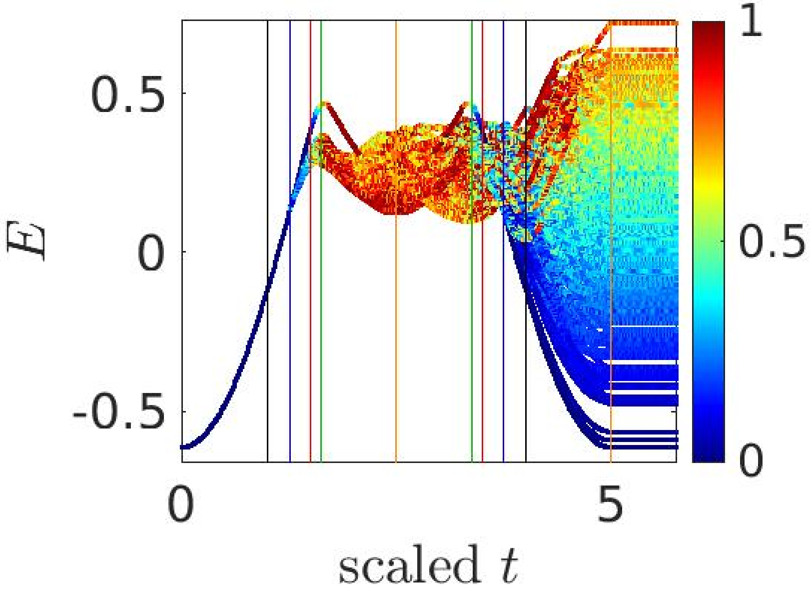
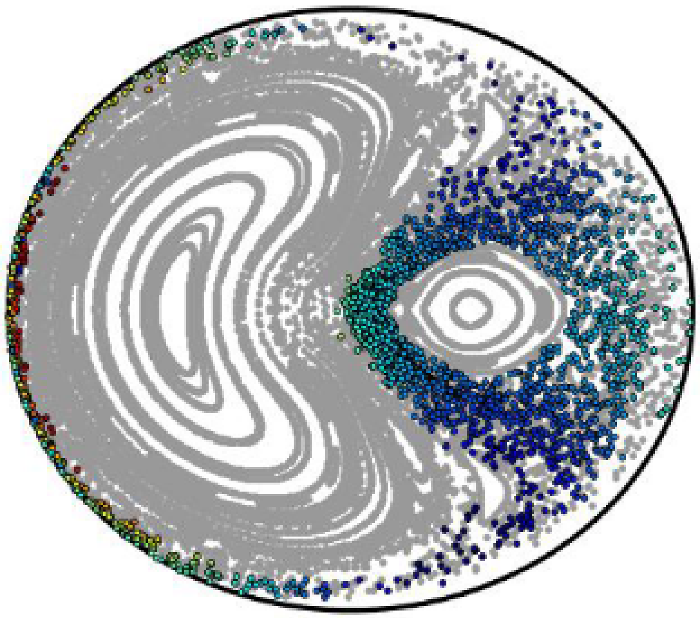
# Quantum irreversibility

Doron Cohen, Ben-Gurion University



Yehoshua Winsten, DC,
Quantum irreversibility of quasistatic protocols for finite-size quantized systems, Phys. Rev. A 107, 052202 (2023).
Quasistatic transfer protocols for atomtronic superfluid circuits, Scientific Reports 11, 3136 (2021).

Related publications:

R. Burkle, A. Vardi, DC, J.R. Anglin, Hamiltonian Hysteresis, Phys. Rev. Lett. 123, 114101 (2019).

A. Dey, DC, A. Vardi, Sweep through chaos, Phys. Rev. Lett. 121, 250405 (2018).

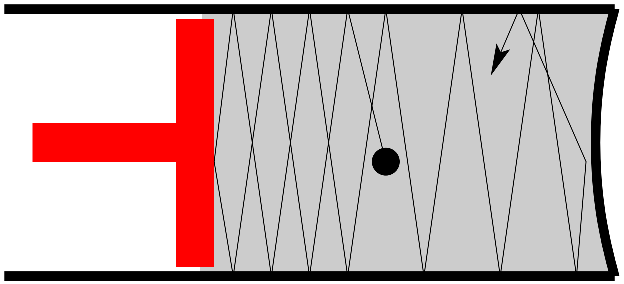
Other related publications, notably by Geva Arwas, are listed in the last slide.

# Adiabaticity and Irreversibility

$H[\boldsymbol{r}, \boldsymbol{p}; X(t)]$

Chaos - motion of particles $(\boldsymbol{r}, \boldsymbol{p})$ inside the box is chaotic.

Quasistatic - the piston position $(X)$ is varied very slowly.



$X(t) \quad = \quad$ piston position

Classical:

- For fully chaotic dynamics the quasistatic limit is adiabatic, hence reversible.
- For mixed phase-space the quasistatic limit is not adiabatic, hence irreversible.

