Some aspects of magnetic reconnection

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Outline

1. Collisionless reconnection: electron inertia versus non-scalar pressure tensor
2. Energetics of forced magnetic reconnection
3. Formation of plasmoids in the resistive MHD reconnection models

Role of a non-scalar electron thermal pressure tensor

the simplest case: electron MHD (EMHD) → immobile heavy ions

Basic equations

$$\frac{\partial \underline{B}}{\partial t} = -c(\underline{\nabla} \times \underline{E})$$

+ Generalized Ohm's law

$$E_\alpha = -\frac{1}{c}(\underline{V} \times \underline{B}) + \underbrace{\frac{1}{ne}\frac{\partial P_{\alpha\beta}}{\partial x_\beta} + \frac{m}{e}\frac{d\underline{V}}{dt}}_{\text{"reconnective" terms}} + \cancel{\eta \underline{j}}$$

collisionless

$$\underline{j} = -ne\underline{V} = \frac{c}{4\pi}(\underline{\nabla} \times \underline{B})$$

$$\underline{V} = \frac{1}{n}\int f \underline{v}\, d^3v$$

$$P_{\alpha\beta} = m\int w_\alpha w_\beta f\, d^3v$$

$$w_\alpha = v_\alpha - V_\alpha$$

Evolution eq. for $P_{\alpha\beta}$ follows from electron kinetic eq.

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_\alpha}(v_\alpha f) + \frac{\partial}{\partial v_\alpha}\left(\frac{F_\alpha}{m} f\right) = 0$$

$$\underline{F} = -e\left[\underline{E} + \frac{1}{c}(\underline{v} \times \underline{B})\right]$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \frac{\partial P_{\alpha\beta}}{\partial t} &= -\frac{\partial}{\partial x_\delta}(P_{\alpha\beta} V_\delta) - P_{\alpha\delta}\frac{\partial V_\beta}{\partial x_\delta} - P_{\beta\delta}\frac{\partial V_\alpha}{\partial x_\delta} \\ &- \frac{e}{m}\left(\varepsilon_{\alpha i k} B_k P_{i\beta} + \varepsilon_{\beta i k} B_k P_{i\alpha}\right) - \frac{\partial}{\partial x_\delta} Q_{\alpha\beta\delta} \end{aligned}}$$

$$Q_{\alpha\beta\gamma} = m \int w_\alpha w_\beta w_\gamma f \, d^3 v \Rightarrow \text{heat flux tensor}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \frac{\partial Q_{\alpha\beta\gamma}}{\partial t} &= -\frac{\partial}{\partial x_\delta}(Q_{\alpha\beta\gamma} V_\delta) \\ &- Q_{\alpha\beta\delta}\frac{\partial V_\gamma}{\partial x_\delta} - Q_{\alpha\gamma\delta}\frac{\partial V_\beta}{\partial x_\delta} - Q_{\beta\gamma\delta}\frac{\partial V_\alpha}{\partial x_\delta} \\ &+ \frac{1}{n m_e}\left(P_{\alpha\beta}\frac{\partial P_{\gamma\delta}}{\partial x_\delta} + P_{\alpha\gamma}\frac{\partial P_{\beta\delta}}{\partial x_\delta} + P_{\beta\gamma}\frac{\partial P_{\alpha\delta}}{\partial x_\delta}\right) \\ &- \frac{e}{m_e c} B_k \left(\varepsilon_{\alpha i k} Q_{i\beta\gamma} + \varepsilon_{\beta i k} Q_{i\alpha\gamma} + \varepsilon_{\gamma i k} Q_{i\alpha\beta}\right) \\ &- \frac{\partial}{\partial x_\delta} \Gamma_{\alpha\beta\gamma\delta} \end{aligned}}$$

Implications for a planar force-free magnetic equilibrium

$$B_{0y}^2(x) + B_{0z}^2(x) = const = B_0^2$$

Near the field reversal:

$$B_{0y}(x) \approx B_0 \alpha x \Rightarrow j_y = \frac{cB_0}{4\pi}\alpha^2 x \Rightarrow$$

non-gyrotropic pressure tensor $\leftarrow \frac{dV_{0y}}{dx} = \frac{cB_0\alpha^2}{4\pi n e} \neq 0$

