Asymptotic modeling of the formation and saturation of mirror structures

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Outline

1. Satellite observations
2. Numerical simulations of the Vlasov-Maxwell equations
3. Modeling the formation and saturation of the structures
4. Summary

1. Satellite observations

Magnetic structures (**humps** or **holes**)
that are quasi-stationary in the plasma frame,
with **no or little change in the magnetic field direction** are commonly observed in the
solar wind and the planetary magnetosheaths.

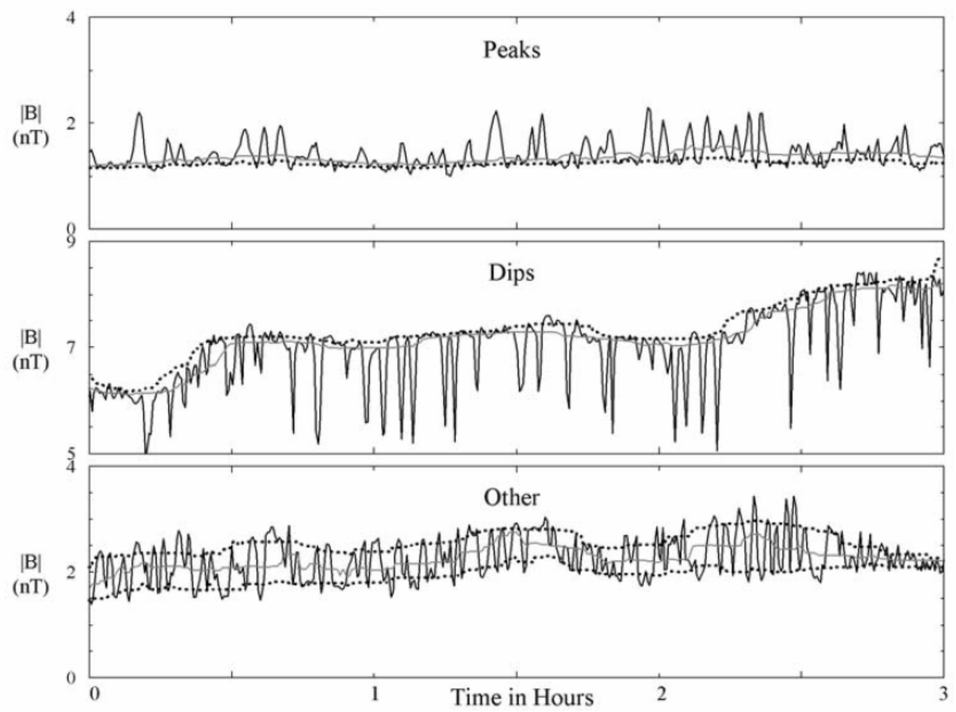


Figure 1. Each panel shows 3 hours of Galileo magnetometer field magnitude data (solid black line), appropriate quartiles (dotted), and the median value (solid gray) computed using 20 min sliding windows with single sample shifts. The panels show examples of "peaks" (top), "dips" (middle), and "other" (bottom) structures.

Joy et al. J. Geophys. Res. **111**, A12212 (2006)

Structures observed in the Jovian magnetosheath

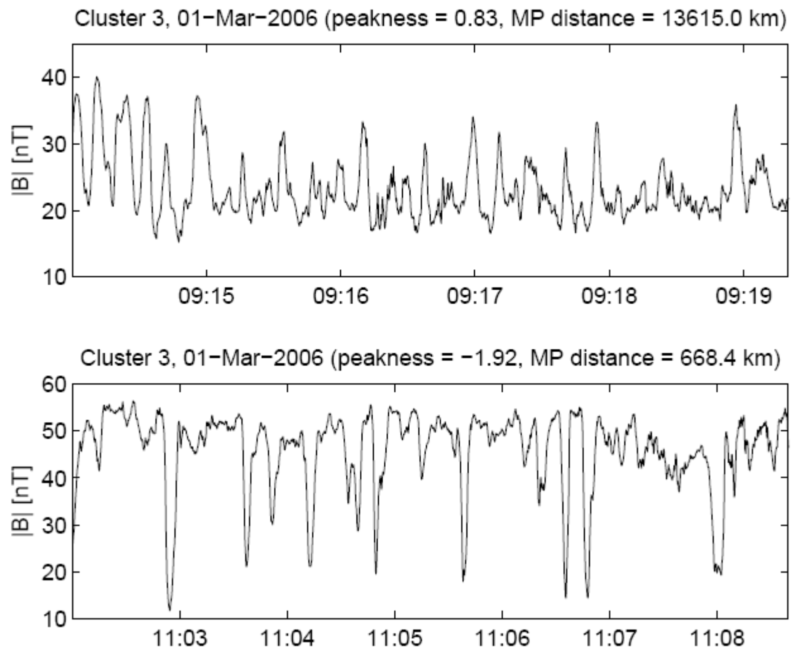


Figure 1. An example of mirror mode structures of the two types. Top panel: peaks (peakness = 0.83), bottom panel: dips (peakness = -1.92).

Soucek, Lucek & Dandouras JGR **113**, A04203 (2008)

Structures observed in
the terrestrial magnetosheath

Usually viewed as **nonlinear mirror modes**

Main properties of observed structures:

- Structures are quasi-static in the plasma frame (propagating drift mirror modes exist in density gradients)

- Small change in the magnetic field direction

- Observed in regions displaying: ion temperature anisotropy $T_{i\perp} > T_{i\parallel}$
  β of a few units

  (conditions met under the effect of plasma compression in front of the magnetopause).
  Not always in a mirror unstable regime.

- Magnetic fluctuations mostly affect the parallel component.

- Cigar-like structures, quasi-parallel to the ambient field, with a transverse scale of a few Larmor radii.

- Density is anticorrelated with magnetic field amplitude.

Origin of these structures is still not fully understood.

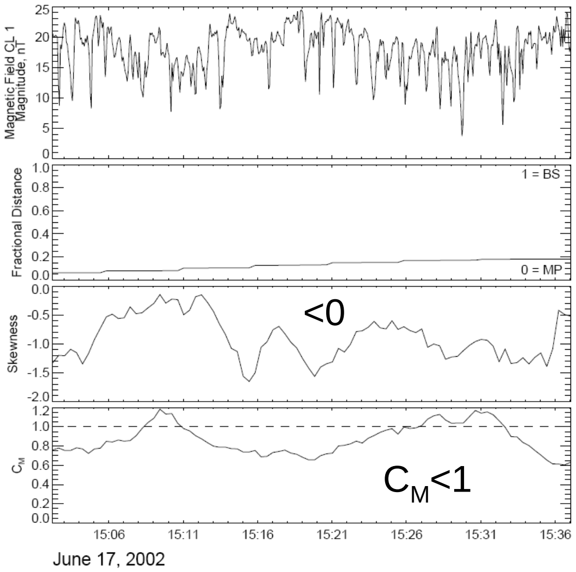
Usually viewed as

- nonlinearly saturated states of the mirror instability *(Venelov & Sagdeev 1958),*