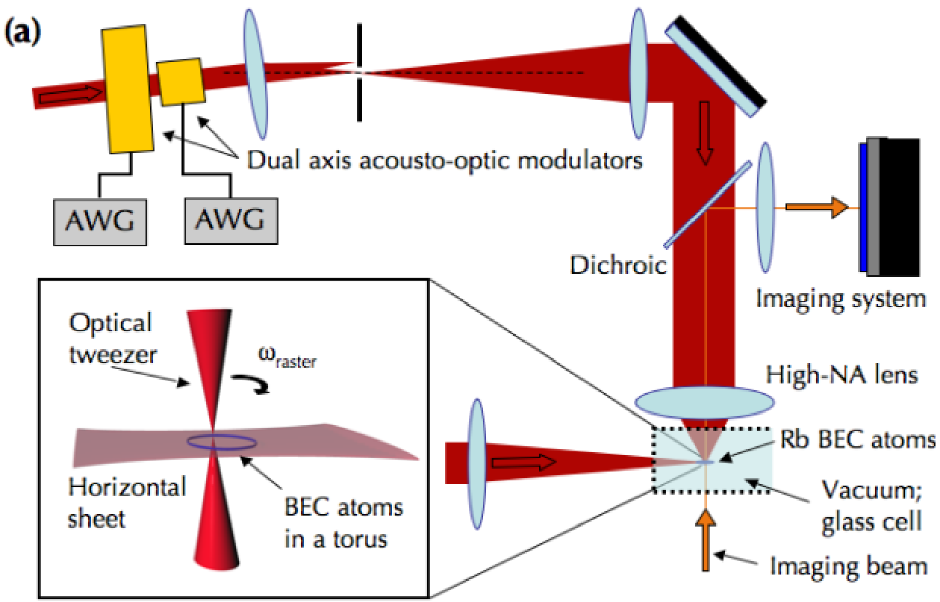
Quantum:

- The quasistatic limit is always adiabatic [but not accessible in practice].
- What happens outside of the QM-adiabatic regime? QCC? or New regime?

# Atomtronics

**Roadmap on Atomtronics: State of the art and perspective**

L. Amico,[1,2,3,4,a)] M. Boshier,[5] G. Birkl,[6] A. Minguzzi,[7] C. Miniatura,[4,2,8,9,10] L.-C. Kwek,[2,4,11] D. Aghamalyan,[12,2] V. Ahufinger,[13] D. Anderson,[14,15] N. Andrei,[16] A. S. Arnold,[17] M. Baker,[18] T.A. Bell,[18] T. Bland,[19,20] J.P. Brantut,[21] D. Cassettari,[22] W.J. Chetcuti,[23,1] F. Chevy,[24] R. Citro,[25] S. De Palo,[26] R. Dumke,[2,9,4] M. Edwards,[27] R. Folman,[28] J. Fortagh,[29] S. A. Gardiner,[30] B.M. Garraway,[31] G. Gauthier,[18] A. Günther,[29] T. Haug,[2] C. Hufnagel,[2] M. Keil,[28] P. Ireland,[22] M. Lebrat,[32] W. Li,[2,8] L. Longchambon,[33] J. Mompart,[13] O. Morsch,[34] P. Naldesi,[35,3] T.W. Neely,[18] M. Olshanii,[36] E. Orignac,[37] S. Pandey,[38] A. Pérez-Obiol,[39] H. Perrin,[33] L. Piroli,[40] J. Polo,[41] A.L. Pritchard,[18] N. P. Proukakis,[19] C. Rylands,[42] H. Rubinsztein-Dunlop,[18] F. Scazza,[43] S. Stringari,[44] F. Tosto,[2] A. Trombettoni,[45,46] N. Victorin,[35] W. von Klitzing,[38] D. Wilkowski,[4,2,47,9] K. Xhani,[19,48] and A. Yakimenko[49]



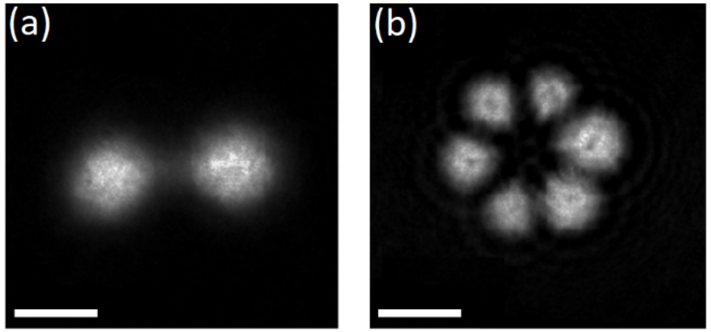




FIG. 1. Example geometries of time averaged optical dipole potentials - (a) BEC trapped in dumbbell potential, with two reservoirs connected through a channel of tunable length and width (b) Ring lattice of BECs. The scale bar on each image indicates 50 $\mu$m.

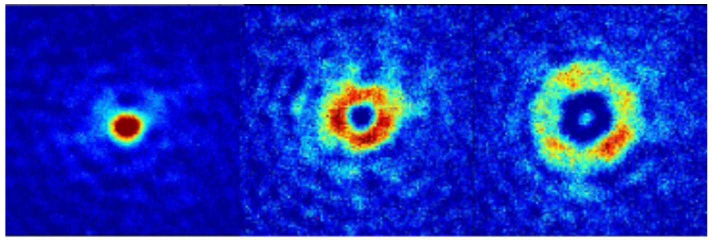


FIG. 15. Time-of-flight images of (a): a non-rotating ring, (b) and (c): rotating rings with different circulations. The rotation is imparted by a rotating 7 µm-waist blue-detuned vertical Gaussian beam.

Toroidal or Lattice rings can be "painted". Currents can be measured.

Stability condition for superflow?

Geve Arwas, DC (SREP 2015, NJP 2016, PRB 2017, PRA 2019).

# The Bose Hubbard Hamiltonian

The system consists of $N$ bosons in $L$ sites.
Optionally we can add a gauge-field $\Phi$.

