Magnetorotational processes in core-collapse supernovae.

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Outline

• Introduction
• Core-collapse supernova mechanisms and simulations.
• Magnetorotational (MR) mechanism of supernova explosion.
• MR supernova with initial quadrupole field, dipole field.
• Magnetorotational instability (MRI) in MR supernova.
• MR supernova with different core masses and rotation rates.
• Comparison of MR supernova simulations with other authors.
• What has to be done?
• Conclusions.
**Introduction**

Core collapse supernova explosions mechanisms and simulations

Spherically symmetrical model (neutrino deposition, shock wave formation). (Simulations by Colgate, White; Ivanova, Imshennik, Nadezhin).

Shock stalls at 100-200km. No explosion.

Observations show: supernova explosions are asymmetrical (SN 1987A).

**Neutrino convection and hydrodynamic instabilities:**

Neutrino convection *inside* protoneutron star --> increase of the energy of radiated neutrino. Does not help to explosions.

Neutrino convection *after* shock front (detailed simulations does not lead to the supernova explosion Mueller, Janka)

**Prononeutron star fragmentation mechanism** Imshennik, 3-D nature, fast rotation.

**Standing Accretion Shock Instability (SASI)** (Blondin, Mezzacappa, Janka, Yamada…). Self-consistent simulations do not give explosion with sufficient level of confidence.

**Acoustic supernova** (Barrows) neutron star oscillations. Energy is too small for explosions. (Papers by K. Sato group, our results)

**Magnetorotational supernova** (Bisnovatyi-Kogan, 1970). Rotation + magnetic field
Magnetorotational mechanism for the supernova explosion Bisnovatyi-Kogan (1970)(original article was submitted: September 3, 1969)

Amplification of magnetic fields due to differential rotation, angular momentum transfer by magnetic field. Part of the rotational energy is transformed to the energy of explosion

First 2D calculations: LeBlanck&Wilson (1970) (original article was submitted: September 25, 1969) -> too large initial magnetic fields. $E_{\text{mag}} \sim E_{\text{grav}}$ ⇒ axial jet


It is popular now!

The realistic values of the magnetic field are: $E_{\text{mag}} \ll E_{\text{grav}}$ ($E_{\text{mag}}/E_{\text{grav}} = 10^{-8}-10^{-12}$)

Small initial magnetic field -is the main difficulty for the numerical simulations.

The hydrodynamic time scale is much smaller than the magnetic field amplification time scale (if magnetorotational instability is neglected).

Explicit difference schemes can not be applied. (CFL restriction on the time-step).

Implicit schemes should be used.

The main difference between bounce shock, neutrino driven mechanisms and MR supernovae: magnetic field works like a piston. This MHD piston supports the supernova MHD shock wave for some time.
Basic equations: MHD +self-gravitation, infinite conductivity:

\[
\begin{align*}
\frac{dx}{dt} &= \mathbf{u}, \quad \frac{d\rho}{dt} + \rho \text{div}\, \mathbf{u} = 0, \\
\rho \frac{du}{dt} &= -\text{grad} \left( p + \frac{\mathbf{H} \cdot \mathbf{H}}{8\pi} \right) + \frac{1}{4\pi} \text{div}(\mathbf{H} \otimes \mathbf{H}) - \rho \text{grad}\Phi \\
\rho \frac{d\varepsilon}{dt} + p \text{div}\, \mathbf{u} + \rho F(\rho, T) &= 0, \\
\Delta \Phi &= 4\pi G\rho, \\
\rho \frac{d}{dt} \left( \frac{\mathbf{H}}{\rho} \right) &= \mathbf{H} \cdot \nabla \mathbf{u}.
\end{align*}
\]

Additional condition div\(\mathbf{H}=0\)

Axis symmetry (\(\frac{\partial}{\partial \phi} = 0\)) and equatorial symmetry (\(z=0\)) are supposed.

Notations:

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla, \quad \mathbf{x} = (r, \varphi, z), \quad \mathbf{u} - \text{velocity}, \quad \rho - \text{density}, \quad p - \text{pressure},
\]

\(\mathbf{H}\) – magnetic field, \(\Phi\) – gravitational potential, \(\varepsilon\) – internal energy,

\(G\) – gravitational constant.
Presupernova Core Collapse

Equations of state take into account degeneracy of electrons and neutrons, relativity for the electrons, nuclear transitions and nuclear interactions. Temperature effects were taken into account approximately by the addition of radiation pressure and an ideal gas

Neutrino losses were taken into account in the energy equations.

A cool white dwarf was considered at the stability limit with a mass equal to the Chandrasekhar limit.

To obtain the collapse we increase the density at each point by 20% and we also impart uniform rotation on it.
Initial state

\[ M = 1.2042 \cdot M_{\text{sun}}, \] spherically symmetrical stationary state, initial angular velocity \( 2.519 \) (1/sec)

Initial temperature distribution \( T = \delta \rho^{2/3} \)

\[
\frac{E^{\text{rot}}}{E^{\text{grav}}} = 0.571\% \\
\frac{E^{\text{int}}}{E^{\text{grav}}} = 72.7\%
\]
Maximal compression state

Max. density = $2.5 \cdot 10^{14} \text{g/cm}^3$
Angular velocity (central part of the computational domain). Rotation is differential.