$P_{yy}^{(0)} \neq P_{xx}^{(0)}$ – anisotropy in the plane perpendicular to the magnetic field $B_{0z} \approx B_0$

$$P_{yy}^{(0)} = P_{zz}^{(0)} = P_{xx}^{(0)}\left(1 + \alpha^2 \frac{c^2}{\omega_{pe}^2}\right)$$

Normally $\alpha \frac{c}{\omega_{pe}} \ll 1 \rightarrow$ magnetic shear length exceeds electron inertial skin-depth c/ω_{pe}

Note, that $Q_{\alpha\beta\gamma}^{(0)} = 0$

Tearing instability in such a field

pressure tensor linear perturbation $P^{(1)}_{\alpha\beta}$ inside the current sheet (neglecting first $Q_{\alpha\beta\gamma}$)

$$\gamma P^{(1)}_{\alpha\beta} = -P^{(0)}_{\alpha\rho}\frac{\partial V_\beta}{\partial x_\rho} - P^{(0)}_{\beta\rho}\frac{\partial V_\alpha}{\partial x_\rho} - \omega_B\left(\varepsilon_{\alpha iz}P^{(1)}_{i\beta} + \varepsilon_{\beta iz}P^{(1)}_{i\alpha}\right)$$

separation into $\underbrace{(P^{(1)}_{xz}, P^{(1)}_{yz})}$ and $\underbrace{(P^{(1)}_{xx}, P^{(1)}_{yy}, P^{(1)}_{xy})}$

contributing to (reconnection) $\partial\Psi_1/\partial t$

contributing to $\frac{\partial B^{(1)}_z}{\partial t}$ (advection of Ψ_1)

$$\frac{\partial\Psi_1}{\partial t} = -cE_z = -\frac{c}{ne}\left(\frac{\partial P^{(1)}_{xz}}{\partial x} + \frac{\partial P^{(1)}_{yz}}{\partial y}\right) + \dots$$

$$\frac{\partial B_{1z}}{\partial t} = c\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) = \frac{c}{ne}\left[\frac{\partial}{\partial y}\left(\frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y}\right) - \frac{\partial}{\partial x}\left(\frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{xy}}{\partial x}\right)\right] + \dots$$

$$\boxed{\begin{aligned} \Psi_1 &= \underbrace{d_e^2\Psi_1''}_{\text{inertial term}} + \underbrace{\beta d_e^4\Psi_1^{(IV)}}_{\text{due to } P^{(1)}_{\alpha\beta}} - \underbrace{\frac{ikd_e x}{(\gamma\tau_{He})}b}_{\text{Hall term}} \\ b &= d_e^2 b'' + \beta d_e^4 b^{(IV)} + \frac{ikd_e x}{(\gamma\tau_{He})}\Psi_1'' \end{aligned}}$$

$\beta \to 0$ limit $\Rightarrow P^{(1)}_{\alpha\beta} \to 0$

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"standard" EMHD inertial regime

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$$\boxed{\gamma \sim \tau_{Ae}^{-1} d_e^3, \quad (\Delta x) \sim l\, d_e^2} \qquad d_e = \frac{c}{\omega_{pe} l} \ll 1$$

$\Downarrow$

pressure tensor effect becomes significant for $\beta > d_e^2 \ll 1$

Does $Q_{\alpha\beta\gamma}$ play a role?

$$\gamma P^{(1)}_{xz} = -P_0 \frac{\partial V_z}{\partial x} + \omega_B P^{(1)}_{yz} - \frac{\partial}{\partial x} Q^{(1)}_{xxz}$$

$$\gamma P^{(1)}_{yz} = -P_0 \frac{\partial V_z}{\partial y} - \omega_B P^{(1)}_{xz} - \frac{\partial}{\partial y} Q^{(1)}_{yyz}$$

$\Downarrow$

contribution of $Q_{\alpha\beta\gamma}$ becomes important when $Q^{(1)}_{xxz}, Q^{(1)}_{yyz} > P_0 V_z$

Linearized eqs. for $Q^{(1)}_{\alpha\beta\gamma}$

$$\gamma Q^{(1)}_{\alpha\beta\gamma} = \frac{P_0}{n m_e}\left[\delta_{\alpha\beta} \frac{\partial P^{(1)}_{\gamma\kappa}}{\partial x_\kappa} + \delta_{\alpha\gamma} \frac{\partial P^{(1)}_{\beta\kappa}}{\partial x_\kappa} + \delta_{\beta\gamma} \frac{\partial P^{(1)}_{\alpha\kappa}}{\partial x_\kappa}\right]$$