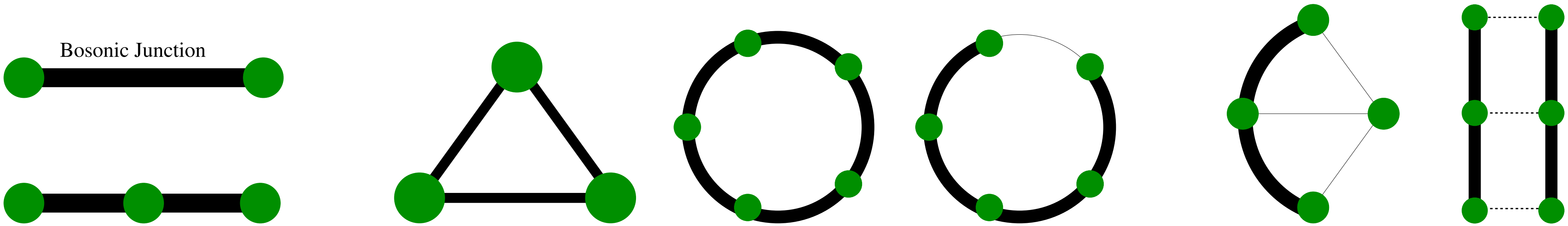
$$\mathcal{H}_{\mathrm{BHH}} = \frac{U}{2}\sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j - \sum_{\langle i,j\rangle} \frac{K_{ij}}{2} a_i^\dagger a_j$$

$$u \equiv L\frac{NU}{K} \qquad \text{[classical, stability, supefluidity, self-trapping]}$$

$$\gamma \equiv \frac{LU}{NK} = \frac{u}{N^2} \qquad \text{[quantum, Mott-regime]}$$

The two dimensionless parameters have a well defined value also in the GP/continuum limit.

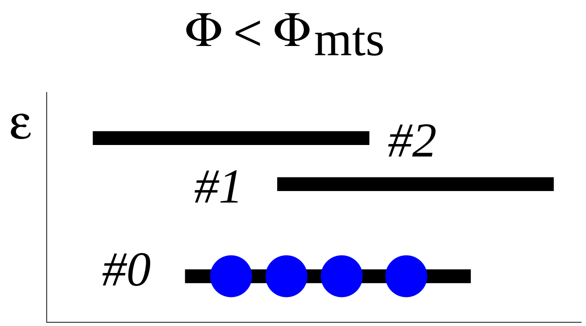
$$L = 2, 3, 4, 5, 6$$

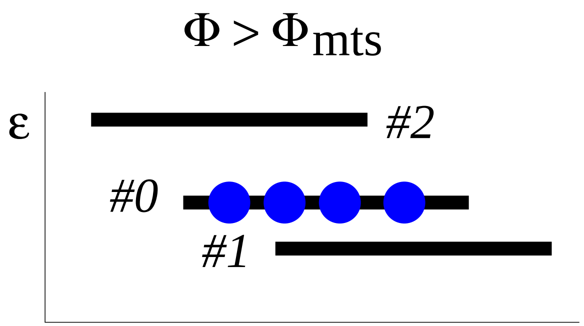


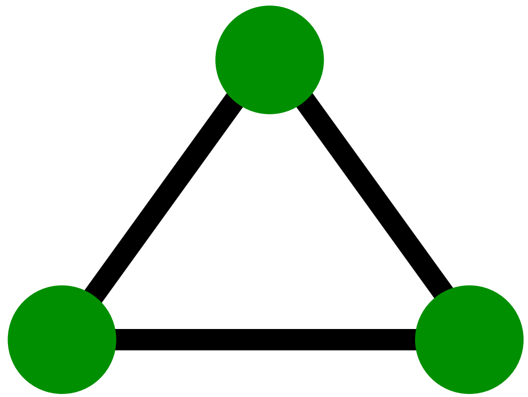
# The Bose Hubbard Ring Circuit

In the rotating reference frame we have a Coriolis force,
which is like magnetic field $\mathcal{B} = 2\mathrm{m}\Omega$.
which implies an effective flux $\Phi = \text{area} \times \mathcal{B}$

$$\mathcal{H} \;=\; \sum_{j=1}^{L}\left[\frac{U}{2}a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2}\left(\mathrm{e}^{i(\Phi/L)}a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/L)}a_j^\dagger a_{j+1}\right)\right] \qquad\qquad L=3$$



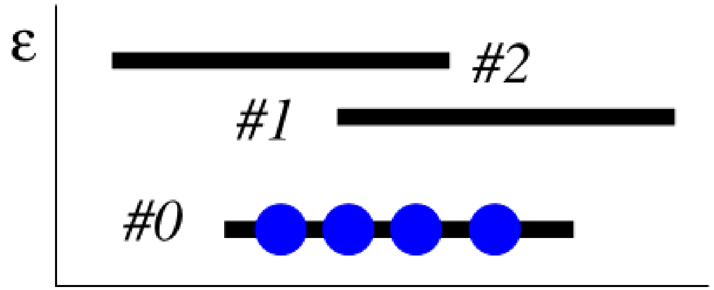
$$\mathcal{H} \;=\; \sum_{k}\epsilon_k(\Phi)\,b_k^\dagger b_k \;+\; \frac{U}{2L}\sum{}' \, b_{k_4}^\dagger b_{k_3}^\dagger b_{k_2} b_{k_1}$$

## The Hamiltonian - semiclassical perspective

We set $b_k = \sqrt{n_k} e^{i\varphi_k}$, where $n_0 + n_1 + n_2 = N$. [Initially $n = 0$]

$n = (n_1 + n_2)$ = the depletion coordinate

$M = (n_1 - n_2)$ = the population imbalance




$$\mathcal{H}(\varphi, n; \phi, M) \;=\; \mathcal{H}^{(0)}(\varphi, n; M) \;+\; \left[\mathcal{H}^{(+)} + \mathcal{H}^{(-)}\right]$$

$$\mathcal{H}^{(0)}(\varphi, n; M) \quad = \quad \mathcal{E}n + \mathcal{E}_\perp M - \frac{U}{12}M^2 \;+\; \frac{U}{3}(N-n)n \;+\; \frac{2U}{6}(N-n)\sqrt{n^2 - M^2}\cos(\varphi)$$

$$\mathcal{H}^{(\pm)} \quad = \quad \frac{U}{3\sqrt{2}}\sqrt{(N-n)(n\pm M)}(n\mp M)\cos\left(\frac{3\phi\mp\varphi}{2}\right)$$

