Dephasing due to the interaction with chaotic degrees of freedom

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We consider the motion of a particle, taking into account its interaction with environmental degrees of freedom. The dephasing time is determined by the nature of the environment and depends on the particle velocity. Our interest is in the case where the environment consists of few chaotic degrees of freedom. We obtain results for the dephasing time and compare them with those of the effective-bath approach. The latter approach is based on the conjecture that the environment can be modeled as a collection of infinitely many harmonic oscillators. The work is related to studies of driven systems, quantum irreversibility, and fidelity. The specific model that we consider requires the solution of the problem of a particle in a box with a moving wall, whose one-dimensional version is related to the Fermi acceleration problem.

I. INTRODUCTION

Determining the dephasing (decoherence) time τ_φ for a particle (x,p) that interacts with an environmental degree of freedom (Q,P) is a central theme in quantum physics. In the absence of such interaction the x motion is coherent, and interference should be taken into account. This means, from a semiclassical point of view, that at least two trajectories $x(\tau) = x_A(\tau)$ and $x(\tau) = x_B(\tau)$ have a leading contribution to the probability to travel, say, from $x(0)$ to $x(t)$, as in the prototype example of the two-slit experiment.

The purpose of this paper is to discuss the case where the particle interacts with few chaotic degrees of freedom. To be more specific we consider a box/piston model that is defined in Sec. II below. It is already known that in the classical descriptions such interaction leads to dissipation, and the motion of the particle is described by the standard Langevin equation [1]. Our aim is to explore the quantum-mechanical consequences of this interaction and, in particular, to determine the dephasing time.

A related aim of this paper is to test the effective-bath conjecture, namely, that any type of environment can be modeled as a large collection of harmonic oscillators. This conjecture is implied by leading order perturbation theory [2]. We shall explain how this conjecture can be applied, in practice, in order to rederive our results for the dephasing time.

Some readers may wonder whether dephasing is not just the entanglement of the particle with some other degrees of freedom. In such a case, even a one-spin environment can provide dephasing (decoherence). However, we would like to adopt a more restrictive definition of dephasing, which involves the notion of irreversibility (see the Appendix).

At first sight it seems that for having irreversibility one needs "infinity." This point of view is emphasized in Ref. [3]: Irreversibility can be achieved by having an infinity of the bath (infinitely many oscillators) or of space (a lead that extends up to infinity). The present paper is based on the observation that also chaos provides irreversibility. We do not need infinity in order to have irreversibility. This conceptual point is further clarified in the Appendix.

The present work is strongly related to so-called studies of quantum irreversibility [4] and fidelity [5], and hence to studies of wave packet dynamics [6], decay of the survival amplitude, and the parametric theory of the local density of states [7–9]. We explain this point in Sec. VII. We would like to emphasize that all these studies share a common conceptual framework. The common idea [10] is that we have to distinguish between perturbative and nonperturbative regimes, and that the semiclassical (sub) regime is contained within the nonperturbative regime.

The study of the box/piston model reduces to the analysis of the problem of a driven particle. Namely, the problem of a particle in a box with a moving wall. This is possibly the simplest example for illustrating the idea of having different dynamical regimes. The velocity of the wall is V. We shall explain that in the one-dimensional (1D) box problem (also known as the problem of infinite well with moving wall [11,12]), there are two quantum-mechanical V regimes: An adiabatic V regime, and a nonperturbative semiclassical V regime. In the general box/piston problem there are three regimes: The additional V regime is a perturbative regime, where we can apply the Fermi golden rule in order to describe the stochastic energy spreading.

II. INTERACTION WITH CHAOS

One can imagine, in principle, a "zoo" of models that describe interaction of a particle with the environment. However, following Caldeira and Leggett [13], the guiding philosophy is to construct "ohmic models" that give Brownian motion (described by the standard Langevin equation) in the classical limit. Three families of models are of particular interest: interaction with chaos, interaction with harmonic bath, interaction with random matrix bath.