"source" terms

$$- \omega_B \left(\varepsilon_{\alpha i z} Q^{(1)}_{i\beta\gamma} + \varepsilon_{\beta i z} Q^{(1)}_{i\alpha\gamma} + \varepsilon_{\gamma i z} Q^{(1)}_{i\alpha\beta}\right)$$

balanced by magnetic isotropization

$$Q^{(1)}_{xyz} = 0; \quad Q^{(1)}_{xxz} = Q^{(1)}_{yyz} = -\frac{P_0^2}{n m_e \omega_B^2} \frac{\partial^2 V_z^{(1)}}{\partial x^2}$$

significant for $\beta \gtrsim d_e^2$

electron Larmor radius $\rho_{Be} \sim (\Delta x)$

Conclusions

1) $\rho_{Be} \ll (\Delta x) \Rightarrow$
- fluid-type description works
- pressure-tensor effects are small (including electron gyroviscosity)
- reconnection is due to the bulk inertia of electrons

2) $\beta > d_e^2 \Rightarrow$ no truncation of fluid equations $\Rightarrow$ kinetic description is necessary

What to expect? numerical simulations point to $(\Delta x) \sim \rho_{Be} \sim \beta^{1/2} d_e$

Why?

$(\Delta x) \gg \rho_{Be} \Rightarrow P_{\perp\parallel}$ approach holds $\Rightarrow$ contradiction

$(\Delta x) \ll \rho_{Be} \Rightarrow$ guide field plays no role $\Rightarrow$ (Δx) is determined by "meandering" of electrons $\Rightarrow$

$(\Delta x) \sim (\rho_{Be} \cdot l)^{1/2} \gg \rho_{Be} \Rightarrow$ contradiction

Energetics of forced magnetic reconnection

Two types of reconnection:

i) spontaneous – via resistive MHD instability (tearing mode)

ii) forced – under external deformation of an MHD stable configuration

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it can act as a trigger for magnetic relaxation

Simple example: sheared force-free magnetic field

$$\underline{B}^{(0)} = \{0,\ B_0 \sin \alpha x,\ B_0 \cos \alpha x\},$$
$$-a \le x \le +a$$

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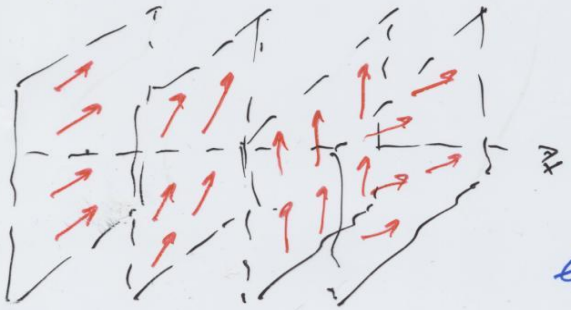

$$\underline{B}^{(0)} = (\nabla \Psi_0 \times \hat{z}) + B_z^{(0)} \hat{z}$$

$$\Psi_0(x) = \frac{B_0}{\alpha} \cos \alpha x;$$
$$B_z^{(0)} = \alpha \Psi_0$$

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linear force-free field

Tearing perturbation

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a new force-free equilibrium

$\psi(x,y) = \psi_0(x) + \psi_1(x)\cos ky$

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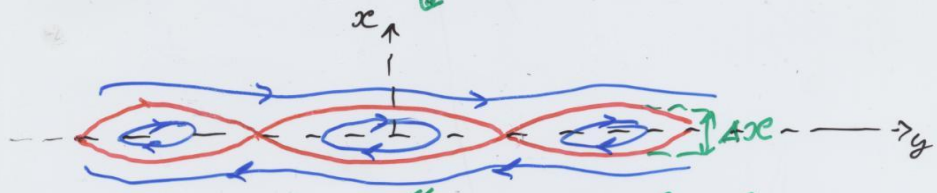

$\Delta x \sim [\psi_1(0)]^{1/2}$ – width of islands

$\psi_1'' + \varkappa^2\psi = 0$; $\varkappa^2 = \alpha^2 - k^2 > 0$

Stability parameter $\Delta' = -2\varkappa\cot(\varkappa a)$

$\Delta' > 0 \to$ instability for $\varkappa a > \frac{\pi}{2} \Rightarrow \boxed{\alpha > \frac{\pi}{2a}}$

Physical explanation:

magnetic energy is reduced if $\Delta' > 0$

$\boxed{\Delta W_M \propto -\Delta' \psi_1^2(0)}$

Tearing-stable field ($\alpha < \frac{\pi}{2a}$) still has electric current and, hence, possesses some free magnetic energy

Can it be released by reconnection?