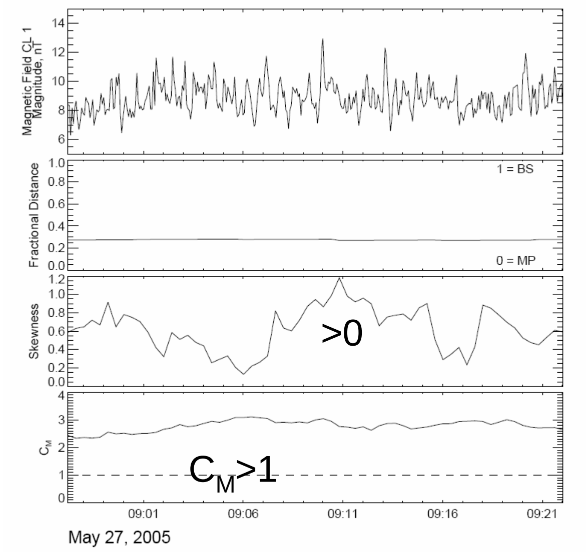
- or (in particular in the solar wind), remnants of mirror structures created upstream of the observation point *(Winterhalter et al. 1995).*

Other interpretations:

- trains of slow-mode magnetosonic solitons *(Stasiewicz 2004)*
- mirror instability is the trigger, generating high amplitude fluctuations that evolve such as to become nonlinear solutions of isotropic or anisotropic plasma equations *(Baumgärtel, Sauer & Dubinin 2005)*



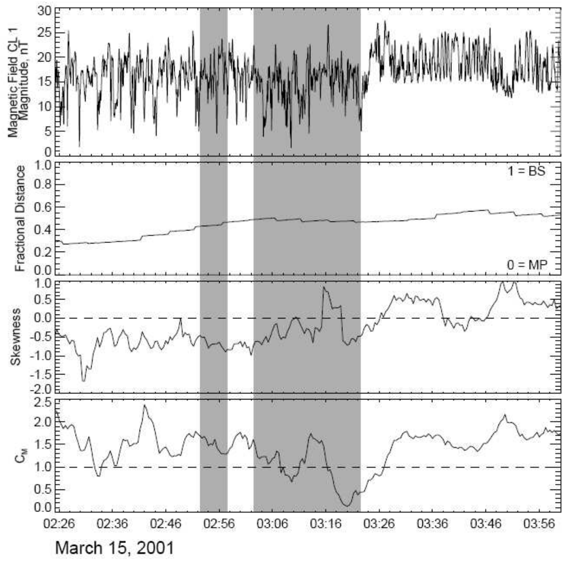
Skewness

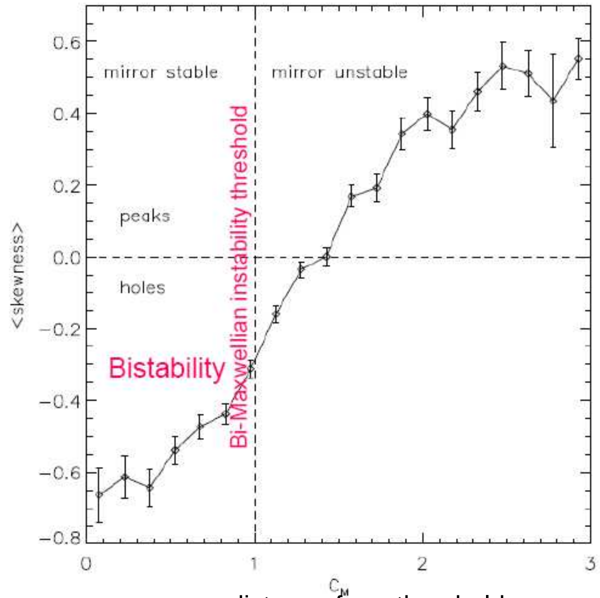


C_M <1 : subcritical
C_M >1 : supercritical
(for bi-Maxwellian equilibrium)

Magnetic holes: mostly in subcritical regime

Magnetic humps: in supercritical regime





distance from threshold

$$C_M = \beta_{p\perp}\left(\frac{T_{p\perp}}{T_{p\parallel}} - 1\right)$$

Génot et al., Ann. Geophys. ***27,*** *601 (2009).*

Similar conclusions in *Soucek et al.JGR* ***113***, *A04203 (2008)*

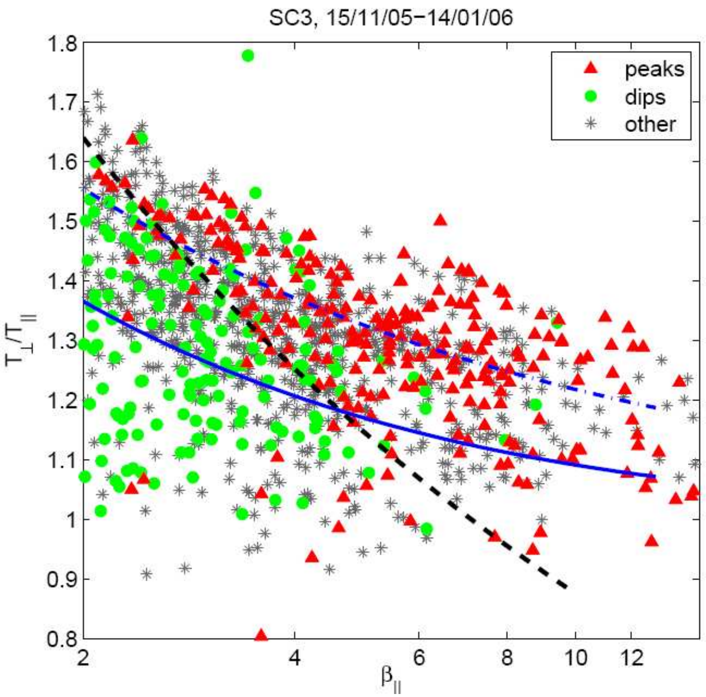


Figure 3. Distribution of mirror modes of different types in the anisotropy-beta plane. Red triangles denote peaks with $\mathcal{P} > 0.3$, green squares dips ($\mathcal{P} < -0.6$) and the remaining ambiguous mirror mode events are marked by grey stars.

Solid blue line: theoretical (bi-Maxwellian) mirror threshold

$$\frac{T_\perp}{T_{||}} > 1 + 1/\beta_\perp$$

Dashed-dotted blue line: empirical marginal stability

$$\frac{T_\perp}{T_{||}} = 1 + \frac{a}{\beta_{||}^b} \qquad \begin{aligned} a &= 0.83 \\ b &= 0.58 \end{aligned}$$

Black dashed line: fitted boundary between peaks and dips

$$\frac{T_\perp}{T_{||}} = \frac{2.15}{\beta_{||}^{0.39}}$$

"Peaks are typically observed in an unstable plasma, while mirror structures observed deep within the stable region appear almost exclusively as dips".

Soucek, Lucek & Dandouras, JGR **113**, A04203 (2008)

Solar wind: "Although the plasma surrounding the holes was generally stable against the mirror instability, there are indications that the *holes may have been remnants of mirror mode structures* created upstream of the points of observation" (Winterhalter et al. 1995).

Steady state stability consistent with energetic arguments in the context of MHD with suitable equation of state *(Passot et al. PoP 13, 102310, 2006)*

QUESTION: Dynamics leading to these structures?