TIME = 4.15163360 (0.14340067 sec)
The period of rotation of the young neutron star is about 0.001-0.003 sec

Rapidly rotating

pre-SN

Crab pulsar P=33ms – rapid rotation at birth

Core collapse

in binaries
Initial magnetic field – quadrupole-like symmetry

Toroidal magnetic field amplification.

pink – maximum_1 of $Hf^2$ blue – maximum_2 of $Hf^2$
Maximal values of $Hf=2.5 \cdot 10^{16} G$

After SN explosion at the surface of neutron star $H=2 \cdot 10^{14} G$
Temperature and velocity field

Specific angular momentum
Time evolution of different types of energies
Ejected energy and mass

Ejected energy \( 0.6 \cdot 10^{51} \text{ erg} \)

Ejected mass \( 0.14 \text{M}_\odot \)

Particle is considered “ejected” –

if its kinetic energy is greater than its potential energy
Initial magnetic field – dipole-like symmetry

Magnetorotational explosion for the dipole-like magnetic field
Ejected energy and mass (dipole)

Ejected energy \( \approx 0.5 \cdot 10^{51} \text{erg} \)
Ejected mass \( \approx 0.14M_\odot \)

Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy
Magnetorotational supernova in 1D
(no MRI)
Bisnovaty-Kogan et al. 1976, Ardeljan et al. 1979

\[ t_{\text{explosion}} \sim \frac{1}{\sqrt{\alpha}}, \quad \left( \alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}} \right) \]

Example: \( \alpha = 10^{-2} \Rightarrow t_{\text{explosion}} = 10, \)

\( \alpha = 10^{-12} \Rightarrow t_{\text{explosion}} = 10^6 !!! \)

**FIG. 3.** Shape of a field line in the region near the core at the time \( t_{\alpha} = 7 \)
for \( \alpha = 10^{-2} \) (dashed line) and \( \alpha = 10^{-4} \) (solid line).
Magnetorotational explosion for the different $\alpha = \frac{E_{mag0}}{E_{grav0}} = 10^{-2} - 10^{-12}$

Magnetorotational instability (MRI) $\Rightarrow$ mag. field grows exponentially

Dependence of the explosion time from $\alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}}$

$t_{\text{explosion}} \sim -\log(\alpha)$ (for small $\alpha$)

Example:

$\alpha = 10^{-6} \Rightarrow t_{\text{explosion}} \sim 6,$

$\alpha = 10^{-12} \Rightarrow t_{\text{explosion}} \sim 12.$
Magnetorotational instability

Central part of the computational domain. Formation of the MRI.
Toy model for MRI in the magnetorotational supernova

\[ \frac{dH_\varphi}{dt} = H_r \left( r \frac{d\Omega}{dr} \right); \quad \text{at the initial stage of the process } \quad H_\varphi < H_\varphi^*: \quad H_r \left( r \frac{d\Omega}{dr} \right) = \text{const}, \]

beginning of the MRI => formation of multiple \textit{poloidal} differentially rotating vortexes

\[ \frac{dH_r}{dt} = H_{r0} \left( \frac{d\omega_v}{dl} \right), \quad \text{in general we may approximate:} \quad \left( \frac{d\omega_v}{dl} \right) \approx \alpha (H_\varphi - H_\varphi^*). \]

Assuming for the simplicity that \((r \frac{d\Omega}{dr}) = A\) is a constant during the first stages of MRI, and taking \(H_\varphi^*\) as a constant we come to the following equation:

\[ \frac{d^2}{dt^2} \left( H_\varphi - H_\varphi^* \right) = A H_{r0} \alpha (H_\varphi - H_\varphi^*) \]

\[ \downarrow \]

\[ \begin{cases} 
H_\varphi = H_\varphi^* + H_{r0} e^{\sqrt{A\alpha H_{r0}} \left( t - t^* \right)}, \\
H_r = H_{r0} + \frac{H_{r0}^{3/2} \alpha^{1/2}}{\sqrt{A}} \left( e^{\sqrt{A\alpha H_{r0}} \left( t - t^* \right)} - 1 \right). 
\end{cases} \]
MR supernova – different core masses and rotation rates


Dependence of the MR supernova explosion energy on the core mass and initial angular momentum

Solid line – initial angular velocity = 3.53s\(^{-1}\)
Dashed line - initial angular velocity = 2.52s\(^{-1}\)
The magnetorotational supernova explosion is always asymmetrical.

while

Jet, kick and axis of rotation are aligned in MR supernovae.

Evidence for alignment of the rotation and velocity vectors in pulsars

“We present strong observational evidence for a relationship between the direction of a pulsar's motion and its rotation axis. We show carefully calibrated polarization data for 25 pulsars, 20 of which display linearly polarized emission from the pulse longitude at closest approach to the magnetic pole...

we conclude that the velocity vector and the rotation axis are aligned at birth“.
First 3D simulations of MR supernova (simplified)


(strong initial magnetic field, simple EoS, no neutrino transport)

Rotational axis and jet axes are aligned!

1 million seconds Chandra survey of Cas A. Second jet was found.
Some comments of MR supernova simulations by other groups.

Different equations of state.

Eulerian variables scheme - strongly collimated jets. *Sato et al.*
Lagrangian variables scheme (mild collimation). *Barrows et al.*

MRI appearance depends on spatial resolution. If the best size of the cell is less ~120m -> MRI can appear.
What has to be done?

Implementation of modified (Shen et al.) equation of state (in process).

To take into account transfer of momentum from neutrino in presence of strong magnetic field. (Gvozdev, Ognev).

More detailed neutrino transport simulations (for example by Monte-Carlo method).

Simulations of mirror symmetry violation (asymmetrical jet formation).

Simulations of MR supernova in Full General Relativity.
Conclusions

- Magnetorotational mechanism (MRM) produces enough energy for the core collapse supernova.
- The MRM is weakly sensitive to the neutrino cooling mechanism.
- MR supernova shape depends on the configuration of the magnetic field and is always asymmetrical.
- MRI develops in MR supernova explosion.
- One-sided jets and rapidly moving pulsars can appear due to MR supernovae.
- 3D simulations of MR supernova with full physics are necessary.
Thank you!