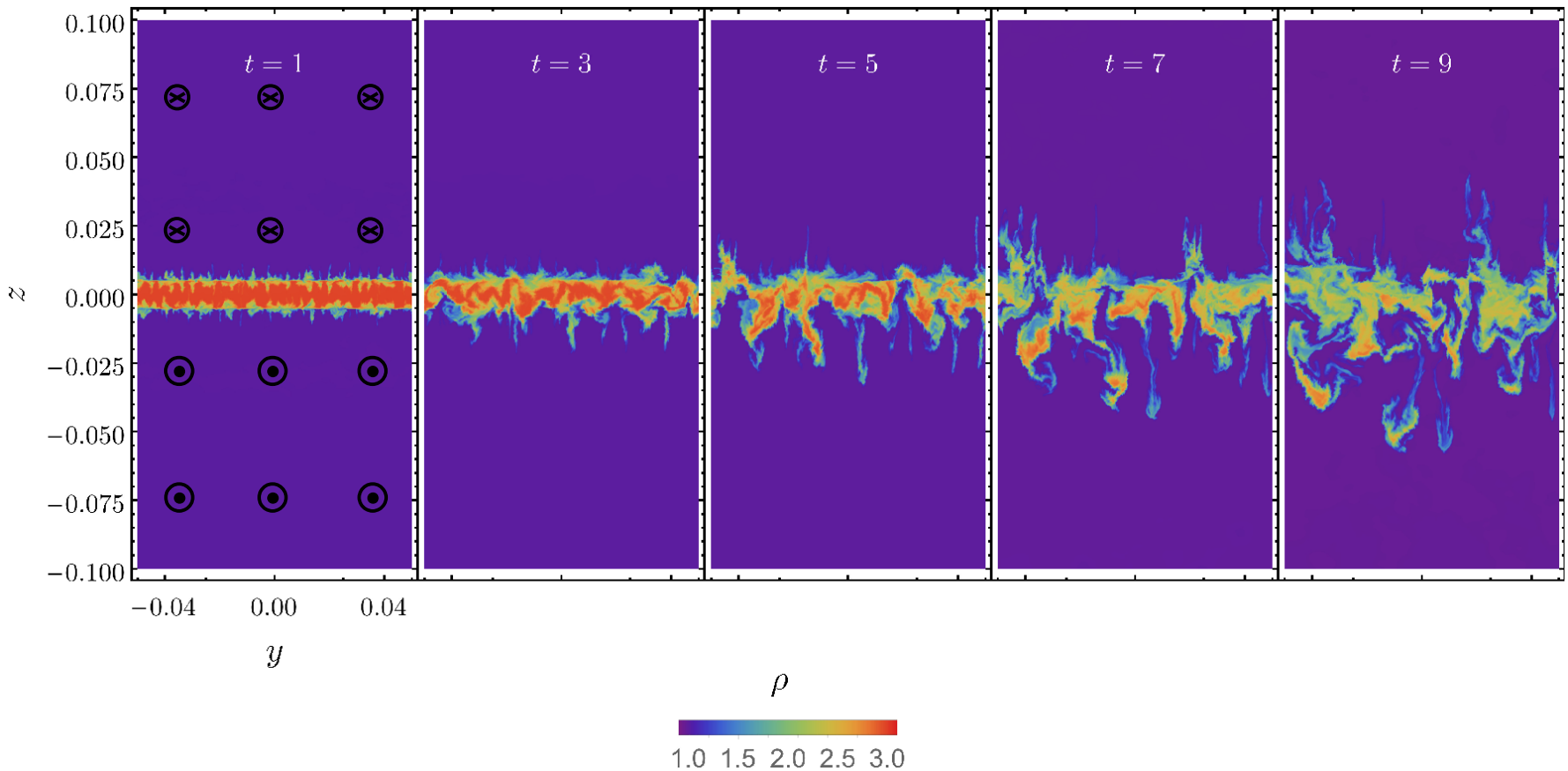
Magnetic field dissipation heats up the plasma



$k\Delta \approx 1\ ,\ m_0 > 1$

- Mode with wavelength comparable to the thickness of the current sheet is the most destructive.