2. Numerical simulations of the Vlasov-Maxwell equations

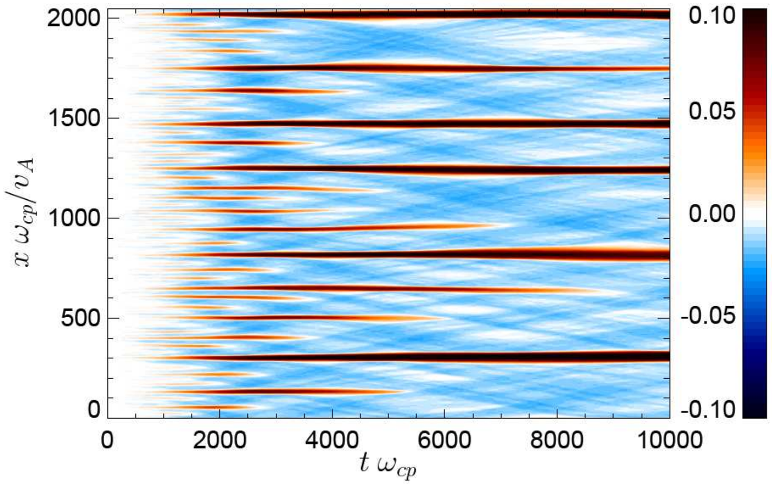
Shed light on the time evolution and on the origin of the structures.

Mirror unstable regime near threshold in a large domain

1D simulation: $\theta_{kB} = 72.8°$ (most unstable direction)

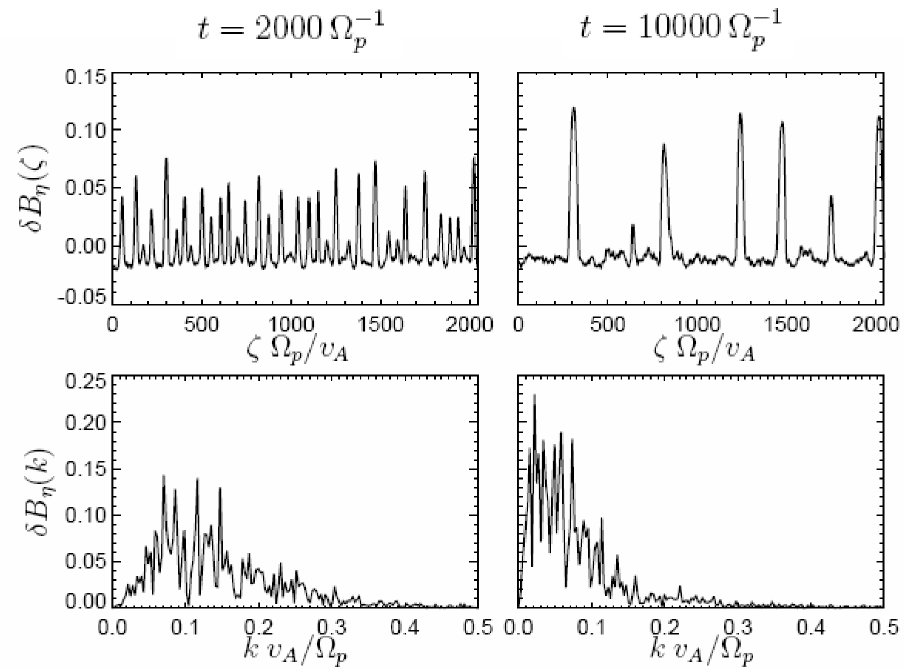
$\beta_{p\|} = 1 \; \beta_{p\perp} = 1.857 \quad \beta_e = 10^{-2}$

With a PIC code in a large domain:
Domain size= 2048 c/ωpi
Growth rate: 0.005 Ωp
1024 cells with 500 000 particles/cell



Color plot of the fluctuations of the magnetic field component B_η perpendicular to the direction ζ of spatial variation, as a function of ζ and t.

Califano et al., JGR 113, A08219 (2008).



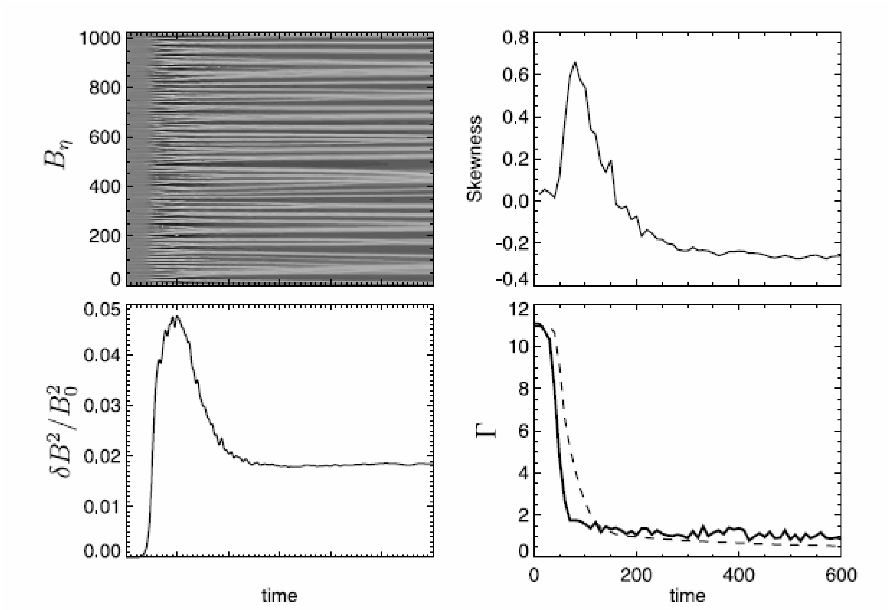
A large number of modes are excited.
Humps form and undergo coarsening.

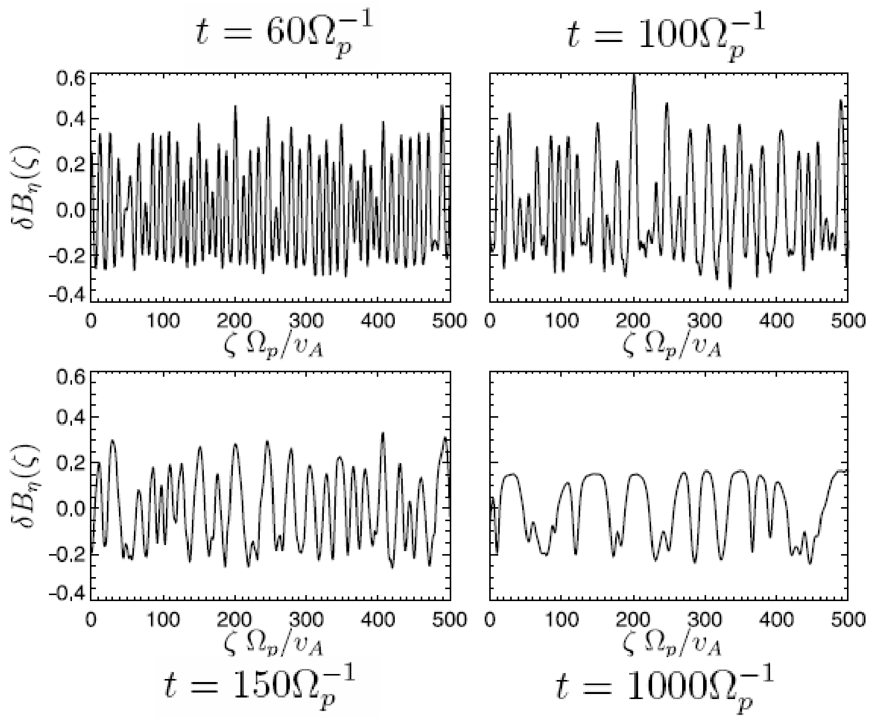
Initial conditions far from threshold

($\beta_{||p}$=1, $T_\perp / T_{//}$=4, θ=50.5)