The second type of modeling leads to the introduction of the diffusion-localization-dissipation (DLD) model [14]. The familiar Zwanzig-Caldeira-Leggett (ZCL) model [13] can be regarded as a special limit of the DLD model. The physics of the ZCL and DLD models is illustrated in Figs. 1(a) and 1(b), respectively. The ZCL model describes a motion under the influence of a fluctuation homogeneous field of force. In the case of the DLD model the fluctuation field is further characterized by a finite correlation distance. We shall come back
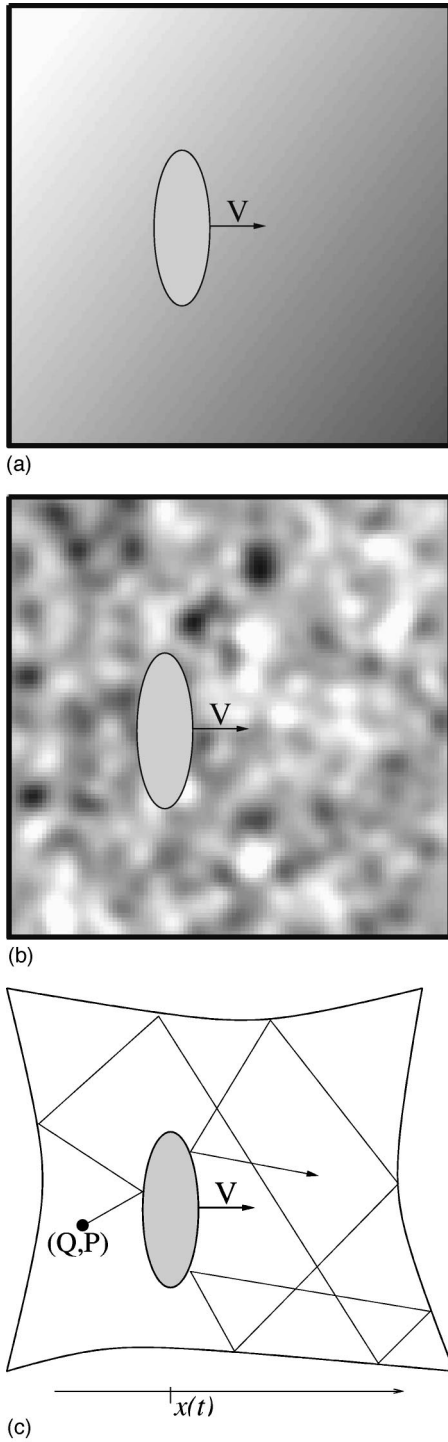
(a)

(b)

(c)

FIG. 1. (a) The Brownian particle in the ZCL model experiences a fluctuating homogeneous field of force. (b) In case of the DLD model the fluctuating field is further characterized by a finite correlation distance. (c) The Brownian motion is induced due to the interaction with chaotic degrees of freedom. $x(t)$ is the (classical) position of the Brownian particle. In (a) and (b), The background image is a "snapshot" of the fluctuating environment. Namely, the gray levels correspond to the "height" of an instantaneous potential experienced by the Brownian particle.

to these models in Sec. X. For completeness we note that random-matrix modeling of the environment, in the regime where it has been solved [18], leads to the same results as those obtained for the DLD model.

In this paper we are going to consider the case where the particle x interacts with few chaotic degrees of freedom. It is well known that *classically* such an interaction has the same effect as that of coupling to an ohmic bath [1]. Quantum mechanically much less is known [19].

We shall analyze the prototype toy model illustrated in Fig. 1(c). The dynamical variable x represents the position of a large ("Brownian") particle. The motion of this particle is affected by collisions with a small "gas" particle. Thus Q is the position of the gas particle and P is the conjugate momentum. The motion of the gas particle is assumed to be chaotic and its collisions with the Brownian particle are assumed to be elastic. The typical time between successive collisions will be denoted by $\tau_0 = L/v_E$, where v_E is the typical velocity of the gas particle. The typical kinetic energy of the gas particle $E = \frac{1}{2}mv_E^2$ has the same significance as the temperature T in the ZCL/DLD models. (For sake of exact comparison we should assume that E has a canonical distribution, but for the purpose of this presentation we prefer to assume that it has some well-defined microcanonical value).

