RECENT PROGRESS IN STUDYING KINK OSCILLATIONS OF CORONAL LOOPS

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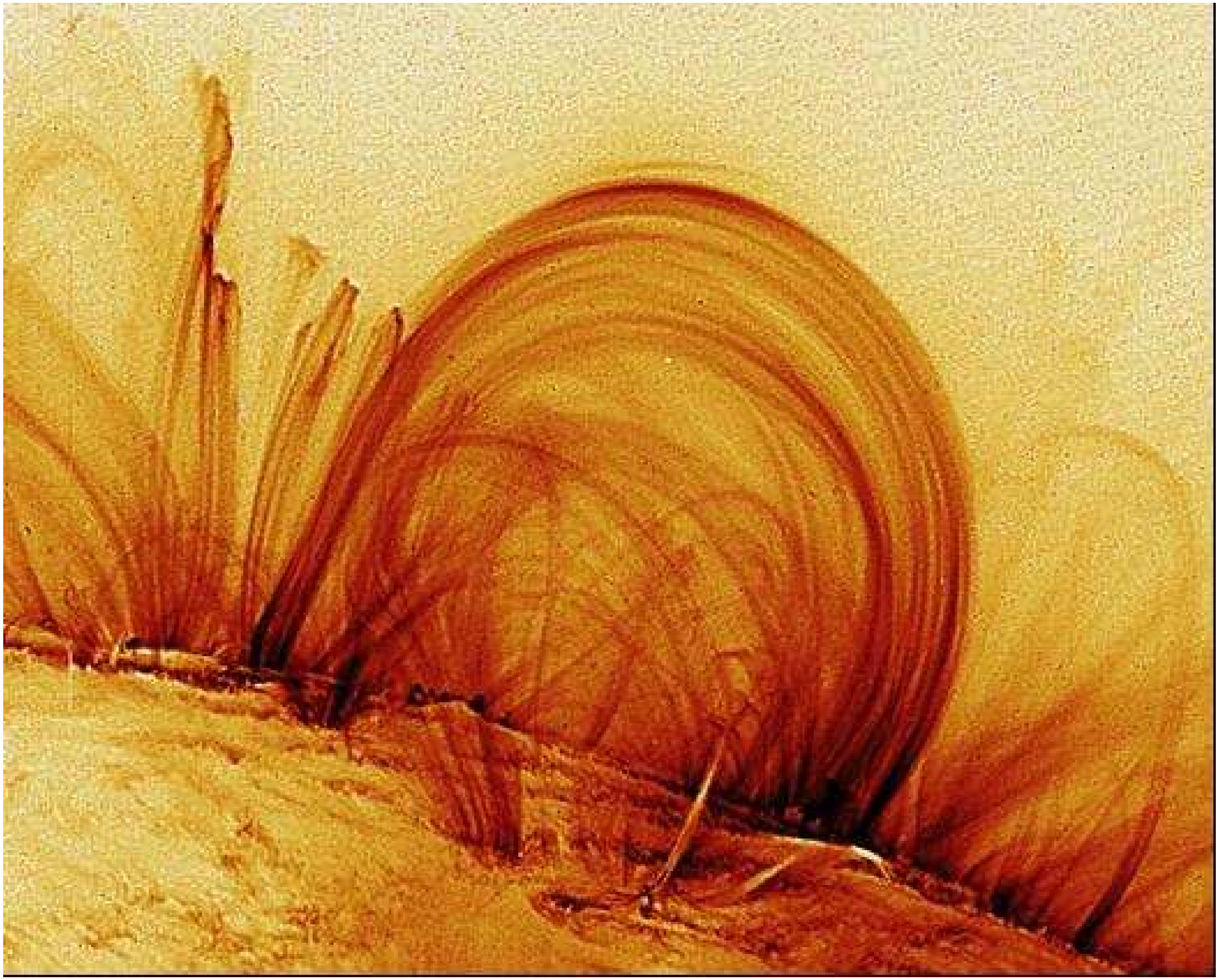


1. INTRODUCTION

Solar corona is highly inhomogeneous and magnetic structured.

Ones of important structures are coronal magnetic loops.

On 14 July 1998 **TRACE** (Transition Region and Corona Explorer) observed transverse oscillations of coronal loops.

In the first theoretical interpretation of this phenomenon coronal loops were considered as straight magnetic cylinders with equilibrium quantities constant inside and outside.

Later more sophisticated models taking into account such effects as the density variation along and across the loop, the loop curvature, the variation of the loop radius and the deviation of the loop cross-section from circular were developed.