$$k_v = 10^{-7}\ ,\ m_0 = 10\ ,\ g = 0.1$$



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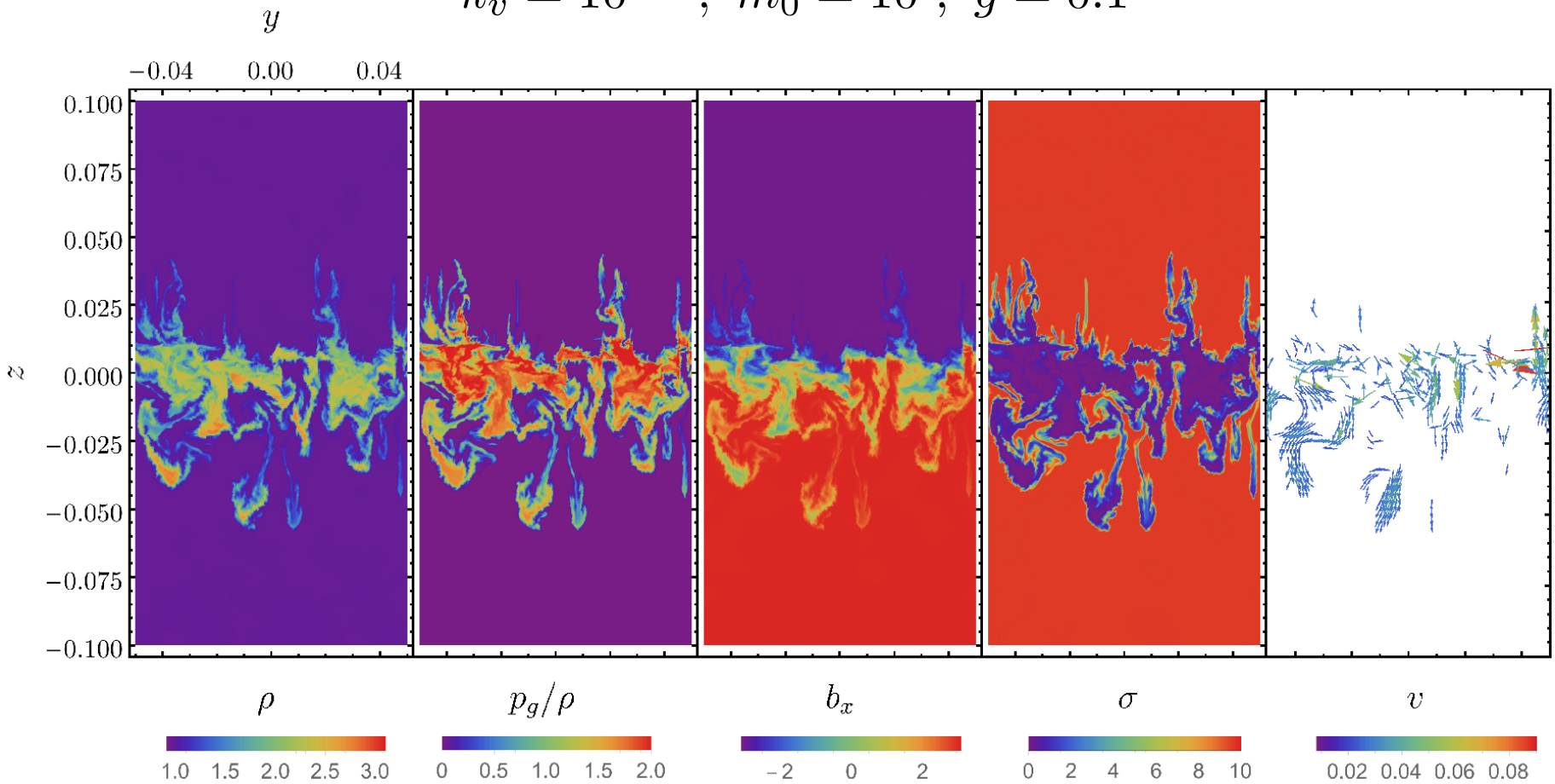
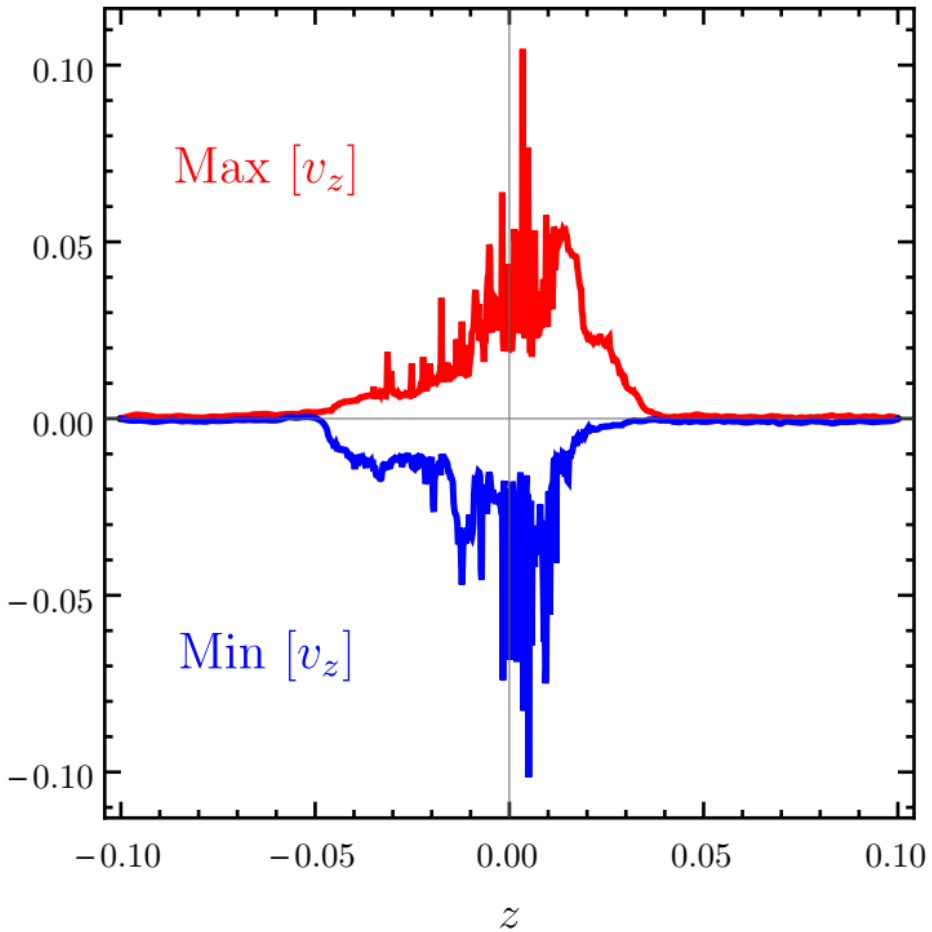