III. DYNAMICAL REGIMES

The results of the forthcoming analysis depend on the typical velocity $V = |\dot{x}|$ of the Brownian particle. A fixed assumption of this paper is that V is slow in a classical sense, meaning $V \ll v_E$. In the quantum-mechanical analysis we are going to distinguish the following quantal V regimes:

$$\text{adiabatic regime} \quad V \ll \left(\frac{\lambda_E^{d-1}}{A} \right)^{3/2} \frac{\hbar}{mL}, \tag{1}$$

$$\text{nonperturbative regime} \quad V \gg \frac{\hbar}{mL}, \tag{2}$$

where d is the dimensionality of the box and A is the effective area of the $(d-1)$ dimensional surface of the Brownian particle. The de Broglie wavelength λ_E of the gas particle is defined as in Eq. (12). For $d>1$ the above two V regimes are well separated, and we have a third "perturbative" regime where V is small $[\ll \hbar/(mL)]$ but nonadiabatic.

The naive semiclassical expectation, regarding the system of Fig. 1(c) is to have a loss of coherence upon collision. In other words, we expect to have

$$\tau_\varphi = \tau_0. \tag{3}$$

We want to go beyond this naive expectation; to obtain specific results for the dephasing time; and to compare them with the prediction that is based on the effective-bath conjecture.

Specifically, we are going to claim that the naive semiclassical result is valid only in the nonperturbative regime. Otherwise, in the perturbative regime, we get that the dephasing time is much larger,

$$\tau_\varphi \approx \left(\frac{L\lambda_E^2}{v_E V^2} \right)^{1/3}. \qquad (4)$$

Finally, in the adiabatic regime, to the degree that adiabaticity is maintained, there is no dephasing at all. However, in practice one should take mainly Landau-Zener transitions into account [15] (see also Sec. 20 of [16]). This leads to a finite dephasing time $\tau_\varphi \propto (1/V)^{1+(\beta/2)}$, where the value of β depends on the level spacing statistics.

IV. DEFINITION OF THE DEPHASING FACTOR

The Hamiltonian of the system and environment can be written as

$$\mathcal{H}_{\text{total}}(x,p,Q,P) = \mathcal{H}_0(x,p) + \mathcal{H}(Q,P;x). \qquad (5)$$

Using this Hamiltonian we should be able to calculate, in principle, any transport property. To test whether "interference" is present we should be able to control this interference. For example, in a two-slit geometry we control the relative position of the detector, while in the Aharonov-Bohm ring geometry (see the Appendix) we control the magnetic flux via the ring.

A more restricted definition of dephasing can be obtained within the framework of the Feynman-Vernon formalism [2]. After elimination of the bath degrees of freedom, one ends up with a *double* path integral for the transport. This expression is not very illuminating unless the x motion can be treated semiclassically [17]. In such a case it becomes a double sum over "classical" trajectories, and we can interpret the "off diagonal" terms as responsible for the interference effect. Due to the elimination of the bath degrees of freedom, each interference term is multiplied by a factor that is known as the "influence functional." The absolute value of the influence functional is defined as the "dephasing factor."

The influence functional is traditionally calculated for linear coupling to harmonics oscillators, assuming that they are initially prepared in a canonical thermal state. However, we would like to consider the case of interaction with chaotic degrees of freedom, and we would like to assume that the "environment" is initially prepared in a pure state Ψ_0. Thus the "bath" is characterized by its microcanonical energy E rather by its temperature T. (Obviously one can obtain the thermal case by canonical averaging over E.)

In order to calculate the influence functional, one considers the evolution that is generated by the time-dependent Hamiltonian

$$\mathcal{H} = \mathcal{H}(Q,P;x(\tau)) \qquad (6)$$

for the particular (interfering) trajectories $x(\tau) = x_A(\tau)$ and $x(\tau) = x_B(\tau)$ that connect $x(0)$ and $x(t)$. The initial preparation of the environment is represented by a wave function Ψ_0, while the final state is either $\Psi(t) = \Psi_A$ or $\Psi(t) = \Psi_B$. The overlap of the two possibilities is known as the influence functional

$$c(t) = F[x_A, x_B] = \langle \Psi_B | \Psi_A \rangle. \qquad (7)$$

The dephasing factor is defined as the absolute value of the influence functional.

We have introduced above the alternate notation $c(t)$ in order to emphasizes the time dependence of the wave function overlap. In order to make $c(t)$ a well-defined quantity, one should predefine the statistical properties of the interfering trajectories as a function of t. See [20] for mathematical details. For the purpose of the present paper one can assume that typical interfering trajectories are ballistic and characterized by their velocity V, and by their (maximal) separation $A \sim Vt$. This motion scheme provides "maximal dephasing." For other schemes of motion the maximal separation scales differently. For example, for two-slit geometry A is the separation between the slits. For diffusive trajectories $A \propto \sqrt{t}$. Note, however, that a full statistical specification is needed in each case [20].