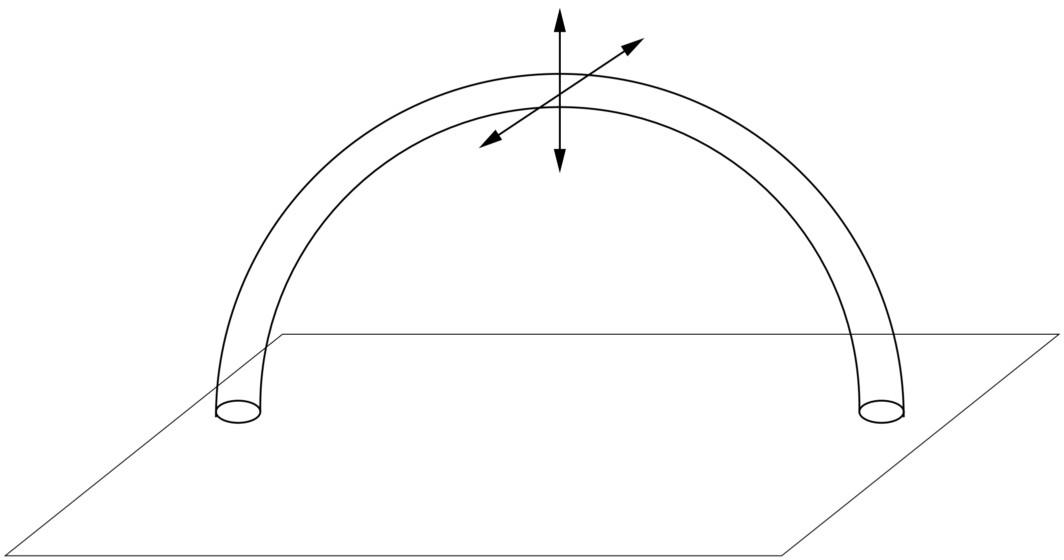
We discuss two problems related to theory of transverse coronal loop oscillations:

- Vertical and horizontal oscillations of coronal loops.
- Oscillations of non-planar loops.

2. VERTICAL AND HORIZONTAL OSCILLATIONS OF CORONAL LOOPS

Most of observed transverse coronal loop oscillations were horizontally polarized. However vertically polarized oscillations were also observed.

Van Doorsselaere et al. 2004, *Astron. Astrophys.* 424, 1065

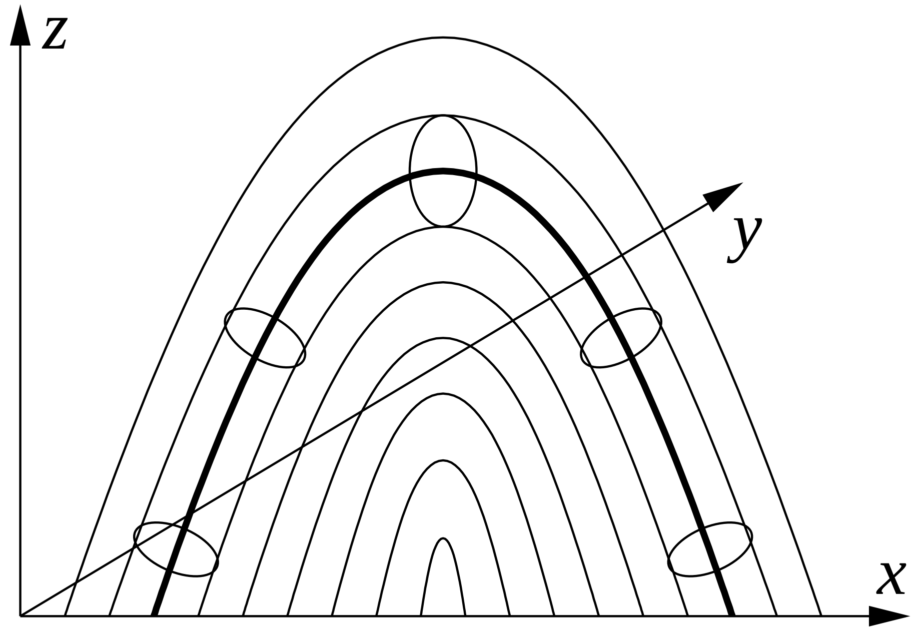


The loop has shape of half-torus. Plasma is homogeneous inside and outside the loop.