$k\Delta \approx 1\ ,\ m_0 > 1$: High Turbulent Velocity

Turbulent velocity of fluid elements in units of c

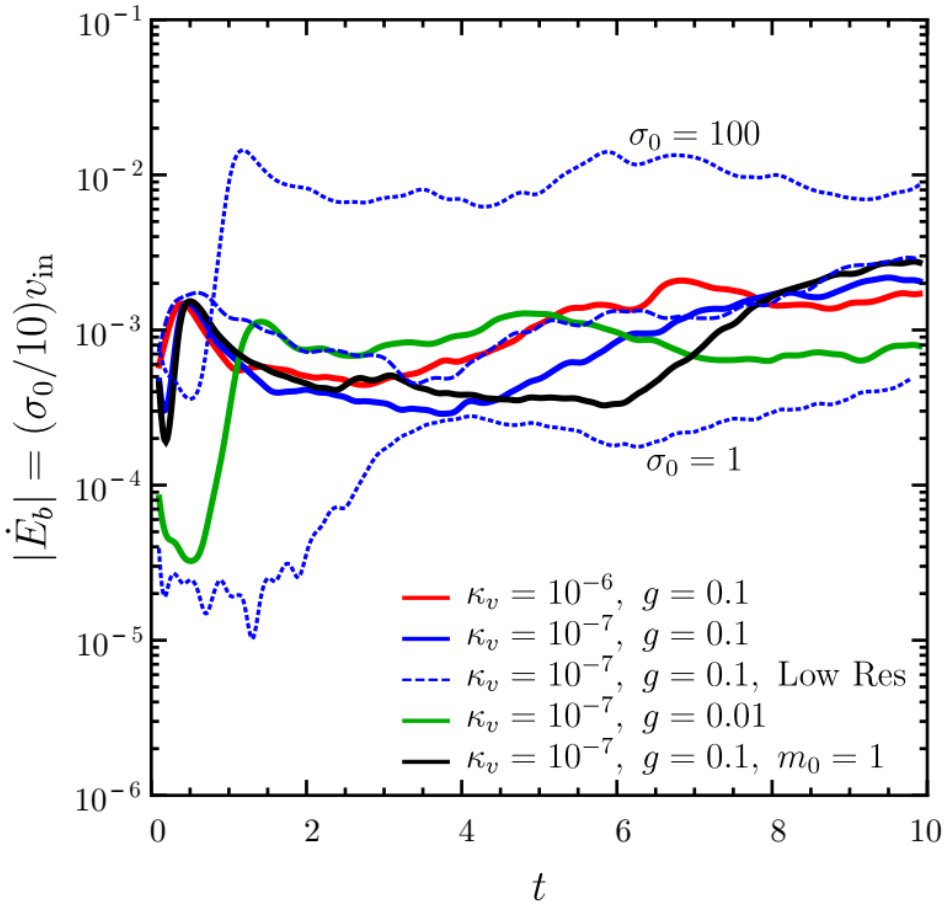


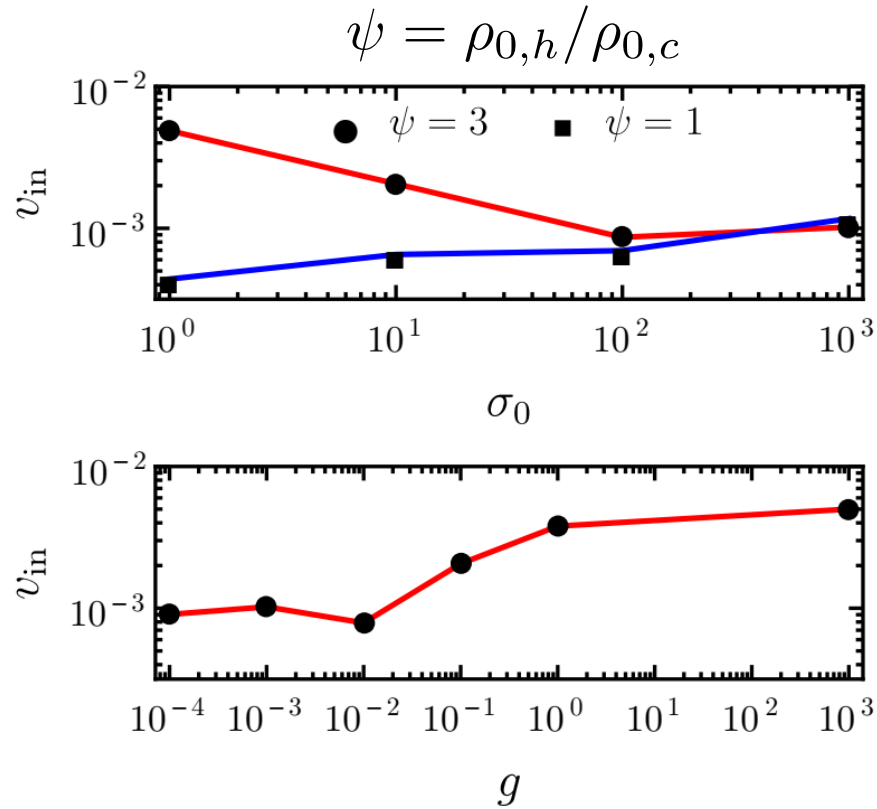
- The turbulent velocity peaks in the region (z = 0) where the fluid undergoes maximum mixing.
- Does it mean that the reconnection rate proceeds at 10% of c ? NO!