V. REMARK: INTERPRETATION OF THE DEPHASING FACTOR

From the definition of the influence functional it is clear that it reflects the probability to "leave a trace" in the environment. In case of the DLD model (see Sec. X) this trace can be further interpreted as leaving an excitation along the way. For a critical discussion of this point see Appendix C of [20]. In the more general case the notion of leaving a trace is somewhat ambiguous. All we can say is that decoherence means that the environment is left in different (orthogonal) states depending on the trajectory that is taken by the particle.

The law of action and reaction holds also in the world of decoherence studies. Feynman and Vernon have realized that the dephasing factor can be reinterpreted as representing the effect of a c-number noise source. From this point of view the decoherence is due to the "scrambling" of the relative phase by this noise. Hence the reason for using the term "dephasing" as a synonym for "decoherence."

The advantage of the latter point of view is that it can give further insight regarding the physics of our results. Namely, we shall see in Sec. XI that Eqs. (3) and (4) have effective DLD-model and ZCL-model interpretations, respectively. From the particle's dynamics point of view this corresponds to "scattering mechanism" and to "spreading mechanism" as discussed in [20].

VI. DETERMINATION OF THE DEPHASING TIME

Within the semiclassical framework, the problem of dephasing reduces to the more restricted problem of studying the dynamics of a time-dependent Hamiltonian [Eq. (6)].

By definition, in order to have coherence ($|c(t)| \sim 1$), the wave function $\Psi(t)$ should contain a component that is independent of the particular way in which $x(\tau)$ evolves from the initial $x(0)$ to the final $x(t)$. Loss of coherence means $|c(t)| \ll 1$, which can be written as $t < \tau_\varphi$. This constitutes a practical definition of the dephasing time τ_φ.

It should be clear that the toy system of Fig. 1(c) is completely equivalent to the toy system that is illustrated in Fig. 2. Namely, what we have to analyze is the evolution that is
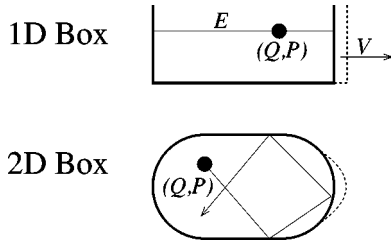
FIG. 2. A "gas" particle in a box is driven by moving the wall. This is essentially the same as Fig. 1(c), but now the moving wall is *not* regarded as a dynamical entity. (a) illustrates the case of one-dimensional box, while (b) is for *chaotic* box. (Hence, in the latter case, the box should be at least two dimensional.)

generated by $\mathcal{H}(Q,P;x(\tau))$, where x controls the deformation of the boundary. In particular, we should determine whether $\Psi(t)$ possesses a trajectory-independent component that is determined only by the endpoints $x(0)$ and $x(t)$.

The rest of this paper is organized as follows. In Sec. VIII we illuminate the key ingredients in the analysis by considering a one-dimensional example. In Sec. IX we outline the derivation of Eqs. (3) and (4) using the core-tail picture that has been developed in [16], and that is supported by our recent numerical studies [8,9,21]. In Sec. XI, we compare the results to those of the effective-bath approach.

VII. DIGRESSION: RELATION TO FIDELITY AND LDOS STUDIES

The definition of $c(t)$ can be rewritten formally as

$$c(t) = \langle \Psi_0 | U[x_B(\tau)]^{-1} U[x_A(\tau)] | \Psi_0 \rangle, \qquad (8)$$

where $U[x(\tau)]$ is the evolution operator due to a driving "pulse" $x(\tau)$. Thus $c(t)$ can be reinterpreted as the probability amplitude to come back to the initial state at the end of a "driving cycle." This quantity, which quantifies the "fidelity" of the driving cycle, has been suggested by Ref. [4] to be a measure for quantum irreversibility.