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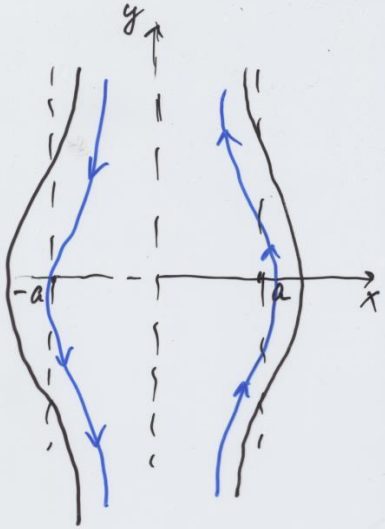
Yes, if reconnection is triggered externally

Forced magnetic reconnection

(Hahm and Kulsrud, 1985)



$x_b = \pm(a + \delta\cos ky)$

$\delta \ll a$ – linear treatment for a slightly deformed new equilibrium

$$\Psi(x,y) = \frac{B_0}{\alpha}\cos\alpha x + \Psi_1(x)\cos ky$$

Boundary condition:

$\Psi(x = x_b) = \Psi_0(a)$

$\Psi_1(a) = B_0\,\delta\sin(\alpha a)$

Force-free equilibrium condition:

$\Psi_1'' + \varkappa^2\Psi_1 = 0$

2 possible solutions with different magnetic topology

1) Regular solution: $\Psi_1^{(1)}(x) = B_0\,\delta\,\frac{\sin(\alpha a)}{\cos(\varkappa a)}\cos(\varkappa x)$

magnetic islands are present $\leftarrow$ $\Psi_1(0) \neq 0$

not-possible in ideal MHD

2). Ideal MHD solution: $\psi_1(0) = 0 \Rightarrow$ no reconnection!

$$\psi_1^{(i)}(x) = B_0 \delta \frac{\sin(\alpha a)}{\sin(\varkappa a)} \, |\sin(\varkappa x)|$$

$\Downarrow$

discontinuity of $\frac{d\psi_1}{dx}$ at $x=0 \Rightarrow$ current sheet is formed at $x=0$

Forced reconnection: transition from $\psi_1^{(i)}$ to $\psi_1^{(r)}$ due to a finite plasma resistivity on a tearing time-scale.

How does magnetic energy change?

$$\Delta W_M = -2 \int_0^t \left\langle \left(\frac{B^2}{2\mu_0}\right)_{x=x_s} \cdot \frac{d\delta}{dt} \cos ky \right\rangle dt$$

$\Downarrow$

1) $$\Delta W_M^{(i)} = \frac{B_0^2}{2\mu_0} \delta^2 \left[\varkappa \frac{\sin^2(\alpha a)\cos(\varkappa a)}{\sin(\varkappa a)} - \alpha \cos(\alpha a)\sin(\alpha a) \right]$$

$\Delta W_M^{(i)}$ is positive $\Rightarrow$ magnetic energy of the system initially increases due to the external deformation

weak shear: $\alpha a \ll 1$, $ka \ll 1$

$$\Delta W_M^{(i)} = \frac{B_0^2}{2\mu_0} \delta^2 \alpha^2 a \frac{(ka)^2}{3}$$

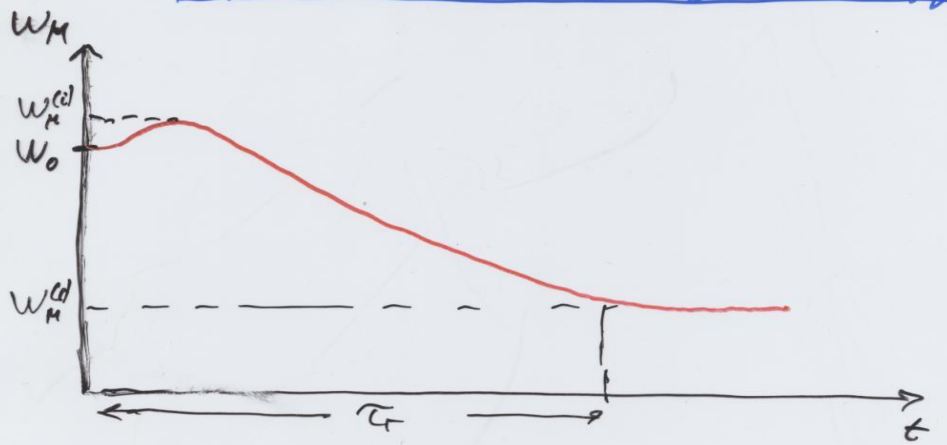
2). $$\Delta W_M^{(r)} = -\frac{B_0^2}{2\mu_0}\delta^2\left[\varkappa \sin^2(\alpha a)\frac{\sin(\varkappa a)}{\cos(\varkappa a)} + \alpha\cos(\alpha a)\sin(\varkappa a)\right]$$

