Fluctuation dissipation phenomenology away from equilibrium

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The fluctuation-diffusion-dissipation relation

Rate of energy absorption (work):
\[ A(\varepsilon) = \partial_\varepsilon D_\varepsilon + \beta(\varepsilon) D_\varepsilon, \quad \dot{W} = \langle A \rangle \]

\[ D_\varepsilon = \int_0^\infty \frac{d\omega}{2\pi} \omega^2 \tilde{C}_\varepsilon(\omega) \tilde{S}(\omega) \]

Derivation:
\[ \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon} \left( g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon} \left( \frac{1}{g(\varepsilon)} \rho \right) \right) = -\frac{\partial}{\partial \varepsilon} \left( A(\varepsilon) \rho - \frac{\partial}{\partial \varepsilon} [D(\varepsilon) \rho] \right) \]


D. Cohen (1999) - adding FDT perspective + addressing the quantum case.

Thermalization of two subsystems

Rate of energy transfer:

\[ A(\varepsilon) = \partial_\varepsilon D_\varepsilon + (\beta_1 - \beta_2) D_\varepsilon, \]

\[ D_\varepsilon = \int_0^\infty \frac{d\omega}{2\pi} \omega^2 \tilde{S}^{(1)}(\omega) \tilde{S}^{(2)}(\omega) \]

After canonical preparation of the two subsystems:

\[ \langle A(\varepsilon) \rangle = \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \langle D_\varepsilon \rangle \]

MEQ version: Hurowitz, Cohen (EPL 2011)
NFT version: Bunin, Kafri (arXiv 2012)

Derivation:

The diffusion is along constant energy lines: \( \varepsilon_1 + \varepsilon_2 = \mathcal{E} \)

The proper Liouville measure is:

\[ g(\varepsilon) = g_1(\varepsilon)g_2(\mathcal{E} - \varepsilon) \]
Demonstration of diffusion: driven Bose-Hubbard trimer

Quantum vs Classical

Quantum chaos $\sim$ QCC
[Cohen (PRL 1999)]

Originally demonstrated for RMT model
[Cohen, Kottos (PRL 2000)]

[Tikhonenkov, Vardi, Anglin, Cohen (arXiv 2012)]
The “sparsity” of weakly chaotic driven systems

\[ \mathcal{H}_{\text{total}} = \text{diag}\{E_n\} - f(t)V_{nm} \]

\[ w_{n,m}^\nu = \nu |V_{nm}|^2 \]

[not a Gaussian matrix...]

[log-wide distribution]

[median \ll \text{mean}]

Stotland, Budoyo, Peer, Kottos, Cohen (2008),
Stotland, Cohen, Davidson (2009),
Stotland, Kottos, Cohen (2010),
Stotland, Pecora, Cohen (2010,2011)
The NESS of a “sparse” system

\[ w_{nm} = w_{nm}^\beta + w_{nm}^\nu = w_{nm}^\beta + \nu g_{nm} \]

\[ \frac{w_{nm}^\beta}{w_{mn}^\beta} = \exp \left[ -\frac{E_n - E_m}{T_B} \right], \quad g_{nm} = g_{mn} \]

\( w^\nu \) by themselves - induces diffusion / ergodization

\( w^\beta \) by themselves - leads to equilibrium

Combined - leads to NESS

Linear response and traditional FD: \( \nu \times \{ g \} \ll \{ w^\beta \} \)

Glassy response and Sinai physics: [within a wide crossover regime]

Semi-linear response and Saturation: \( \nu \times \{ g \} \gg \{ w^\beta \} \)
FD phenomenology for a “sparse” system

\[ w_{nm} = w_{nm}^\beta + w_{nm}^\nu = w_{nm}^\beta + \nu g_{nm} \]

\[ \dot{W} = \text{rate of heating} = \frac{D(\nu)}{T_{\text{system}}} \]

\[ \dot{Q} = \dot{W} = \frac{1/T_B}{D_B^{-1} + D(\nu)^{-1}} \]

Hence at the NESS:

\[ T_{\text{system}} = \left(1 + \frac{D(\nu)}{D_B}\right)T_B \]

\[ \dot{Q} = \dot{W} = \frac{1/T_B}{D_B^{-1} + D(\nu)^{-1}} \]

Experimental way to extract response:

\[ D(\nu) = \frac{\dot{Q}(\nu)}{\dot{Q}(\infty) - \dot{Q}(\nu)} D_B \]

\[ D(\nu) \text{ exhibits LRT to SLRT crossover} \]

\[ D(\nu) = \left[ \frac{w_n}{w_\beta + w_n} \right] \left[ \frac{1}{w_\beta + w_n} \right]^{-1} \]

\[ D[\text{LRT}] = \frac{g_n \nu}{[1/g_n]^{-1}} \nu \quad \text{[weak driving]} \]

\[ D[\text{SLRT}] = \frac{1}{g_n} \nu \quad \text{[strong driving]} \]

Expressions above assume n.n. transitions only.
Conclusions

(*) Wigner (∼ 1955): “The perturbation is represented by a random matrix whose elements are taken from a Gaussian distribution.” Not always...

1. “weak quantum chaos” \implies log-wide distribution, “sparsity” and “texture”
2. The heating \sim a percolation process.
3. Resistors network calculation to get the response coefficient.
4. RMT modeling \sim generalization of the VRH estimate.
5. Experimental fingerprint: semi-linear response characteristics.

6. SLRT applies if the driving is stronger then the background relaxation.
7. The stochastic NESS has glassy characteristics (wide distribution of microscopic temperatures).
8. Definition of effective NESS temperature, and extension of the F-D phenomenology.
9. For very strong driving - quantum saturation of the NESS temperature (T \to T_\infty).

10. Topological aspects: The emergence of the Sinai regime.
11. Topological term in the formula for the heating rate.

12. Applications: beyond the “Drude formula” and beyond the “Wall formula”.
**Perspective and references**

The classical LRT approach: Ott, Brown, Grebogi, Wilkinson, Jarzynski, Robbins, Berry, Cohen

The Wall formula (I): Blocki, Boneh, Nix, Randrup, Robel, Sierk, Swiatecki, Koonin

The Wall formula (II): Barnett, Cohen, Heller [1] - regarding $g_c$

Semi Linear response theory: Cohen, Kottos, Schanz... [2-6]

Billiards with vibrating walls: Stotland, Cohen, Davidson, Pecora [7,8] - regarding $g_s$

Sparsity: Austin, Wilkinson, Prosen, Robnik, Alhassid, Levine, Fyodorov, Chubykalo, Izrailev, Casati

Random networks: Mott; Miller, Abrahams; Ambegaokar, Halperin, Langer; Pollak [...]

Random site model: Alexander, Bernasconi, Schneider, Orbach; Amir, Oreg, Imry [...]

Extensions related to: Sinai; Derrida,Pomeau; Burlatsky, Oshanin, Mogutov, Moreau, [...]