To be more precise, the original definition of the fidelity in Ref. [4] assumes rectangular "pulses." This means, without loss of generality, that $x_A(\tau) = 0$ and $x_B(0 < \tau < t) = A$ = const. Thus the time variation of $c(t)$ depends on the amplitude A of the "perturbation." Further simplification is obtained if Ψ_0 is assumed to be an eigenstate of the unperturbed Hamiltonian $\mathcal{H}_0 = \mathcal{H}(Q,P;0)$. In such a case $c(t)$ is known as the survival amplitude and we get

$$|c(t)|^2 = \left| \left\langle \Psi_0 \left| \exp\left(-\frac{i}{\hbar} \mathcal{H} t \right) \right| \Psi_0 \right\rangle \right|^2, \qquad (9)$$

where $\mathcal{H} = \mathcal{H}(Q,P;A)$ is the perturbed Hamiltonian. The study of the survival probability is also known as "wave packet dynamics." Note that the survival probability is the Fourier transform of the local density of states (LDOS). Hence the study of wave packet dynamics can be reduced to LDOS study. For an introduction to this subject see [6].

The fidelity in general and the survival amplitude in particular, have similar physics [28]. In Ref. [4] it is explained

that if A is small (in the sense of standard first-order perturbation theory), then the decay of $c(t)$ has a Gaussian time dependence due to a statistical effect. For larger A we get exponential time dependence as in Wigner theory. On the other hand in semiclassical circumstances [6] the decay may become *perturbation independent*. (This idea was generalized in Ref. [5]). In such a case the rate of the decay is determined by the stability of the classical motion, and is characterized by the Lyapunov exponent. A unified picture of the crossover from the perturbative A regime to the semiclassical A regime has been presented in [7], and has been generalized in [29]. See also [28].

Does the study of "rectangular pulses" constitute a good bridge for developing a general theory for fidelity? The answer is definitely not. An essential ingredient in the theory of driven systems is the rate V in which the parameter x is being changed in time. Thus, rather than talking about A regimes, we should talk about V regimes, as in the present paper. The general theory becomes more complicated [10,19,16], but the physical picture is similar in spirit. The box/piston model that we are going to analyze in the following sections is possibly the simplest demonstration for the applicability of the ideas that were presented in [10,19,16]. More generally we should talk about (V,A) regimes, as in the theory of periodically driven mesoscopic systems [22]. The latter issue is beyond the scope of the present paper.

VIII. ANALYSIS OF THE 1D BOX PROBLEM

Consider the one-dimensional system of Fig. 2(a). The classical analysis of the dynamics is trivial. Each time the gas particle collides with the moving wall it loses energy: Upon collision its velocity undergoes a change $v \mapsto -v + 2V$ and therefore the change in energy is $dE_{col} = -2mvV$.

Now we want to analyze the dynamics quantum mechanically. This turns out to be less trivial. Past studies of this model [11,12], which are related to the interest in the Fermi accelerator problem [23], were focused on the issue of finding stationary solutions. To the best of our knowledge, the time-dependent picture has not been explored.

Let $|n(x)\rangle$ denote the eigenstates of the box Hamiltonian $\mathcal{H}(Q,P;x)$. The expansion of the wave function in this x-dependent basis is $\Psi(t) = \Sigma_n a_n(t)|n(x(t))\rangle$. The expansion coefficients $a_n(t)$ are the probability amplitudes to find the particle in the energy level n after time t. One easily obtains the equation

$$\frac{da_n}{dt} = -\frac{i}{\hbar} E_n a_n - \frac{V}{L} \sum_{m(\neq n)} \frac{2nm}{n^2 - m^2} a_m. \qquad (10)$$

Let us assume that the initial preparation is $a_n(0) = \delta_{nm}$. The mean level spacing for the 1D box is $\Delta = \pi \hbar v_E / L$. If $dE_{col} \ll \Delta$ one finds out, by inspection of Eq. (10), that the dynamics is adiabatic, meaning that $a_n(t) \sim \delta_{nm}$. On the other hand, if $dE_{col} \gg \Delta$, one expects to find a semiclassical transition $E \mapsto E + dE_{col}$.

How can we explain the $E \mapsto E + dE_{col}$ transition from the quantum-mechanical point of view? For this purpose we can

adopt the core-tail picture of Ref. [16]. The core-tail picture is a generalization of the Fermi-golden-rule picture: The "core" consists of the levels that are mixed nonperturbatively; the "tail" is formed by first-order transitions from the core.