$\Delta W_M^{(r)}$ is negative $\Rightarrow$ magnetic energy relaxes below its initial magnitude

$$\left.\begin{matrix}\alpha\ell \ll 1\\ \varkappa a \ll 1\end{matrix}\right\} \Rightarrow \Delta W_M^{(r)} \approx -\frac{B_0^2}{2\mu_0}\delta^2\alpha^2 a \Rightarrow |\Delta W_M^{(r)}| \gg |\Delta W_M^{(i)}| \Rightarrow$$

external perturbation acts as a trigger for internal magnetic relaxation

Temporal evolution of magnetic energy



τ_r - reconnection time

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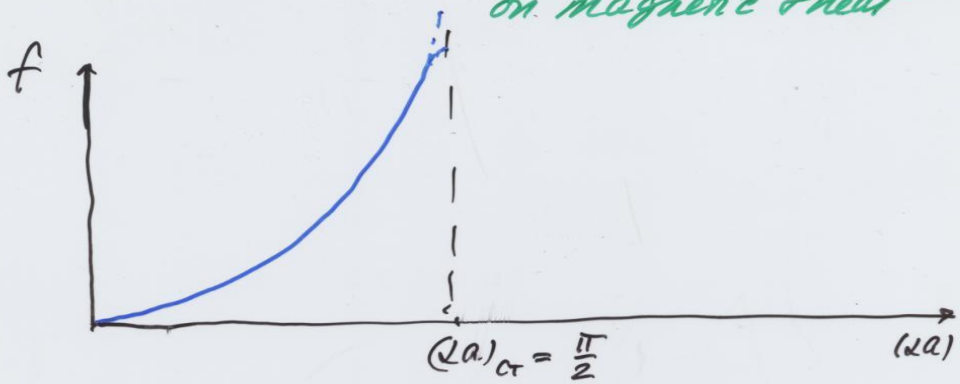
$\tau_r \sim \tau_A S^{3/5}$ for the standard resistive MHD

Released magnetic energy:

$$\Delta W_M = \frac{B_0^2}{2\mu_0} a \left(\frac{\delta}{a}\right)^2 \underbrace{f(\alpha a)}_{\text{dependance on magnetic shear}}$$



The energy effect of forced reconnection is strongly amplified for a marginally stable magnetic configuration

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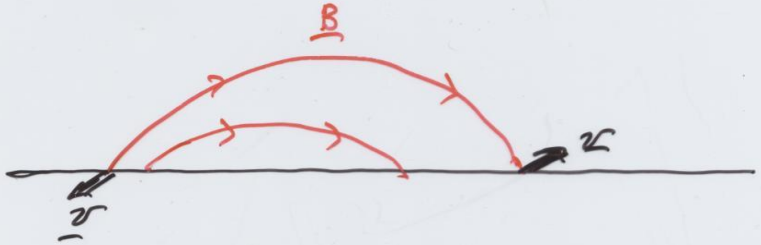
a link between the forced and spontaneous reconnection

Possible implications for solar corona

Another scenario of how magnetic energy could be stored in the corona without being quickly released by reconnection.

1) magnetic energy is accumulated in the corona due to one kind of deformation (for example, shearing)



2). Then, at some instant, another deformation occurs (for example, emergence of new flux).

forced reconnection takes place, and initially stored magnetic energy is released.

Formation of plasmoids: "comeback" of the resistive MHD reconnection

Classical Sweet-Parker model →
highly elongated current sheet

$l \sim L S^{-1/2}$

$S = \frac{L \cdot v_A}{\eta}$ – Lundquist number

$S \sim 10^{12}$ in the solar corona

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reconnection time $\tau_r \sim \tau_A S^{1/2}$
is too large!!