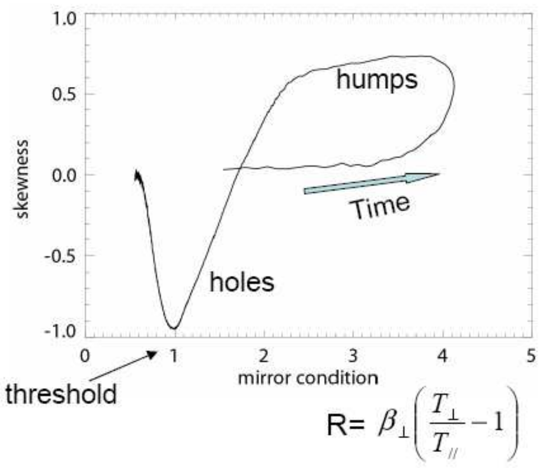
After a while skewness becomes negative.
Humps eventually transforms into dips.

No such transition at larger β (e.g. $\beta_{p\,\parallel} = 2$).





(quarter of the box)



Magnetic holes also formed in PIC simulations in an expanding domain modeling the magnetosheath *(Hellinger & Travnicek 2007).*

3. Reductive perturbative expansion: Isolates the mirror mode dynamics

(Kuznetsov et al. PRL 2007, Califano et al. JGR 2008)

Cold electrons

A (long-wavelength) reductive perturbative expansion near threshold can be performed for any (frozen) smooth equilibrium distribution function $f(v_\parallel^2, v_\perp)$ provided $\tilde{v} > 0,\ \tilde{r}^2 > 0,$ and $\chi > 0$).

$$\partial_t b = \sqrt{\frac{2}{\pi}}\tilde{v}\,(-\mathcal{H}\partial_z)\left(\Gamma b + \frac{3}{2}\tilde{r}^2\Delta_\perp b - \chi\frac{\partial_z^2}{\Delta_\perp}b - \Lambda b^2\right)$$

$$b = \delta B_z(\boldsymbol{r}, t)/B_0$$

(normalized parallel magnetic perturbation)

Linear growth rate: $\gamma_{\boldsymbol{k}} = \sqrt{\frac{2}{\pi}}|k_\parallel|\tilde{v}\left(\Gamma - \frac{3}{2}\tilde{r}^2 k_\perp^2 - \frac{k_\parallel^2}{k_\perp^2}\chi\right)$

Instability condition: $\Gamma = \beta_\Gamma - \beta_\perp - 1 > 0$

$$\beta_\Gamma = -\frac{mn}{p_B}\int \frac{v_\perp^4}{4}\frac{\partial f}{\partial v_\parallel^2}d^3v$$

$$\beta_\perp = \frac{mn}{p_B}\int \frac{v_\perp^2}{2} f d^3v$$

$$\beta_\parallel = \frac{mn}{p_B}\int v_\parallel^2 f d^3v$$

$$\tilde{r}^2 = -\frac{mn}{24 p_B}\frac{1}{\Omega^2}\int\left(v_\perp^6\frac{\partial f}{\partial v_\parallel^2} + 3v_\perp^4 f\right)d^3v$$

$$\chi = 1 + \frac{1}{2}(\beta_\perp - \beta_\parallel)$$

$$\tilde{v}^{-1} = -\sqrt{2\pi}\frac{mn}{p_B}\int \frac{v_\perp^4}{4}\delta(v_\parallel)\frac{\partial f}{\partial v_\parallel^2}d^3v$$

(Hellinger et al. 2009)

Nonlinear coupling:

$$\Lambda = \beta_\Lambda - 2\beta_\Gamma + \frac{\beta_\perp}{2} + \frac{1}{2}$$

with $\beta_\Lambda = \frac{mn}{p_B}\int \frac{v_\perp^6}{8}\frac{\partial^2 f}{\partial (v_\parallel^2)^2}d^3v$

For a bi-Maxwellian distribution

, $\beta_\Lambda = 3/2\beta_\perp^3/\beta_\parallel^2$

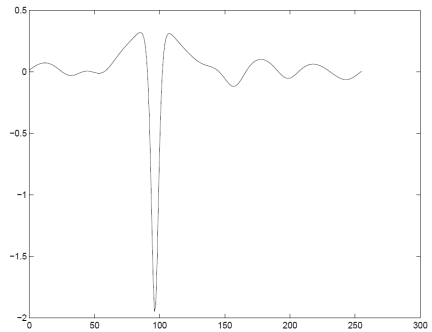
Thus **Λ** >0 and the model predicts formation of magnetic holes, while humps are observed in the simulations.

For a bi-Maxwellian distribution:

$$\beta_\perp = mnv_{th\perp}^2/p_B$$
$$\beta_\parallel = mnv_{th\parallel}^2/p_B$$
$$\tilde{v} = v_{th\parallel}/\beta_\Gamma$$
$$\beta_\Gamma = \beta_\perp^2/\beta_\parallel$$
$$\tilde{r} = v_{th\perp}(\beta_\Gamma - \beta_\perp)^{1/2}/\Omega$$

Instability condition:

$$\Gamma^* \equiv \beta_\perp\left(\frac{\beta_\perp}{\beta_\parallel} - 1\right) - 1 > 0$$



Profile of 1D solution near collapse

Subcritical bifurcation

(Kuznetsov et al. 2008)

Solution above threshold blows up in a finite time

Possible saturation mechanism: particle trapping

Difficult to retain particle trapping within a systematic reductuve perturbative analysis

At the level of the asymptotic equation, particle trapping can be phenomenologically interpreted as a renormalization of the time derivative (It indeed corresponds to a quenching of the Landau resonance).

This effect was introduced at the level the asymptotic equation by prescribing a flattening of the parallel distribution function on a range that extends with the strength of the magnetic perturbation (*Pokhotelov et al. JGR 2008)*