The analysis is carried out in two steps. The first step is to analyze the *parametric evolution* that is associated with Eq. (10). This means to solve Eq. (10) without the first term in the right-hand side (RHS) (This is the so-called sudden limit.) Obviously the resultant $\tilde{a}_n(t)$ is a function of $\delta x = Vt$, while V by itself makes no difference. The solution depends only on the end points $x(0)$ and $x(t)$. The second step is to analyze the actual time evolution. This means to take into account the effect of the first term in the RHS of Eq. (10), and to understand how the resultant $a_n(t)$ differs from $\tilde{a}_n(t)$.

By careful inspection of Eq. (10) one observes that a level is mixed with the next level whenever the wall is displaced an additional distance $\lambda_E/2$. This effect can be regarded as "parametric." Further inspection reveals that this nonperturbative (parametric) effect modulates the core-to-tail transition amplitude (see remark [24]). The modulation frequency is $\omega = 2 \times 2\pi/(\lambda_E/V)$. This frequency drives a core-to-tail resonance transition $|dE| = \hbar\omega$, in agreement with the semiclassical expectation.

IX. ANALYSIS OF THE GENERAL BOX PROBLEM

The strength of the core-tail picture is that it can be used to analyze the more general case, which is illustrated in Fig. 2(b). We assume that the motion of the particle inside the box is chaotic in the classical limit. The quantum-mechanical analysis follows the same steps as in the 1D problem, and requires the use of results that we have obtained in previous publications (mainly [9]). In order to keep the presentation illuminating, and trying to avoid duplications, we shall just sketch the derivation.

On the basis of the analysis of Ref. [9] we recall that there are two relevant parametric scales,

$$\delta x_c^{\mathrm{qm}} \approx \left(\frac{\lambda_E^{d-1}}{A} \right)^{1/2} \times \lambda_E, \qquad (11)$$

$$\delta x_c^{\mathrm{cl}} = \lambda_E = (2\pi\hbar)/(mv_E). \qquad (12)$$

The first parametric scale tells us what is the displacement δx that is required to mix neighboring levels. The second parametric scale determines what is the "linear" regime of this deformation process, and marks the parametric crossover from the perturbative to the (nonuniversal) semiclassical regime. For more details see [9].

The existence of two distinct parametric scales implies [16] that there are three V regimes in the problem: The most trivial one is the adiabatic regime [Eq. (1)], which is defined via the requirement

$$Vt_H \ll \delta x_c^{\mathrm{qm}}, \qquad (13)$$

where $t_H = 2\pi\hbar/\Delta$ is the Heisenberg time for recurrences. From this definition it is clear that quantum recurrences start before the probability goes to other levels. As a results, in leading order description, the probability remains concentrated all the time in the initial level. From now on we assume without saying that we are in the nonadiabatic V regime(s).

The nonperturbative semiclassical regime [Eq. (2)] is defined by the requirement

$$V\tau_0 \gg \delta x_c^{\mathrm{cl}}. \qquad (14)$$

If V is nonadiabatic on the one hand, and not large in the sense of Eq. (2) on the other hand, then, using the terminology of [16], we are in the (extended) perturbative regime. In the 1D case δx_c^{qm} and δx_c^{cl} coincide, and therefore the perturbative regime is absent. We turn now to explain the mechanism for energy spreading in the (extended) perturbative regime. We shall call it the Fermi-golden-rule picture. After that we explain how to obtain the semiclassical picture that arises in the nonperturbative regime.

After displacement $\delta x = Vt$ of the wall, the number of levels that become mixed nonperturbatively [16] is $(\delta x/\delta x_c^{\mathrm{qm}})^2$. (In general [8], there may be nonuniversal deviations from this quadratic growth, leading straightforwardly to possible generalization of our results.) Thus we can define the core width (in energy units) as

$$\Gamma(\delta x) = \left(\frac{\delta x}{\delta x_c^{\mathrm{qm}}} \right)^2 \times \Delta, \qquad (15)$$

where Δ is the mean level spacing. As explained in Ref. [9], this nonperturbative mixing proceeds as long as $\delta x < \delta x_c^{\mathrm{cl}}$ and provided $\Gamma \ll \hbar/t$.

In the (extended) perturbative regime the inequality $\Gamma \ll \hbar/t$ breaks down before $\delta x \sim \delta x_c^{\mathrm{cl}}$. This determines the dephasing time of Eq. (4). Coherence is maintained for $t < \tau_\varphi$ because most of the probability is still concentrated in the core, whose evolution is of parametric nature. In other words, as long as the core is not resolved ($\Gamma \ll \hbar/t$), its evolution depends only on the end points $x(0)$ and $x(t)$.

