Thermalization of mesoscopic subsystems

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The fluctuation-diffusion-dissipation relation

Rate of energy absorption (work):

\[ A(\varepsilon) = \partial_\varepsilon D_\varepsilon + \beta(\varepsilon) D_\varepsilon, \quad \dot{W} = \langle A \rangle \]

\[ D_\varepsilon = \int_{0}^{\infty} \frac{d\omega}{2\pi} \omega^2 \tilde{C}_\varepsilon(\omega) \tilde{S}(\omega) \]

Derivation:

\[ \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon} \left( g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon} \left( \frac{1}{g(\varepsilon)} \rho \right) \right) = -\frac{\partial}{\partial \varepsilon} \left( A(\varepsilon) \rho - \frac{\partial}{\partial \varepsilon} [D(\varepsilon) \rho] \right) \]


D. Cohen (1999) - adding FDT perspective + addressing the quantum case.

Thermalization of two subsystems

Rate of energy transfer [FPE version]:

\[ A(\varepsilon) = \partial_\varepsilon D_\varepsilon + (\beta_1 - \beta_2) D_\varepsilon \]

\[ D_\varepsilon = \int_0^\infty \frac{d\omega}{2\pi} \omega^2 \tilde{S}^{(1)}(\omega) \tilde{S}^{(2)}(\omega) \]

Derivation:

The diffusion is along constant energy lines: \( \varepsilon_1 + \varepsilon_2 = \mathcal{E} \)

The proper Liouville measure is:

\[ g(\varepsilon) = g_1(\varepsilon)g_2(\mathcal{E} - \varepsilon) \]

Note: After canonical preparation of the two subsystems:

\[ \langle A(\varepsilon) \rangle = \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \langle D_\varepsilon \rangle \]

MEQ version: Hurowitz, Cohen (EPL 2011)

NFT version: Bunin, Kafri (arXiv 2012)
The minimal model for a subsystem

The FPE description makes sense if each subsystem is chaotic and hence ergodic.

**Minimal models:** Billiard; 2deg oscillator; 3site Bose-Hubbard model.

\[
H = \frac{U}{2} \sum_{i=0,1,2} a_i^\dagger a_i^\dagger a_i a_i + \frac{K}{2} \sum_{i=1,2} (a_i^\dagger a_0 + a_0^\dagger a_i)
\]

\[
K = K_0 + \varepsilon f(t)
\]

\[
\mathcal{H} = \mathcal{H}_0 + f(t)W
\]

**Note on linear response:** Driven integrable system (e.g. ”kicked rotor”) - quasi-linear behavior shows up only for large driving amplitude \(\varepsilon > \varepsilon_c\).
Demonstration of diffusion: driven Bose-Hubbard trimer

Quantum vs Classical
Quantum chaos $\sim$ QCC
[Cohen (PRL 1999)]

Originally demonstrated for RMT model
[Cohen, Kottos (PRL 2000)]
Complexity of phase space - stickiness - beyond FPE

The minimal Fokker-Planck description of thermalization:

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon} \left( g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon} \left( \frac{1}{g(\varepsilon) \rho} \right) \right)$$

$$g(\varepsilon) = g_1(\varepsilon) g_2(\varepsilon - \varepsilon)$$

Complexity of phase space might affect the thermalization.

BEC trimer: long dwell times in sticky regions are reflected in $\varepsilon(t)$
Complexity of phase space - sparsity - beyond LRT

$$\mathcal{H}_{\text{total}} = \text{diag}\{E_n\} - f(t)V_{nm}$$

$$w_{n,m}^\nu = \nu |V_{nm}|^2$$

[not a Gaussian matrix...]

[median $\ll$ mean]

Stotland, Budoyo, Peer, Kottos, Cohen (2008),
Stotland, Cohen, Davidson (2009),
Stotland, Kottos Cohen (2010),
Stotland, Pecora, Cohen (2010,2011)
LRT, SLR, NLR

LRT = linear response theory

\[ D[\tilde{S}(\omega)] = \int_0^\infty \frac{d\omega}{2\pi} \omega^2 \tilde{C}(\omega) \tilde{S}(\omega) \]

\[ D[\lambda \tilde{S}(\omega)] = \lambda D[\tilde{S}(\omega)] \]

\[ D[\tilde{S}(1) + \tilde{S}(2)] = D[\tilde{S}(1)] + D[\tilde{S}(2)] \]

SLR = semilinear response

e.g. due to "series addition" [Cohen, Kottos, Schanz, JPA 2006; Wilkinson, Mehlig, Cohen, EPL 2006]

\[ D[\tilde{S}(\omega)] = \left[ \int_0^\infty d\omega \frac{\tilde{P}(\omega)}{\tilde{S}(\omega)} \right]^{-1} \]

e.g. due to "variable range hopping" [de Leeuw, Cohen, PRE 2012]

\[ D[\tilde{S}(\omega)] \approx \text{EXP}_{d} \left( \frac{1}{s} \right) e^{-\frac{1}{s}} D_{\text{LRT}} \quad \text{[VRH for } s \ll 1, \text{ LRT for large } s] \]

NLR = nonlinear response

e.g. due to non-Ohmic power spectrum [Sela, Aisenberg, Kottos, Cohen, IPA-FTC 2010]

\[ D_\varepsilon[\tilde{S}(\omega)] \sim \left\| \tilde{S}(\omega) \right\|^{\frac{3}{4-\sigma}} \quad \text{[NLR for } \sigma \neq 1, \text{ LRT for } \sigma = 1] \]
Conclusions

1. BEC trimers are the minimal building blocks for thermalization
2. The generic package deal: diffusion, LRT and QCC.
3. FPE based FD phenomenology for mesoscopic thermalization
4. Beyond FPE - statistics of dwell times due to sticky dynamics
5. Beyond LRT - non Ohmic fluctuations - nonlinear response
6. Beyond LRT - sparsity - resistor network picture - semilinear response
7. FD phenomenology for sparse (glassy) systems [Hurowitz, Cohen, EPL 2011]