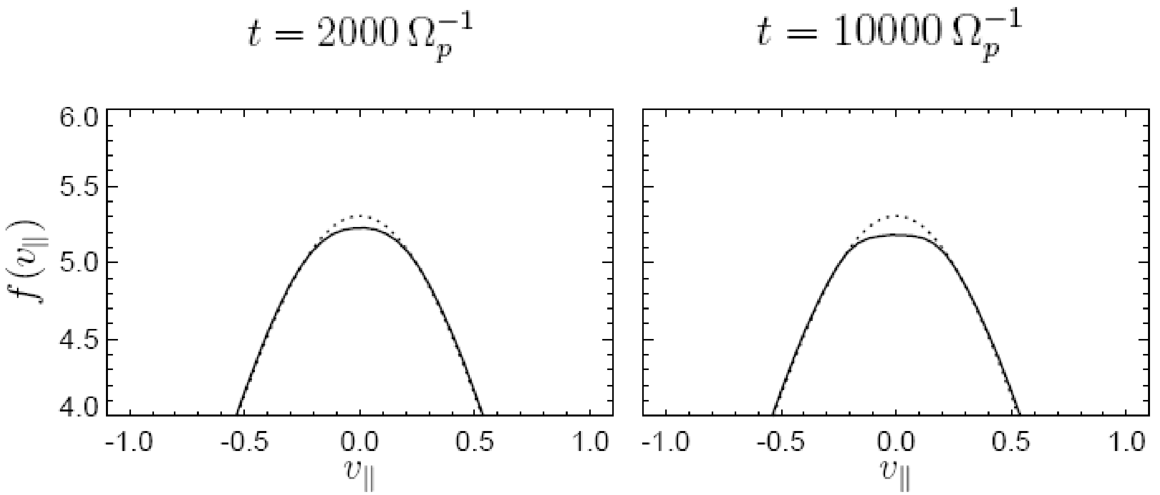
$$\left(1 - \frac{2}{\pi}\arctan\frac{|h|^{1/2}}{\gamma}\right)\frac{\partial h}{\partial \tau} = \hat{k}_{\xi}\left[\left(1 + \frac{\partial^2}{\partial \xi^2}\right)h - h^2\right]$$

(1D model)

Can prevent explosive growth of the amplitude.
The stationary solutions still have the form of KdV solitons.
Only holes can result from this model.

Formation of magnetic hump suggests that the distribution function does not remain bi-Maxwellian

PIC simulation in an extended domain near threshold



Reduced distribution function f as a function of $v_\parallel$ (solid curve) compared to the initial reduced distribution function (dotted curve).

Flattening of the distribution function resulting from diffusion in velocity space.

Possibly described by the quasi-linear theory

Quasi-linear theory *(Shapiro & Shevchenko 1963)*

- Assumes space homogeneity (thus absence of coherent structures); can thus be valid at early times only.
- Requires many modes in interaction, thus an extended domain.
- Mainly associated with a diffusion process in velocity space (dominantly along the ambient field).

$$D_{\|\|} = v_\perp^4 \sum_{\boldsymbol{k}} \frac{|b_{\boldsymbol{k}}|^2}{4} \frac{\gamma_{\boldsymbol{k}} k_\|^2}{k_\|^2 v_\|^2 + \gamma_{\boldsymbol{k}}^2}$$

$$D_{\perp\|} = -2\frac{v_\|}{v_\perp} D_{\|\|}$$

$$D_{\perp\perp} = v_\perp^2 \sum_{\boldsymbol{k}} \gamma_{\boldsymbol{k}} \frac{|b_{\boldsymbol{k}}|^2}{4}$$

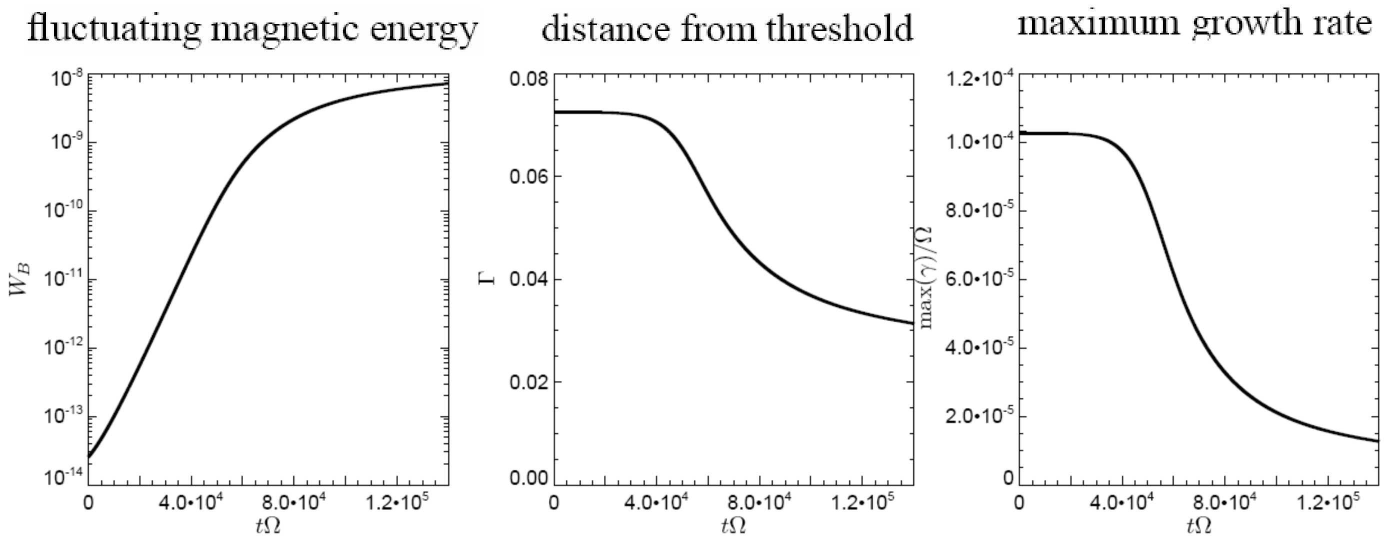
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_\|} D_{\|\|} \frac{\partial f}{\partial v_\|} + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \left(D_{\perp\|} \frac{\partial f}{\partial v_\|} + D_{\perp\perp}\frac{\partial f}{\partial v_\perp}\right)$$

$$\frac{\partial b_{\boldsymbol{k}}}{\partial t} = \gamma_{\boldsymbol{k}} b_{\boldsymbol{k}}$$

$$b_{\boldsymbol{k}} = \delta B_z(\boldsymbol{k})/B_0$$

linear growth rate

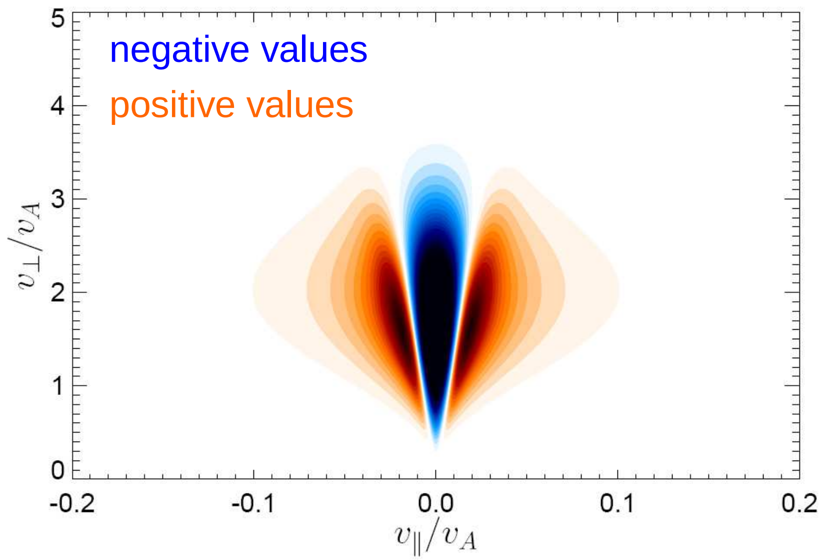
Hellinger & al., GRL, 36, L06103, (2009)