Eigenfrequencies were found analytically using toroidal coordinates. The account of curvature gives correction to oscillation frequency of order $(a/L)^2 \ll 1$. This result was confirmed by numerical solution of the same problem but in stratified atmosphere by

Terradas et al. 2006, *Astrophys. J.* 650, L91

ANOTHER MODEL



Loop is embedded in potential field: $\boldsymbol{B} = \nabla\phi = \nabla \times \psi$

$\phi(x, z)$ is magnetic potential. $\psi(x, z)$ is flux function.

Density is $\rho_i(\phi)$ inside loop, and $\rho_e(\phi)$ outside loop.

Curvilinear coordinates: ψ, y, ϕ. Then polar coordinates in ψy surface:

$$\psi = \psi_0 + r\cos\theta, \quad y = r\sin\theta, \quad \psi = \psi_0 \text{ equation of loop axis}$$

Equation of loop boundary is $r = a$.

Asymptotic expansions with respect to a/L,

$$L = \int_{\phi_1}^{\phi_2} \frac{d\phi}{B^2} \quad \text{is loop length}$$

$\phi = \phi_1$ and $\psi = \phi_2$ at loop footpoints.

To make analytical progress we, in addition, assume that expansion is weak,

$$B^2 = B_0^2[1 + \lambda q(\phi)], \qquad |\lambda| \ll 1, \quad q(\phi) \sim 1$$

Then, in the **first order approximation with respect to** λ, we obtain

$$\frac{d^2\xi}{ds^2} + \frac{\omega_1^2}{C_k^2}\xi = 0, \quad \xi = 0 \ \text{at} \ s = 0, L$$

$$s = \int_{\phi_1}^{\phi} \frac{d\phi}{B^2} \quad \text{is distance along loop axis}$$

$$C_k^2 = \frac{B_0^2}{\mu_0[\rho_i(s) + \rho_i(s)]}$$ is square of kink speed

Second order approximation with respect to λ:

$$\omega = \omega_1 + \lambda\omega_{2\mathrm{v,h}}$$

$$\omega_{2\mathrm{v}} \int_0^L \frac{\xi^2}{C_k^2}\, ds = \int_0^L \frac{q}{8} \left[\frac{\mu_0\omega_1}{B_0^2}(3\rho_i + 2\rho_e)\xi^2 - \frac{1}{\omega_1}\left(\frac{d\xi}{ds}\right)^2 \right] ds$$

$$\omega_{2\mathrm{h}} \int_0^L \frac{\xi^2}{C_k^2}\, ds = \int_0^L \frac{q}{8} \left[\frac{\mu_0\omega_1}{B_0^2}(\rho_i + 2\rho_e)\xi^2 + \frac{1}{\omega_1}\left(\frac{d\xi}{ds}\right)^2 \right] ds$$

Example: EXPONENTIALLY DECAYING FIELD IN ISOTHERMAL ATMOSPHERE

$$\phi = B_0 l e^{-kz} \sin(kx), \quad \psi = B_0 l e^{-kz} \cos(kx)$$

so that $B = B_0(lk)e^{-kz}$. Loop foodpoints are at $z = 0$, $x = \pm x_0$.

Isothermal atmosphere: $\rho_e = \rho_0 e^{-z/H}$, $\rho_i/\rho_e = \zeta > 1$, $\zeta =$ const.

Weak loop expansion $\Longrightarrow (kx_0)^2 = \lambda \ll 1$. Then $l = k^{-1}(1 + \alpha\lambda)$, and

$$q(s) = (s/x_0 - 1)^2 + 2\alpha - 1$$

where $\alpha \sim 1$ is free parameter. Equation of loop axis is

$$z = \frac{1}{2} x_0 \sqrt{\lambda} \left[1 - \left(\frac{x}{x_0} \right)^2 \right]$$

Assume that $x_0 \sim H \Longrightarrow$ loop height $\ll H \Longrightarrow \rho_{i,e} \approx$ const. Then

$$\omega_{2\mathrm{v}} = \frac{C_k[\pi^2(5\zeta+3)(6\alpha+1) - 6(7\zeta+5)]}{24\pi^2 L(\zeta+1)}$$