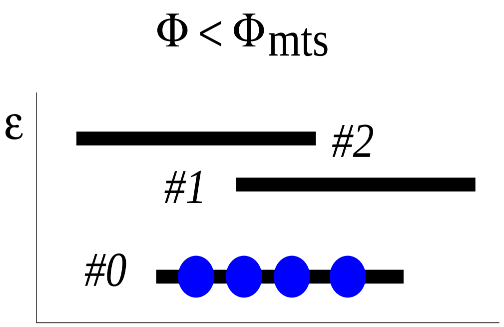
# Quasistatic protocol for an atomtronic superfluid circuit

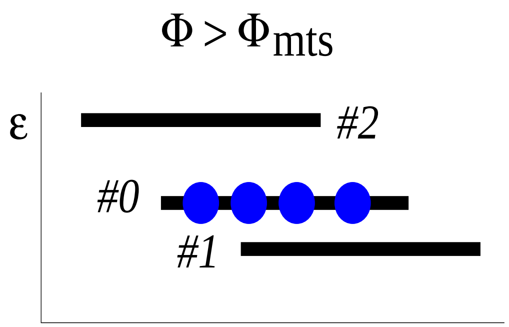
(1) All the particles are condensed into the lowest momentum orbital that has a zero winding number.
(2) The rotation frequency $\Phi$ is gradually changed from 0 to $2.5\pi$, aka sweep process.
(3) Reversed sweep back to $\Phi = 0$.
(4) The final state of the system is probed; the momentum distribution is measured.

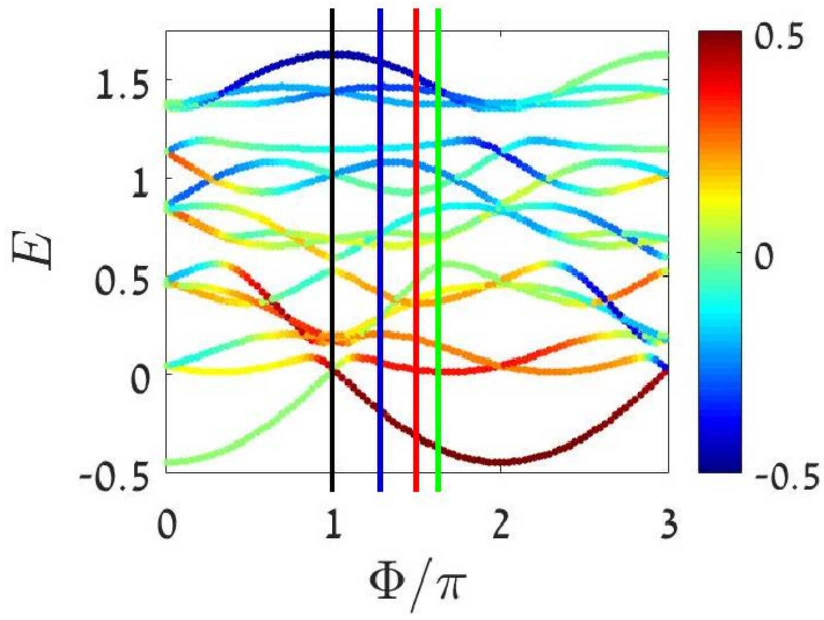
What is the fate of the evolving many-body state?

$n = (n_1 + n_2) =$ depletion coordinate

$M = (n_1 - n_2) =$ occupation imbalance




($u$=2.3, $N$=4, dim=15 levels).

# What is the fate of the evolving many-body state?

Possible answers:

(1) Without interaction: No depletion.

(2) Quantum adiabaticity: Depletion at $\Phi_{\text{mts}}$.

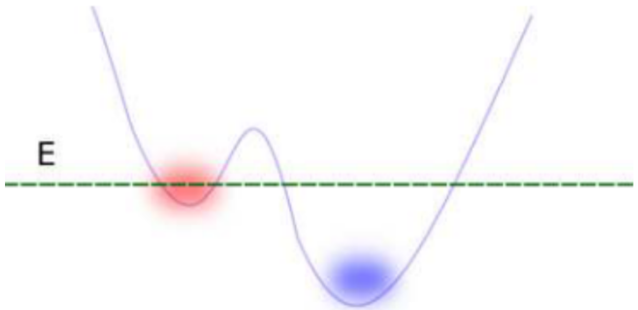
(3) Landau Criterion: Depletion at $\Phi_{\text{stb}}$.

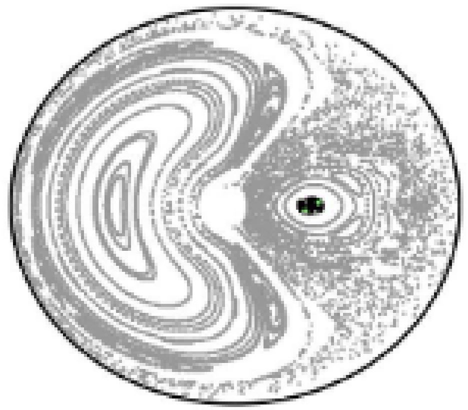
(4) Bogolyubov stability analysis: Depletion starts at $\Phi_{\text{dyn}}$.

(5) Bogolyubov (integrable) approximation: Shuttling at $\Phi_{\text{swp}}$.

(6) Beyond Bogolyubov: Chaos-assisted depeletion.