Tearing instability of S-P current sheet

Loureiro et al....
Phys. Rev. Lett, 103, 105004, 2009
Phys. Plasmas, 14, 100703, 2007

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current sheet breaks-up into a large number of magnetic islands (plasmoids) very quickly (super-Alfvenic time-scale)

$N \sim S^{3/8}$, $\tau \sim \tau_{AL} \cdot S^{-1/4}$, $\tau_{AL} = L/v_A$

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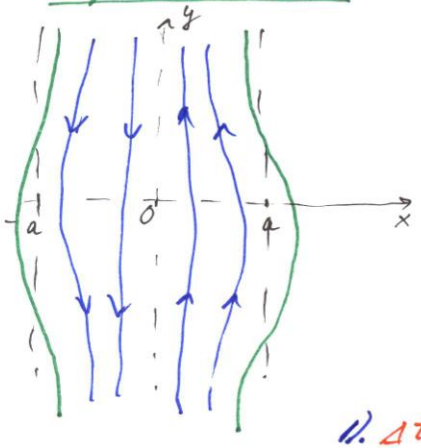
much faster reconnection for $S \gg 1$

How could it manifest itself in a dynamically evolving system?

Simple model: forced reconnection



$x_B = \pm(a + \delta\cos ky)$

$\Downarrow$

external deformation of amplitude δ on the time scale $\Delta t \gtrsim \tau_{Aa} = a/v_A$

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Subsequent evolution of the system

1). $\Delta t \ll t \lesssim \tau_{Aa} S_a^{1/3}$; $S_a = \frac{a \cdot v_A}{\eta} \gg 1$

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"ideal" evolution $\Rightarrow$ ideal solution $\psi_1^{(i)}$, formed with the current sheet width

$\underbrace{l \sim a\,\tau_A/t}$

2). magnetic reconnection, i.e. transition from $\psi_1^{(i)}$ to $\psi_1^{(r)}$ configuration with magnetic islands, proceeds at

$\tau_{Aa} S_a^{1/3} \ll t \sim \underbrace{\tau_{Aa} S_a^{3/5}}_{\text{reconnection time}}$

Ideal phase: current sheet with length $L \sim 1/k \sim a$ and width $l \sim a\,\tau_A/t$ $\Rightarrow$

aspect ratio $L/l \sim t/\tau_A$ is increasing with time $\Rightarrow$

C.S. should become tearing-unstable

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The effect of it become noticable, when $\gamma(t_*)\,t_* \sim 1$

For tearing mode:

$$\gamma_l \sim \tau_{Al}^{-1}\, S_l^{-3/5} (kl)^{2/5} (\Delta')^{4/5}$$

For long-wave perturbation (most unstable)

$$k \sim \frac{1}{a} \Rightarrow kl \sim \frac{l}{a}; \quad \Delta' \sim \frac{1}{kl} \sim \frac{a}{l}$$

$$\tau_{Al} \simeq \tau_{Aa} \cdot \frac{l}{a}; \quad S_l \simeq S_a \cdot \frac{l}{a}$$

$\Downarrow$

$$\gamma_l \sim \tau_{Aa}^{-1}\, S_a^{-3/5} \left(\frac{a}{l}\right)^2 \sim \tau_{Aa}^{-1}\, S_a^{-3/5} \left(\frac{t}{\tau_{Aa}}\right)^2$$

$\Downarrow$

$$\gamma_l(t) \cdot t \sim S_a^{-3/5} \left(\frac{t}{\tau_{Aa}}\right)^3 \Rightarrow \boxed{t_* \sim \tau_{Aa} \cdot S_a^{1/5}} \Rightarrow$$

Such a C.S. cannot survive for $t \gtrsim \tau_{Aa}\, S_a^{1/5}$

No stabilization by the plasma flow
(Bulanov et al, 1979)

$\tau_{rec} \sim 1/t < \gamma_e$ for $t > t_*$

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C.S. breaks-up if $t_* \sim \tau_{Aa} S_a^{1/5} < \tau_{Aa} S_a^{1/3}$ (before forced reconnection occurs)

This is the case for asymptotically large S_a

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This could explain why this effect has not been observed in numerical simulations with $S_a \sim 10^4$

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transition to a faster regime of reconnection could be anticipated for larger S