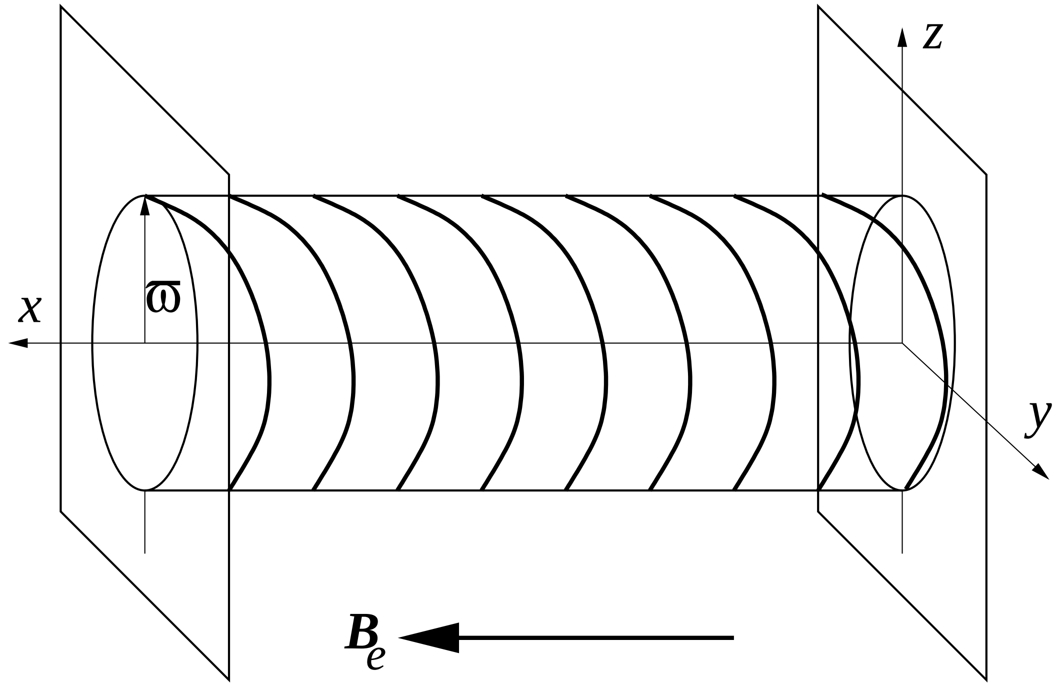
$$\omega_{2\mathrm{h}} = \frac{C_k[\pi^2(3\zeta+5)(6\alpha+1) - 6(\zeta+3)]}{24\pi^2 L(\zeta+1)}$$

where $L \approx 2x_0$.

$$\Delta\omega = \lambda(\omega_{2\mathrm{v}} - \omega_{2\mathrm{h}}) = \frac{\lambda C_k[\pi^2(\zeta-1)(6\alpha+1) - 6(3\zeta+1)]}{12\pi^2 L(\zeta+1)}$$

3. TRANSVERSE OSCILLATIONS OF NON-PLANAR LOOPS

EQUILIBRIUM STATE



We use Cartesian coordinates x, y, z with the vertical z-axis, and auxiliary cylindrical coordinates x, ϖ, φ, where $y = \varpi \cos\varphi$, $z = \varpi \sin\varphi$.

In $\varpi > \varpi_1$ magnetic field is in x-direction and has constant magnitude B_1.

In $\varpi > \varpi_1$ magnetic field is given by

$$B_x = \frac{q^2 B_0}{q^2 + \varpi^2}, \quad B_\varpi = 0, \quad B_\varphi = \frac{q\varpi B_0}{q^2 + \varpi^2}$$

$$\nabla \times \boldsymbol{B} = \frac{2q\boldsymbol{B}}{q^2 + \varpi^2} \quad \Longrightarrow \quad \text{magnetic field is force-free}$$

magnetic pressure is continuous at $\varpi = \varpi_1 \quad \Longrightarrow \quad B_1^2 = \dfrac{q^2 B_0^2}{q^2 + \varpi^2}$

Equation of a magnetic field line is

$$x = q\varphi + x_0, \quad y = \varpi_0 \cos\varphi, \quad z = \varpi_0 \sin\varphi$$