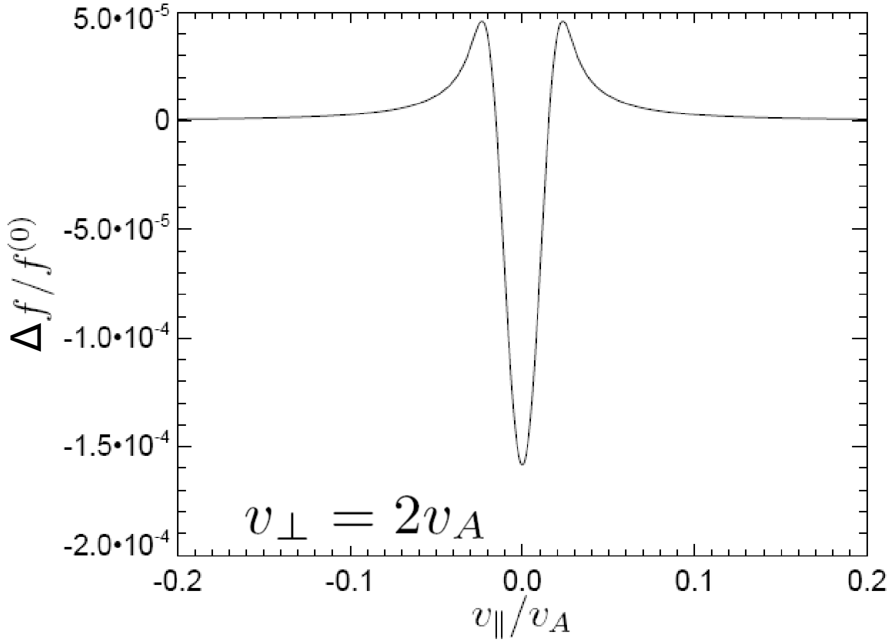


Perturbation of the space-averaged distribution function $\Delta f = f - f^{(0)}$

QL theory

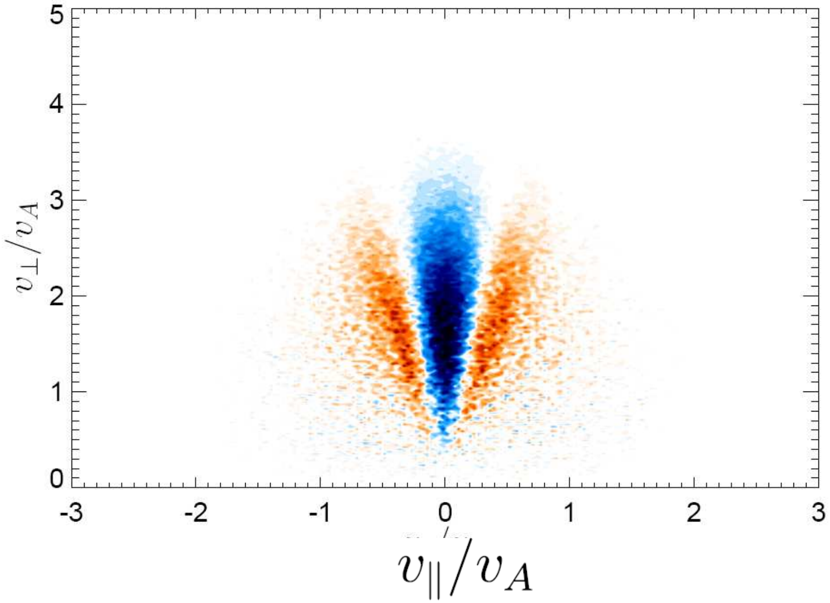
t= 1.4 10^5

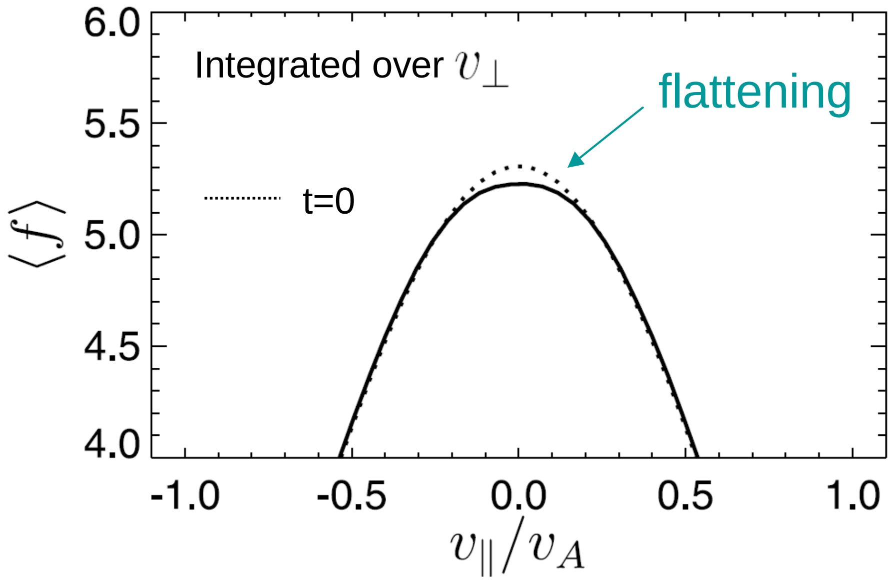




PIC simulation

t=2 10^3





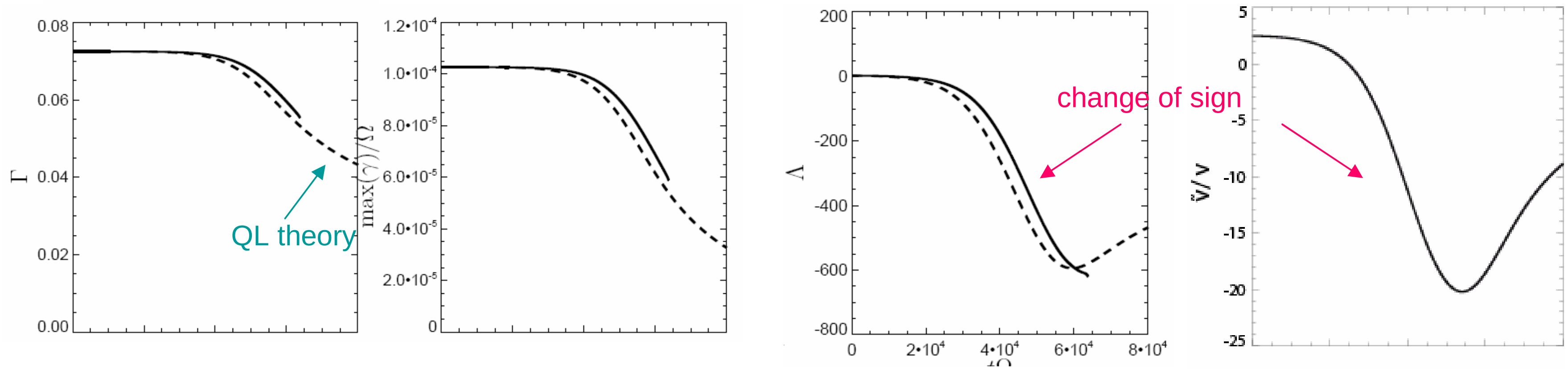
This suggests to couple QL theory and reductive perturbative expansion by estimating the coefficients in the equation for the magnetic fluctuations from the instantaneous QL distribution function (that is sensitive to the magnetic fluctuations).