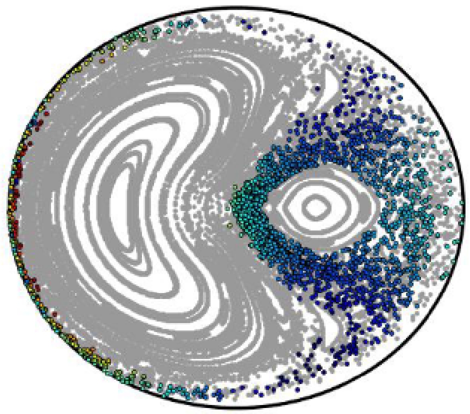
(7) Quantum chaos... leakage through the barrier



The naive two orbital approximation implies no interaction.
Therefore the manybody Landau-Zener paradigm does not apply.

Landau criterion requires energetic metastability
(local minimum of the energy landscape),
while dynamical stability can persist beyond.



Bogolyubov approximation keeps only pair-creation events
and therefore $M = (n_1 - n_2)$ is constant of motion.
Consequently chaos is ignored...

Irreversibility?

# Explicit expressions for the thresholds

The central SP is the global minimum of the energy landscape up to

$$\Phi_{\text{mts}} = \pi$$

The SP is still a local minimum up to

$$\Phi_{\text{stb}} = 3\arccos\left(\frac{1}{6}\left(\sqrt{u^2+9}-u\right)\right)$$

The SP becomes an unstable saddle at

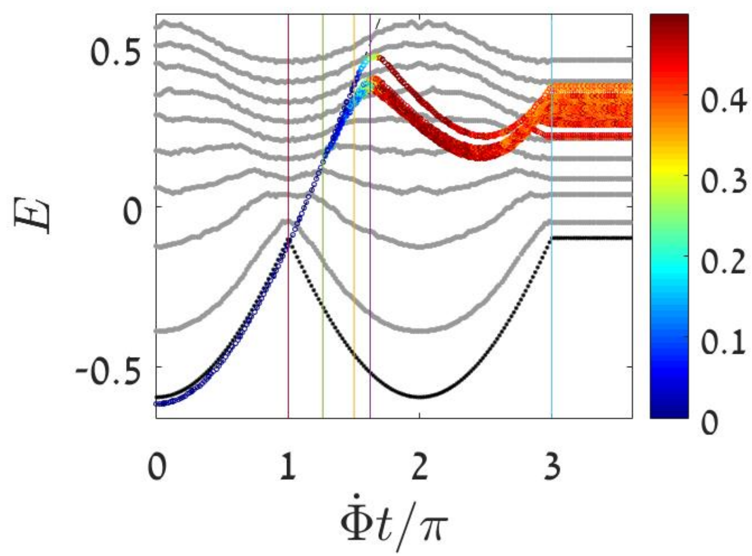
$$\Phi_{\text{dyn}} = \frac{3}{2}\pi$$

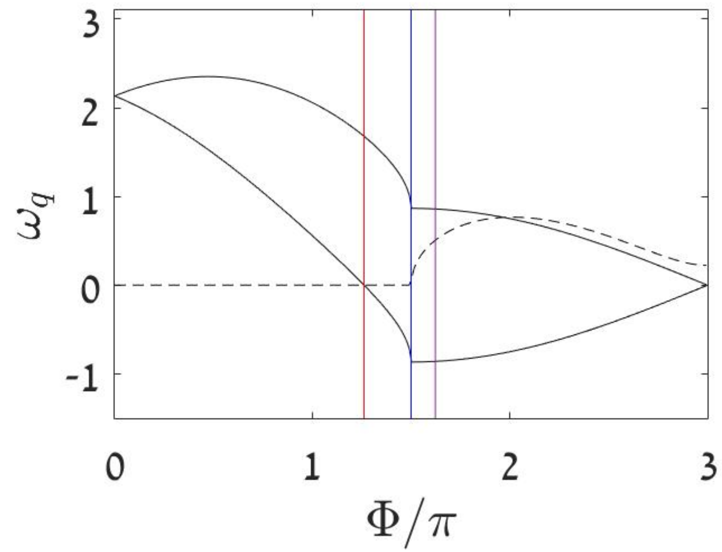
The SP becomes connected to the periphery at the swap transition

$$\Phi_{\text{swp}} = 3\arccos\left(-\frac{1}{18}u\right)$$

The Bogolyubov frequencies:

$$\omega_{\pm} = \pm\frac{\sqrt{3}}{2}\sin\frac{\Phi}{3} + \sqrt{\left(\frac{3}{2}\cos\frac{\Phi}{3}\right)^2 + u\cos\frac{\Phi}{3}}$$