What about the evolution of the tails? A general argumentation (see Sec. 16 of Ref. [16]) implies that as long as $\delta x < \delta x_c^{\mathrm{cl}}$, the core-to-tail transitions are not affected by the local nonperturbative mixing of the levels (no modulation of the core-to-tail transition amplitude). By definition, in the extended perturbative regime the core is resolved much before we get $\delta x \sim \delta x_c^{\mathrm{cl}}$. Consequently, we can use the Fermi-golden-rule picture in order to describe the crossover to stochastic diffusion in energy [10,9,16].

A different picture arises in the nonperturbative regime. From the definition of this regime it follows that we still have $\Gamma \ll \hbar/t$ at the time when $\delta x \sim \delta x_c^{\mathrm{cl}}$. In this case the core saturates to a semiclassically determined profile (see details in Sec. 10 of [9]), having the width

$$\Gamma \sim \hbar/\tau_0. \qquad (16)$$

The time to resolve this width is τ_0. Consequently the dephasing time is simply $\tau_\varphi = \tau_0$, which is the naive semiclassical result. As for the core-to-tail transitions: These are modulated as in the analysis of the one-dimensional case. Consequently the long-time dynamics in the nonperturbative regime is of semiclassical (rather then of the Fermi-golden-rule) nature.

X. THE DLD AND THE ZCL MODELS

Following Feynman and Vernon it is common to model the environment as a huge collection $Q = \{Q_\alpha\}$ of harmonic oscillators. The advantage of such modeling is obviously the ability to make an exact elimination of the environmental degrees of freedom, and to end up with a simple path-integral expression for the (reduced) propagator of the particle.

In case of the ZCL model the interaction of the particle x with the environmental degrees of freedom Q_α is expressed as $\mathcal{H}_{\mathrm{int}} = x \Sigma_\alpha c_\alpha Q_\alpha$, where c_α are coupling constants. Thus, in case of the ZCL model the particle experiences fluctuations of a homogeneous field of force [Fig. 1(a)].

In case of the DLD model the interaction with Q_α is expressed as $\mathcal{H}_{\mathrm{int}} = \Sigma_\alpha c_\alpha Q_\alpha u(x - x_\alpha)$, where $u(r)$ is a short-range interaction and x_α is the location of the α oscillator. Thus, in case of the DLD model the particle experiences fluctuations of a disordered field of force [Fig. 1(b)]. A specific example for the application of the DLD model is the dephasing of an electron due to its Coulomb interaction with the rest of the Fermi sea [17,25].

The long-time classical motion of the Brownian particle, for all three models of Fig. 1, is described by the same Langevin equation $m\ddot{x} = -\eta\dot{x} + \mathcal{F}$, where η is called the friction coefficient, and $\mathcal{F}$ can be regarded as arising from a stochasticlike fluctuating field of force. The fluctuating force is characterized by an intensity ν, which is related to η via a fluctuation-dissipation (FD) relation [26]. In case of the DLD model, the fluctuating field [Fig. 1(b)] is further characterized by a correlation distance ℓ, which is determined by the range of the interaction $u(r)$.

XI. THE EFFECTIVE-BATH CONJECTURE

If we make the conjecture that the system of Fig. 1(c) is effectively described by the DLD model, then we should substitute

$$(\nu)_{\mathrm{effective}} = m^2 v_E^3 / L, \tag{17}$$

$$(\ell)_{\mathrm{effective}} = \delta x_c^{\mathrm{cl}} = \lambda_E. \tag{18}$$

For getting Eq. (17) see details in Sec. 7 of Ref. [16]. For getting Eq. (18) see the detailed discussion in Sec. 11 of [9].

The dephasing time in the high-temperature limit of the DLD model is given by [14]

$$\tau_\varphi = \frac{\hbar^2}{\nu \ell^2} \quad \text{for } T \gg \hbar \frac{V}{L}. \tag{19}$$

Upon substitution of Eqs. (17) and (18) into Eq. (19), we get $\tau_\varphi = \tau_0$, which is the naive semiclassical result. With the identification $E \leftrightarrow T$, the high-temperature condition of Eq. (19) becomes $V \ll v_E$, which is just the classical slowness condition that we assume in any case.