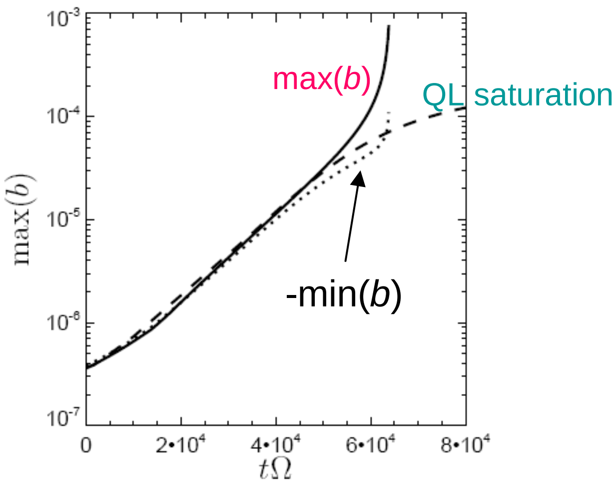
$$\begin{cases} \dfrac{\partial f}{\partial t} = \dfrac{\partial}{\partial v_\parallel} D_{\parallel\parallel} \dfrac{\partial f}{\partial v_\parallel} + \dfrac{1}{v_\perp}\dfrac{\partial}{\partial v_\perp} v_\perp \left(D_{\perp\parallel} \dfrac{\partial f}{\partial v_\parallel} + D_{\perp\perp}\dfrac{\partial f}{\partial v_\perp} \right) \\ \partial_t b = \dfrac{\sqrt{\frac{2}{\pi}}\tilde{v}}{1 + 2\frac{\tilde{v}}{v_\Lambda} b} (-\mathcal{H}\partial_z) \left(\Gamma b + \dfrac{3}{2}\tilde{r}^2 \Delta_\perp b - \chi \dfrac{\partial_z^2}{\Delta_\perp} b - \Lambda b^2 \right) \end{cases}$$

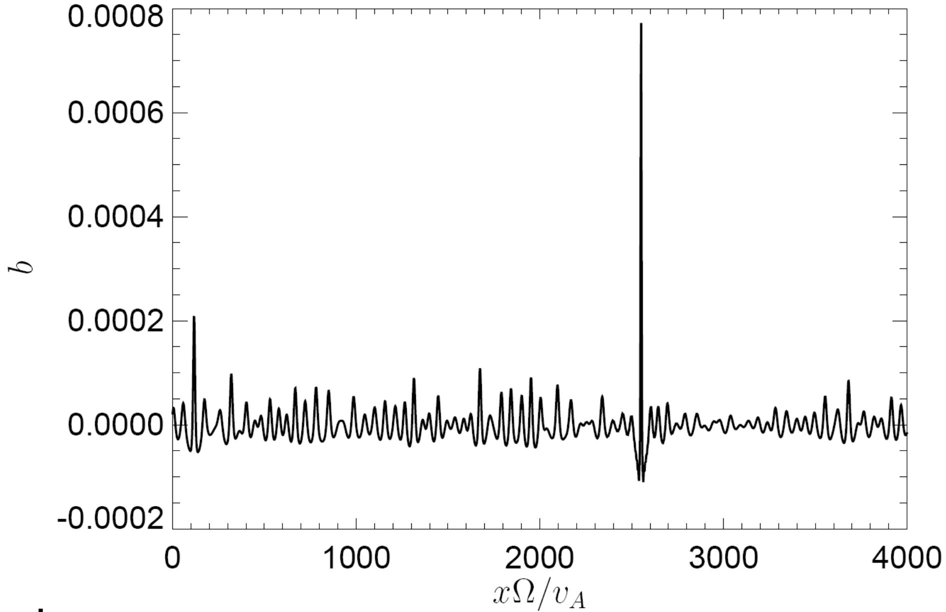
$$v_\Lambda^{-1} = \sqrt{2\pi}\frac{mn}{p_B} \int \frac{v_\perp^6}{8} \delta(v_\parallel) \frac{\partial^2 f}{(\partial v_\parallel^2)^2} d^3 v$$

Because of the quasi-singularity of distribution function resulting from QL evolution, near zero parallel velocity, contributions of the resonant particles are to be taken into account in the estimate of the nonlinear coupling (nonlinear Landau damping), leading to the denominator.

distance from threshold | maximum growth rate | **nonlinear coupling** | **resonance coefficient**







Results of the simulation of coupled system in 1D (in the most unstable direction)

Formation of **magnetic humps**

Saturation by nonlinear Landau damping

Difficult to study saturation by direct integration of the model system (due to numerical limitations).

In order to isolate the saturation effect, we freeze the coefficients after the QL phase
(QL diffusion is expected to be strongly depleted as structures are formed)

1D model after rescaling,

$$\partial_t b = \frac{1}{1+s\alpha b}\left(-\mathscr{H}\partial_\xi\right)\left(\sigma b + \mu \partial_{\xi\xi} b - 3sb^2\right)$$

where $\sigma=+1$ (supercritical) or -1 (subcritical)
$s=+1$ (near a Maxwellian distribution)
or $s=-1$ (due to QL flattening of the distribution function)

The parameters α and μ are taken positive.

The denominator is reminiscent (in a small amplitude expansion) of the *arctan* trapping correction suggested by Pokhotelov et al. (JGR 2008).
The physical mechanism is however different, originating here from nonlinear Landau damping.

The parameter α refers to the contribution of the QL resonance to the nonlinear coupling.

While for $\alpha=0$, the solution blows up in a finite time,
the denominator arrests the collapse at a maximal amplitude given by $1/\alpha$,
leading to the formation of

- magnetic hole solitons for $s=+1$
- magnetic hump solitons for $s=-1$

(Passot et al., 2009)

Saturated solutions in a supercritical regime

Numerical integration of the model equation, starting from a sine wave of amplitude 0.01 in a domain of size 2π leads to a stationary hump solution with a negative value of b in the background.

The problem is numerically (and mathematically) difficult and is still under investigation. Extremely small time steps are required.

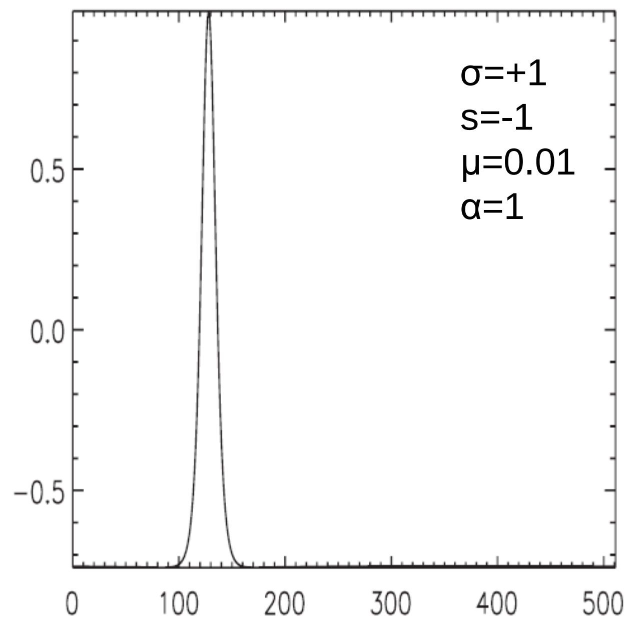
Saturation and stability of the soliton profile:

During the nonlinear phase of amplitude growth, a plateau of negative values gradually develops, that tends to locally reduce the ambient magnetic field, putting the system in a situation similar to the subcritical regime.

