Fluctuation dissipation phenomenology away from equilibrium

Doron Cohen
Ben-Gurion University

Daniel Hurowitz (BGU) [1]
Igor Tikhonenkov (BGU) [2]
Amichay Vardi (BGU) [2]
James R. Anglin (Kaiserslautern) [2]


http://www.bgu.ac.il/~dcohen
The fluctuation-diffusion-dissipation relation

Rate of energy absorption (work):

\[ A(\varepsilon) = \partial_\varepsilon D_\varepsilon + \beta(\varepsilon) D_\varepsilon, \quad \dot{W} = \langle A \rangle \]

\[ D_\varepsilon = \int_0^\infty \frac{d\omega}{2\pi} \omega^2 \tilde{C}_\varepsilon(\omega) \tilde{S}(\omega) \]

Derivation:

\[ \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon} \left( g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon} \left( \frac{1}{g(\varepsilon)} \rho \right) \right) = -\frac{\partial}{\partial \varepsilon} \left( A(\varepsilon) \rho - \frac{\partial}{\partial \varepsilon} [D(\varepsilon) \rho] \right) \]


D. Cohen (1999) - adding FDT perspective + addressing the quantum case.

Thermalization of two subsystems

Rate of energy transfer [FPE version]:

\[ A(\varepsilon) = \partial_\varepsilon D_\varepsilon + (\beta_1 - \beta_2) D_\varepsilon \]

\[ D_\varepsilon = \int_0^\infty \frac{d\omega}{2\pi} \omega^2 \tilde{S}^{(1)}(\omega) \tilde{S}^{(2)}(\omega) \]

Derivation:

The diffusion is along constant energy lines: \( \varepsilon_1 + \varepsilon_2 = \mathcal{E} \)

The proper Liouville measure is: \( g(\varepsilon) = g_1(\varepsilon)g_2(\mathcal{E} - \varepsilon) \)

Note: After canonical preparation of the two subsystems:

\[ \langle A(\varepsilon) \rangle = \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \langle D_\varepsilon \rangle \]

MEQ version: Hurowitz, Cohen (EPL 2011)

NFT version: Bunin, Kafri (arXiv 2012)
The minimal model for a subsystem

The FPE description makes sense if each subsystem is chaotic and hence ergodic.

Minimal models: Billiard; 2deg oscillator; 3site Bose-Hubbard model.

\[
\mathcal{H} = \frac{U}{2} \sum_{i=0,1,2} a_i^\dagger a_i^\dagger a_i a_i + \frac{K}{2} \sum_{i=1,2} (a_i^\dagger a_0 + a_0^\dagger a_i)
\]

\[
K = K_0 + \varepsilon f(t)
\]

\[
\mathcal{H} = \mathcal{H}_0 + f(t)W
\]

Note on linear response: Driven integrable system (e.g. ”kicked rotor”) - quasi-linear behavior shows up only for large driving amplitude \(\varepsilon > \varepsilon_c\).
Demonstration of diffusion: driven Bose-Hubbard trimer

Quantum vs Classical

Quantum chaos $\sim$ QCC
[Cohen (PRL 1999)]

Originally demonstrated for RMT model
[Cohen, Kottos (PRL 2000)]
Complexity of phase space - stickiness - beyond FPE

The minimal Fokker-Planck description of thermalization:
\[
\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \varepsilon} \left( g(\varepsilon) D(\varepsilon) \frac{\partial}{\partial \varepsilon} \left( \frac{1}{g(\varepsilon)} \rho \right) \right)
\]
\[g(\varepsilon) = g_1(\varepsilon) g_2(\varepsilon - \varepsilon)\]

Complexity of phase space might affect the thermalization.

BEC trimer: long dwell times in sticky regions are reflected in \( \varepsilon(t) \)
Complexity of phase space - sparsity - beyond LRT

$\mathcal{H}_{\text{total}} = \text{diag}\{E_n\} - f(t)V_{nm}$

$\omega_{n,m}^\nu = \nu |V_{nm}|^2$

[not a Gaussian matrix...]

[log-wide distribution]

[median $\ll$ mean]
The NESS of a “sparse” system

\[ w_{nm} = w_{nm}^\beta + w_{nm}^\nu = w_{nm}^\beta + \nu g_{nm} \]

Cold bath: \[ \frac{w_{nm}^\beta}{w_{mn}^\beta} = \exp \left[ -\frac{E_n - E_m}{T_B} \right] \]

Hot source: \[ g_{nm} = g_{mn} \]

\( w^\nu \) by themselves - induces diffusion / ergodization

\( w^\beta \) by themselves - leads to equilibrium

Combined - leads to NESS

Linear response and traditional FD: \[ \nu \times \{ g \} \ll \{ w^\beta \} \]

Glassy response and Sinai physics: [within a wide crossover regime]

Semi-linear response and Saturation: \[ \nu \times \{ g \} \gg \{ w^\beta \} \]
FD phenomenology for a “sparse” system

\[ w_{nm} = w_{nm}^\beta + w_{nm}^\nu = w_{nm}^\beta + \nu g_{nm} \]

\[ \dot{W} = \text{rate of heating} = \frac{D(\nu)}{T_{\text{system}}} \]

\[ \dot{Q} = \text{rate of cooling} = \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}} \]

Hence at the NESS:

\[ T_{\text{system}} = \left(1 + \frac{D(\nu)}{D_B}\right)T_B \]

\[ \dot{Q} = \dot{W} = \frac{1/T_B}{D_B^{-1} + D(\nu)^{-1}} \]

Experimental way to extract response:

\[ D(\nu) = \frac{\dot{Q}(\nu)}{\dot{Q}(\infty) - \dot{Q}(\nu)} D_B \]

\[ D(\nu) \text{ exhibits LRT to SLRT crossover} \]

\[ D(\nu) = \left[ \frac{w_n}{w_\beta + w_n} \right]^{-1} \left[ 1 \left( \frac{1}{w_\beta + w_n} \right) \right]^{-1} \]

\[ D[\text{LRT}] = \overline{g_n} \nu \quad \text{[weak driving]} \]

\[ D[\text{SLRT}] = \left[ 1/\overline{g_n} \right]^{-1} \nu \quad \text{[strong driving]} \]

Expressions above assume n.n. transitions only.
Conclusions

1. BEC trimers are the minimal building blocks for thermalization
2. The generic package deal: diffusion, LRT and QCC.
3. FPE based FD phenomenology for mesoscopic thermalization
4. Beyond FPE - statistics of dwell times due to sticky dynamics
5. Beyond LRT - sparsity - resistor network picture - semilinear response
6. FD phenomenology for sparse (glassy) systems