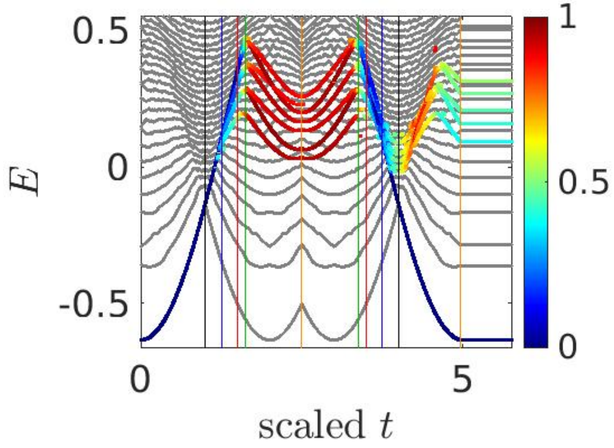


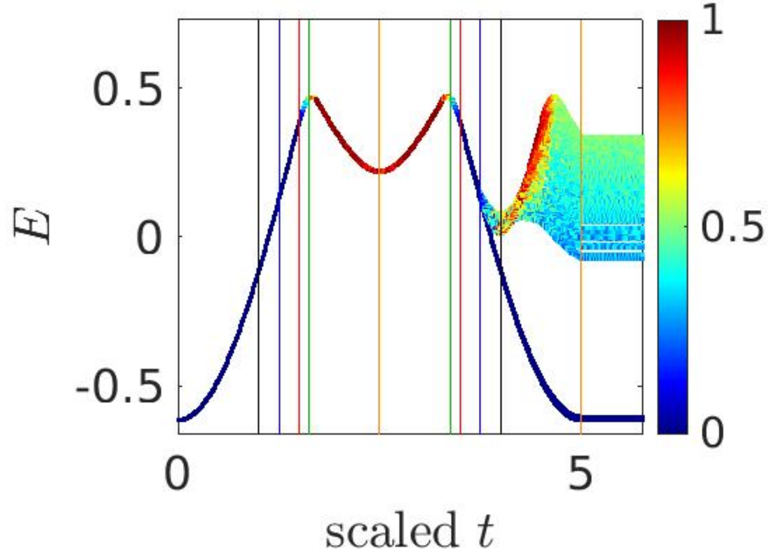




$$\mathcal{H} \approx E_0 + \sum_q \omega_q c_q^\dagger c_q$$

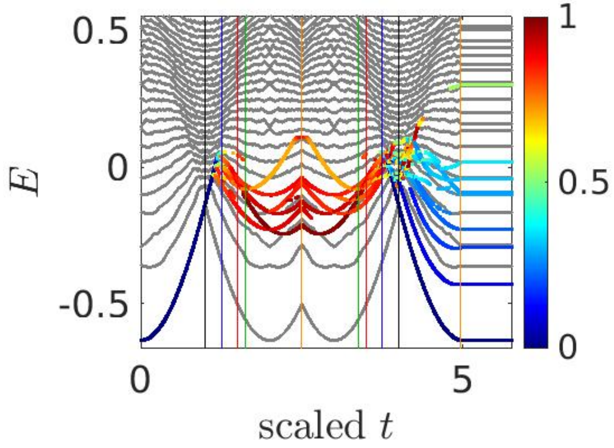
# Simulations

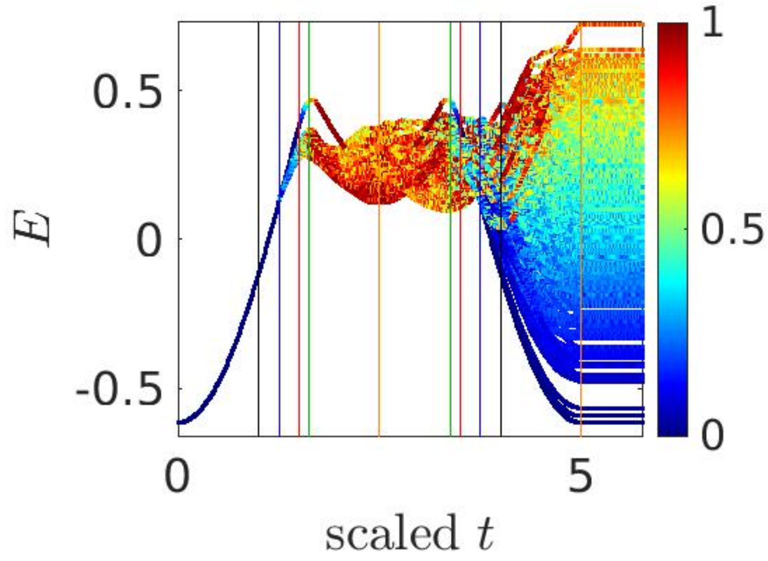
Slow:




Very Slow:



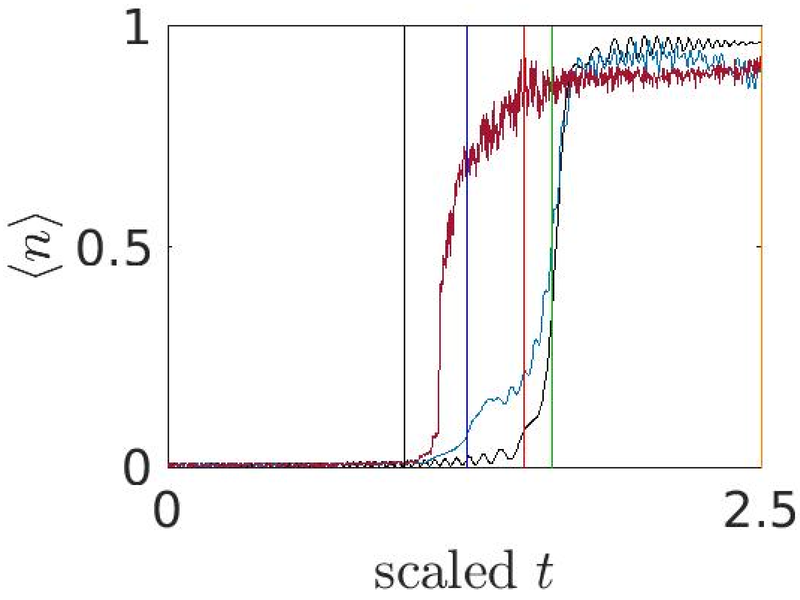

- Thresholds:

$(\Phi_{\mathrm{mts}}, \Phi_{\mathrm{stb}}, \Phi_{\mathrm{dyn}}, \Phi_{\mathrm{swp}})$