$k\Delta \approx 1\ ,\ m_0 > 1$: Rate of Reconnection

- A better measure of the reconnection rate is obtained from the rate at which magnetic energy is dissipated, such that

$$|\dot{E}_b| = 2 v_{\rm in} L_y \frac{b_0^2}{2} = \sigma_0 \rho_{0,c} L_y v_{\rm in} = \left(\frac{\sigma_0}{10}\right) v_{\rm in} \qquad (\rho_{0,c} = 1\ ,\ L_y = 0.1)$$



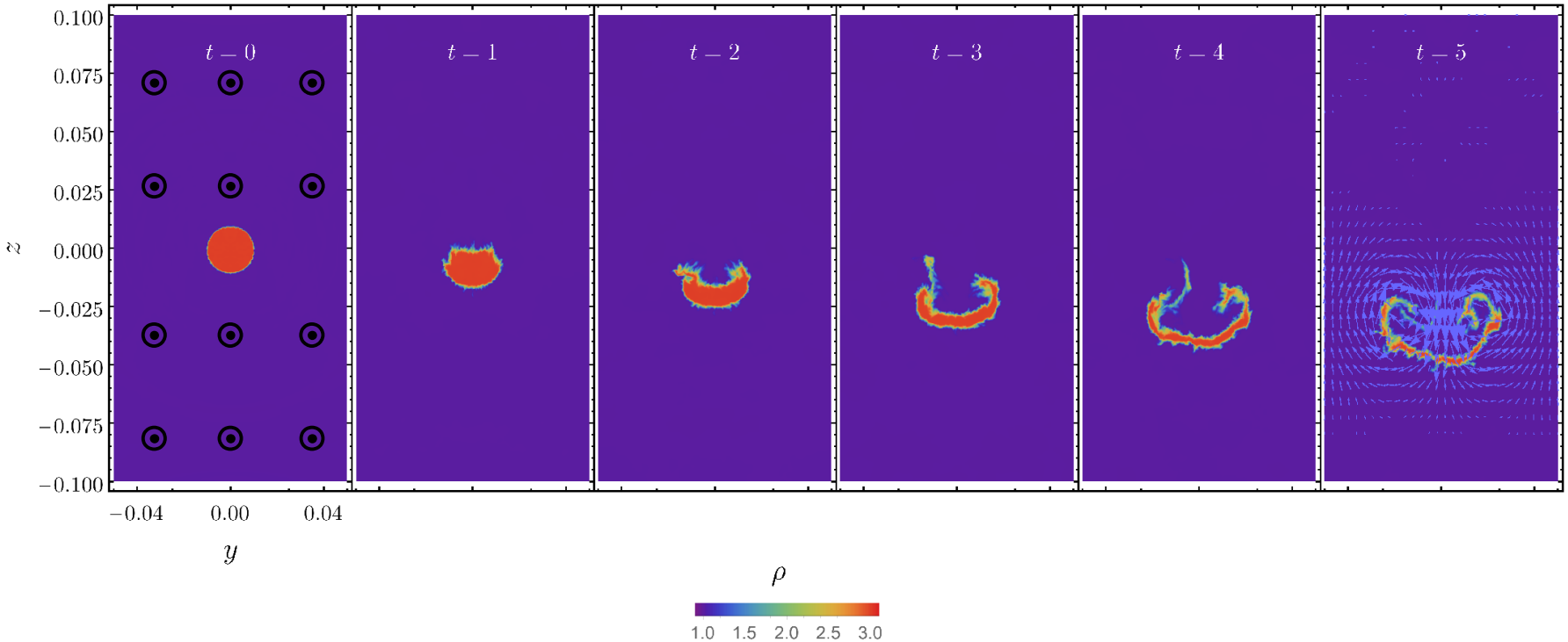


Reconnection rate is two orders of magnitude smaller than that achieved in resistive MHD simulations!!

Why's the Reconnection Rate So Slow?