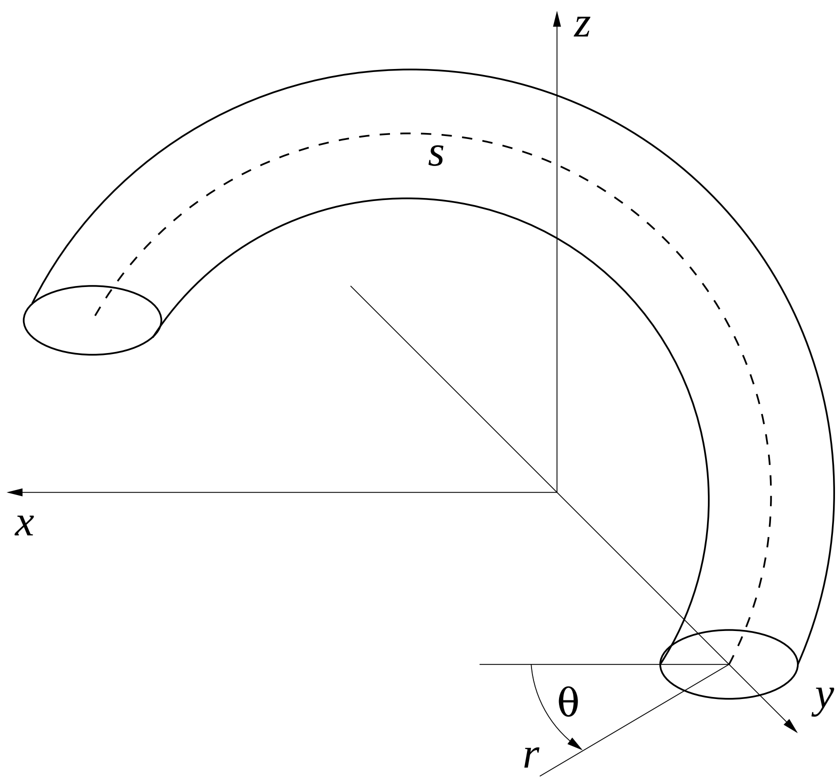
Each magnetic field line is invariant under helical map

$$\varphi \mapsto \varphi + \tilde{\varphi}, \qquad x \mapsto x + q\tilde{\varphi}$$

Loop axis coincides with magnetic filed line defined by

$$x = q\varphi, \quad y = R\cos\varphi, \quad z = R\sin\varphi$$

where $R < \varpi_1$ and $\varpi_1 - R \sim \varpi_1$.



Loop boundary consists of magnetic field lines passing through circle of radius a in plane perpendicular to loop axis at one footpoint. Helical invariance $\Longrightarrow$ loop cross-section at any point is a circle of radius a.

Curviliniar coordinates r, θ, s

In the lowest order approximation with respect to $\epsilon = a/R \ll 1$ this coordinate system reduces to ordinary cylindrical coordinates.

Using asymptotic analysis with ϵ as a small parameter we obtain

$$\frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} - C_k^2 \frac{\partial^2 \boldsymbol{\xi}}{\partial s^2} = 0, \quad \boldsymbol{\xi} = 0 \text{ at } s = 0, L$$

$$C_k^2 = \frac{B^2}{\mu_0[\rho_i(s) + \rho_e(s)]}, \quad \boldsymbol{\xi} = (\xi_r,\, \xi_\theta,\, 0)$$

Consider linearly polarized fundamental eigenmode determined by

$$\xi_r = f(s)\sin(\omega t)\sin(\theta + \theta_0), \qquad \xi_\theta = f(s)\cos(\omega t)\cos(\theta + \theta_0)$$

where

$$\frac{d^2 f}{ds^2} + \frac{\omega^2 f}{C_k^2(s)} = 0, \quad f(0) = f(L) = 0$$

The Cartesian coordinates of $\boldsymbol{\xi}$ are given by

$$\boldsymbol{\xi} = f(s)\left(\frac{R\sin\theta_0}{\sqrt{R^2+q^2}},\ \cos\theta_0\cos\varphi + \frac{q\cos\theta_0\sin\varphi}{\sqrt{R^2+q^2}},\right.$$

$$\left.\cos\theta_0\cos\varphi - \frac{q\sin\theta_0\cos\varphi}{\sqrt{R^2+q^2}}\right)$$

where $\varphi = s/\sqrt{R^2+q^2}$ and θ_0 is the angle between the y-axis and $\boldsymbol{\xi}$ at $s=0$, i.e. it determines the direction of polarization at one footpoint.

At apex point ($\varphi = \pi/2$) we have

$$\boldsymbol{\xi} = f(s)\boldsymbol{e}_0, \quad \boldsymbol{e}_0 = \left(\frac{R\sin\theta_0}{\sqrt{R^2+q^2}},\ \frac{q\cos\theta_0}{\sqrt{R^2+q^2}},\ \cos\theta_0\right)$$

If the line of sight is parallel to $\boldsymbol{e}_0$, than we see the node at the apex.

CONCLUSIONS

- Ratio of frequencies of vertical and horizontal oscillations depends strongly on a model. While these frequencies are practically the same in a half-torus model, they can be quite different in a model of loop embedded in a two-dimensional potential magnetic field.
- Kink oscillations of a non-planar loop are described by the same eigenvalue problem as those of a straight thin magnetic tube.
- Observational evidences of kink oscillations of non-planar loops can be strongly different from those of planar loops. In particular, if we observe only the displacement component perpendicular to the line of sight then the fundamental mode can look like the first overtone.