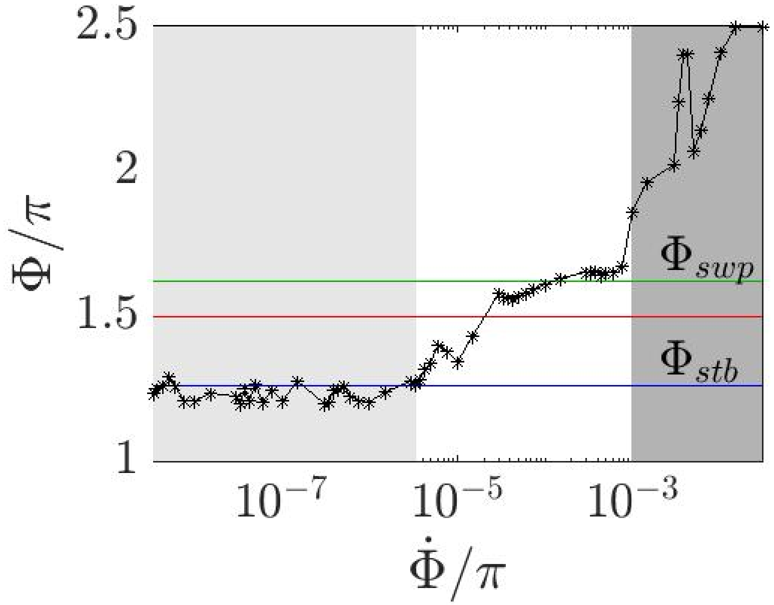
- Branching
- Depletion time
- UQF
- Irreversibility?
- QCC?

# Time of Depletion vs Sweep Rate

Depletion versus time:



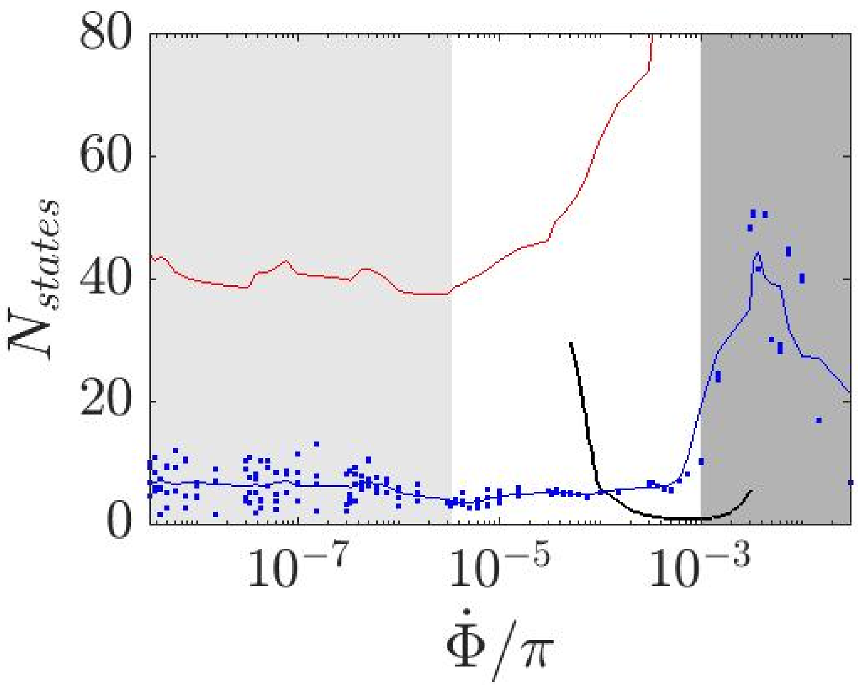

$\Phi$ at the time of depletion:



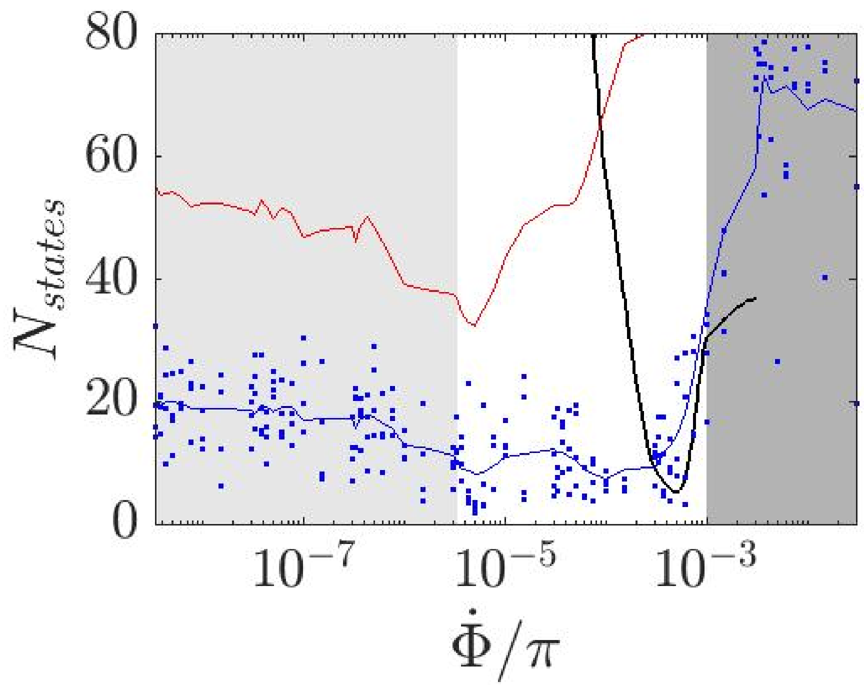

- Slow sweep: [blue] depletion at $\Phi_{\text{dyn}}$, indicating Relay-Shuttling.
- Very Slow sweep: [red] depletion at $\Phi_{\text{stb}}$, indicating Chaos-Assisted mechanism.
- Note agreement of Relay-Shuttling with the Bogolyubov approximation [black].