Equation (19) becomes nonapplicable if $V\tau_\varphi < \ell$. In such a case the dephasing happens before δx gets to ℓ and therefore the DLD model reduces to the ZCL model. One easily verifies that the above distinction between ZCL and non-ZCL regimes formally coincides with Eq. (2): What appears to be nonperturbative in case of Fig. 1(c) appears as a non-ZCL feature in the effective-bath description.

Is it possible to give an effective-bath interpretation to Eq. (4)? The answer is positive. Using the same procedure to estimate τ_φ for the ZCL model as in [20], and upon using the estimate $|x_A(\tau) - x_B(\tau)| \sim Vt$, one obtains

$$\tau_\varphi = \left(\frac{\hbar^2}{\nu V^2}\right)^{1/3}. \tag{20}$$

Substitution of Eq. (17) gives Eq. (4). It is important to realize that these results are the "worst case estimates." We have assumed that only the core component is capable of maintaining coherence.

Using the effective-bath approach it is easier to get a heuristic (phase-space based) understanding of why τ_φ can be much longer compared with the above estimate. In the ZCL regime, the actual value of the dephasing time is quite sensitive to the geometry of the interfering trajectories. See further discussion of dephasing via the "spreading mechanism" in [20].

APPENDIX: IRREVERSIBILITY VERSUS RECURRENCES

In order to avoid confusion it is better to distinguish between the notions of "irreversibility" and "recurrence." In the first part of this appendix we define and discuss the issue of irreversibility, while in its second part we define and discuss the issue of recurrences.

We say that a process is reversible if it is possible to "undo" it. For example: consider a gas inside a cylinder with a piston. Let us shift the piston inside. Due to the compression the gas is heated up. Can we undo the "heating" simply by shifting the piston outside, back to its original position? If the answer is yes, as in the case of a strictly adiabatic process, then we say that the process is reversible.

Consider the prototype example of interference in the Aharonov-Bohm ring geometry. The particle can go from the input lead to the output lead by traveling via either arm of the ring. This leads to interference, which can be tested by measuring the dependence of the transmission on the magnetic flux via the ring. Consider now the situation where

there is a spin degree of freedom in one arm [25]. The particle can cause a spin flip if it travels via this arm. In such a case interference is lost completely. However, this entanglement process is completely reversible. We can undo the entanglement simply by letting the particle interact with the spin for twice the duration. Therefore, according to our restrictive definition, this is not a real dephasing process.

Consider now the situation where a particle gets entangled with bath degrees of freedom. If the bath is infinite, then the entanglement process is irreversible, and therefore it constitutes, according to our definition, a dephasing process. In this paper we have analyzed a more tricky situation where a particle gets entangled with chaotic degrees of freedom. The environment is finite, but due to its chaotic nature we have irreversibility. Hence we can talk about the dephasing process.

Consider an ice cube inside a cup of tea. After some time it melts and disappears. But if we wait long enough we have some probability to see the ice cube reemerging due to recurrences. The issue of recurrences becomes relevant whenever we consider a closed (undriven) system. In other words, whenever we do not try to control its evolution from the outside.

There are recurrences both in classical and quantal physics. In the latter case the tendency for recurrences is stronger due to the quasiperiodic nature of the dynamics. However, if the time scale for recurrences is long enough with respect to other relevant time scales, then we can practically ignore these recurrences. Actually it is useful to regard these recurrences as "fluctuations," and to take the standpoint that our interest is only in some "average" behavior.

If the bath is infinite, then also the time for recurrences of the particle-bath system becomes infinite. On the other hand, if the bath is finite, then we have to consider the recurrences of the particle-bath system. These recurrences can lead back to an unentangled state.

In practice the time to get unentangled by recurrences is extremely large. Assuming a chaotic environment, and ignoring issues of level statistics, the time scale for recurrences is at least the Heisenberg time (inverse of the mean level spacing) of the combined particle-environment system. It scales like $\hbar^{-(d+d_0)}$ where d_0 is the number of degrees of freedom of the particle.

It goes without saying that the above issue of recurrences becomes irrelevant if the x motion is treated classically. There is, however, a twist to this latter statement in the case where the time variation of x is strictly periodic. This is due to dynamical localization effect [27]. Note, however, that dynamical localization is a very fragile effect. Even in the case that it is found, it turns out that it manifests itself only after a time that scales like $\hbar^{-1+2d}$, which is much larger than the Heisenberg time of the environment [22].

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