The solution is then attracted to the negative of the KdV soliton with an amplitude $b_{max}=1/\alpha$.

It is stable due to the presence of the denominator term.

When starting with random initial conditions, which leads to a large number of humps, a coarsening phenomenon is observed.



For s=+1, hole solutions are obtained (change b into –b): regime where QL effects are subdominant, even above threshold.

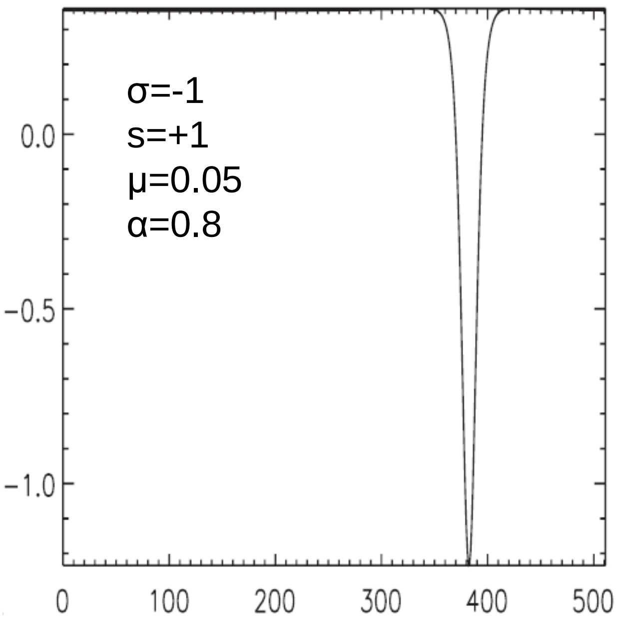
The amplitude of the structures is prescribed by the strength of the early time QL resonance: larger amplitudes are obtained when these effects are smaller.

Subcritical solutions

When σ=-1 with large initial data, no quasi-linear phase: the distribution function remains bi-Maxwellian (s=+1).

The denominator correction (with α small) is to be retained because of the large amplitudes.

Magnetic holes are obtained.



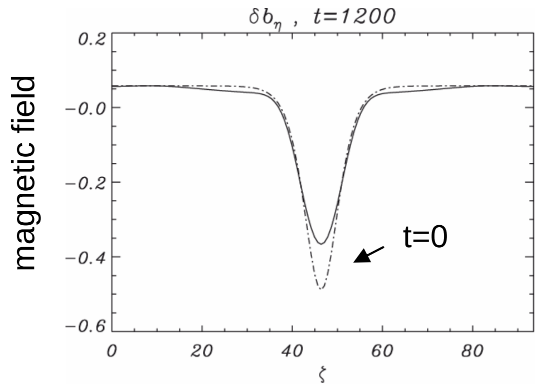
Random initial conditions of amplitude 1.2

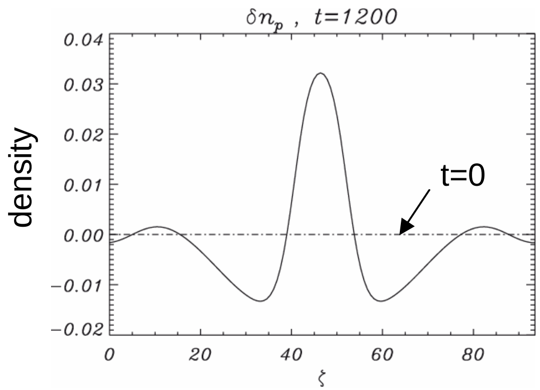
Formation of magnetic holes when starting with large initial perturbations

Vlasov-Maxwell simulation

Domain size: 15x $2\pi\ c/\omega_{pi}$

Subcritical solutions (i.e. below threshold)



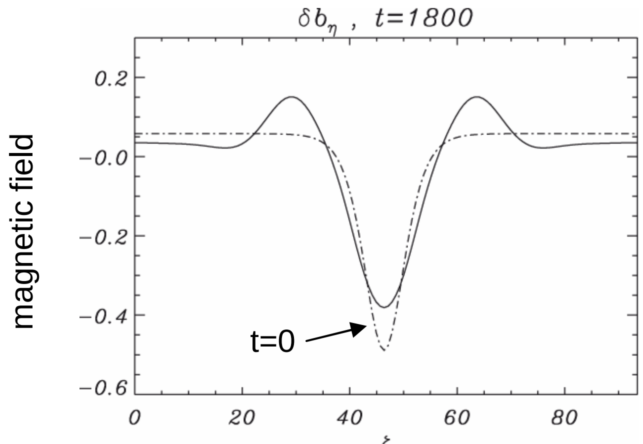


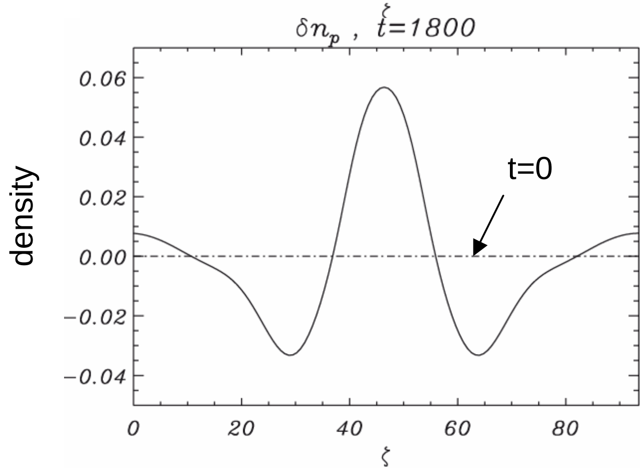
$$\beta_{\|} = 6, \quad \theta = 1.463, \quad T_{\perp} = T_{\|}$$

Large-amplitude magnetic holes
survive even far **below and above threshold**.

Magnetic humps do not survive below threshold

Solution above threshold.





$\beta_{\|}=6$, $T_{\perp}/T_{\|}=1.2$ and $\theta=1.463$

4. Summary

Numerical integrations of VM equations in an extended domain:

- In the supercritical regime, existence of an early phase described by the quasi-linear theory, followed by a regime where coherent structures are formed.

- Structures resulting from the **saturation of the mirror instabilty** are **magnetic humps**.

- Stable solutions in the form of large-amplitude magnetic holes also exist both above and below threshold.

Asymptotic modeling:

- Reductive perturbative expansion of VM eqs near threshold leads, for a bi-Maxwellian (and probably any smooth) equilibrium distribution function, to an equation predicting the formation of magnetic holes, with a finite-time singularity (i.e. large-amplitude structures).

- In fact, the early quasi-linear phase introduces a boundary layer for the distribution function near $v_{//}=0$.
  As a result, the asymptotic equation leads to **magnetic humps.**
  Saturation by nonlinear Landau damping at a level prescribed by strength of the quasi-linear resonance.

Although mirror modes near threshold have zero frequency, are quasi-transverse and at large scale, genuine kinetic effects play a main role, making the dynamics not amenable to a fluid approach.

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