The blob test

$$v_{\text{blob}} = 10^{-2} v_A \ , \ g = 0.1 \ , \ \sigma_0 = 10$$



1) The blob is buoyant: The buoyancy force acts in the direction opposite to gravity, slowing down its descent.

$$\mathbf{F}_b \approx -w_c V_b \mathbf{g} \longrightarrow g_{\text{eff}} = (w_h - w_c) g / w_h$$

$$\approx (w_{h,0} - w_{c,0}) g / w_{h,0}$$

$$= g/2$$

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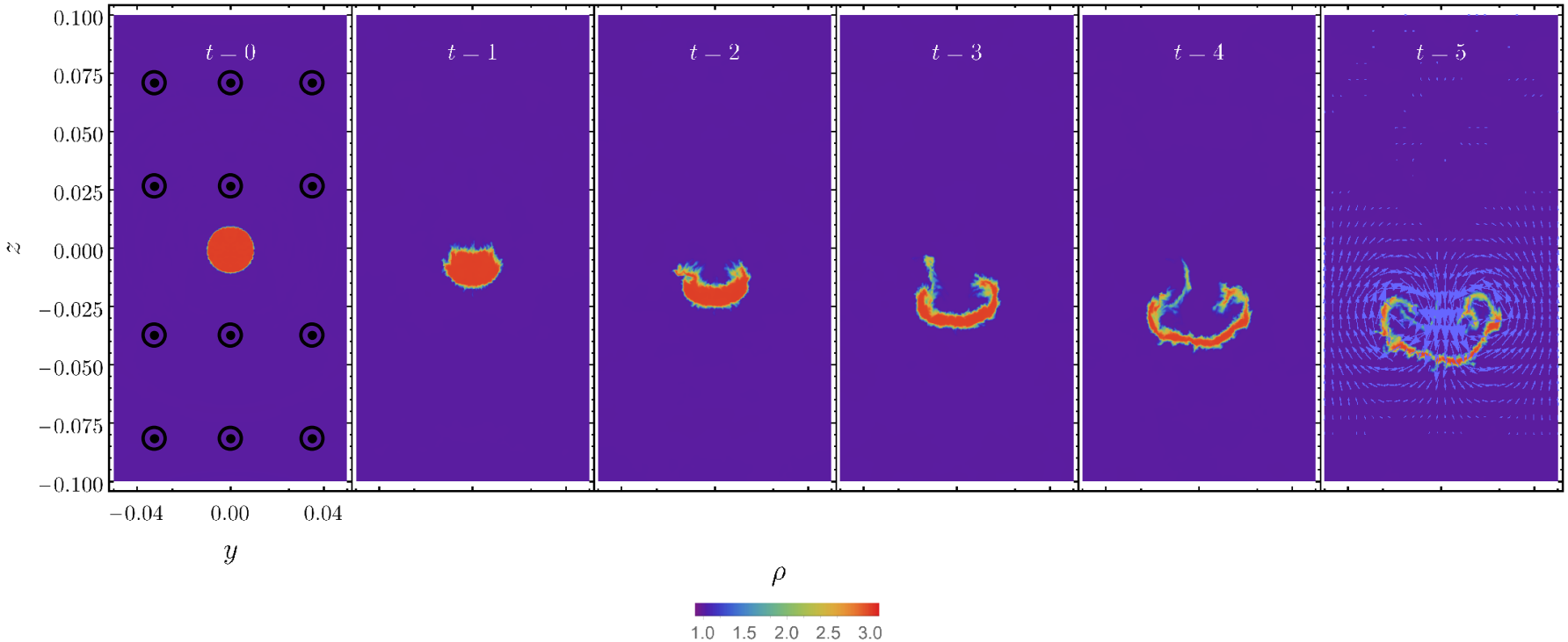

2) Kelvin-Helmholtz instability seeded by the RTI at the interface between the two fluids makes the ram-pressure flattened blob to curl upwards.

- The vortical motion prevents the blob from descending at a faster rate

Conclusions

- The KSI can potentially yield high reconnection rates in a striped wind outflow, which can lead to the bulk acceleration of the flow.
- 2D relativistic MHD simulation of the KSI show the same growth rate of the instability ast predicted by the linear theory (Lyubarsky 2010).
- In 2D, the effective reconnection rate is two orders of magnitude smaller than that achieved in resistive MHD simulations.
- Buoyancy force and Kelvin-Helmholtz instability play a critical role is slowing down the evacuation of hot plasma from the reconnection layer, thus impeding the rate.
- Resistive MHD simulations of the KSI might yield faster reconnection rates.

Thanks!