# Irreversibility vs Sweep Rate

Spreading after the forward sweep:

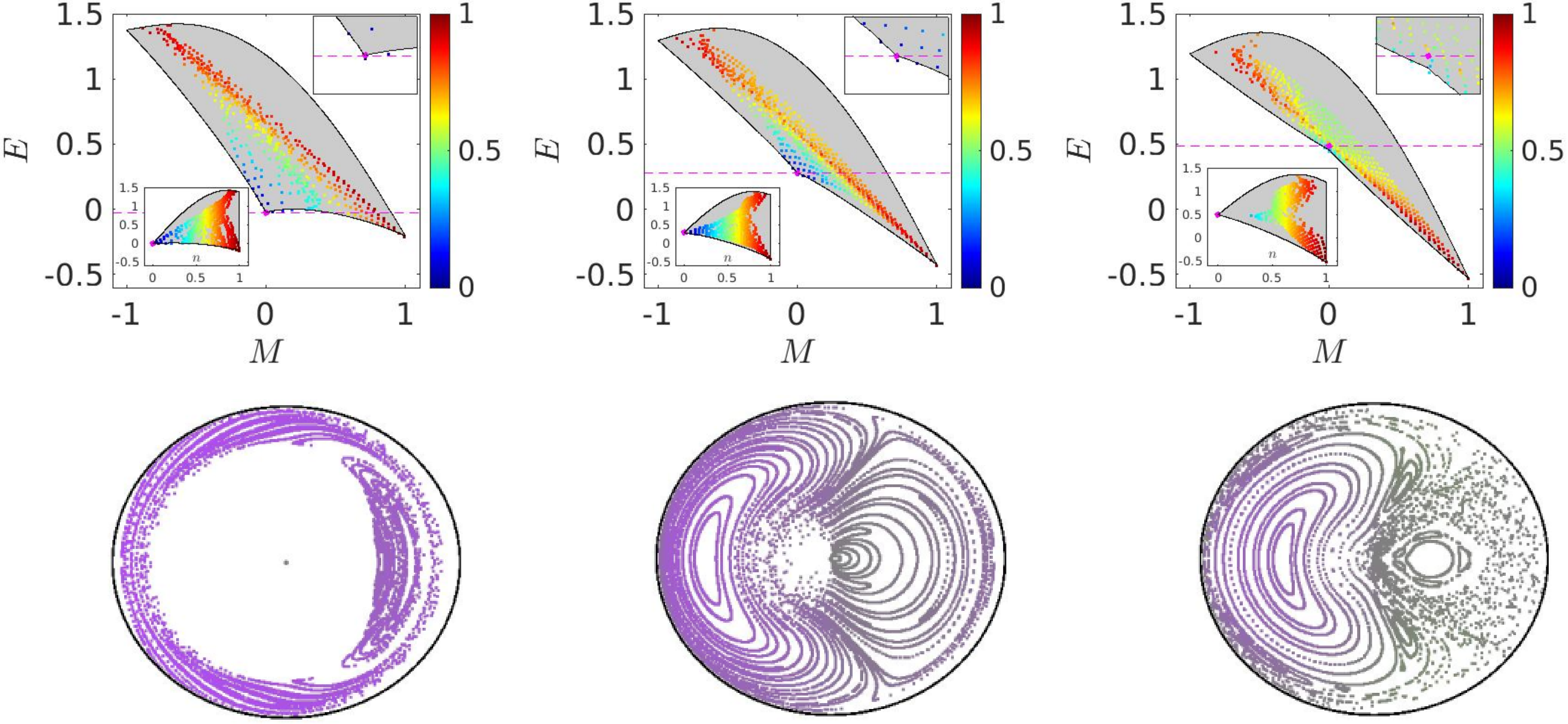



Spreading after the reversed sweep:



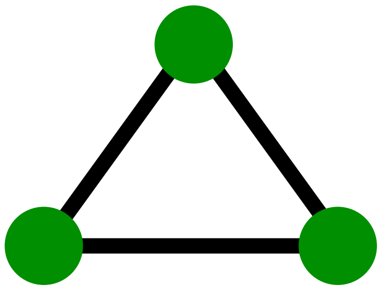

- Optimal sweep rate in the semiclassical simulations [black line].
- Universal Quantum Fluctuations (UQF) in the chaos-assisted-depletion regime.
- Breakdown of Quantum to Classical Correspondence (QCC) [blue vs black].

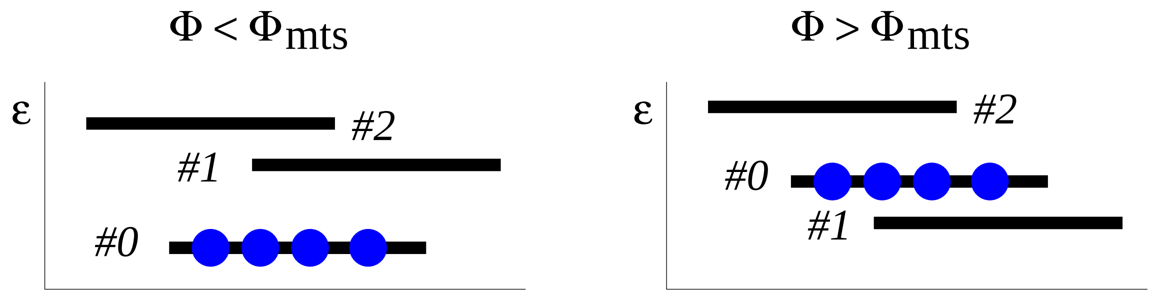
# Phase space structure

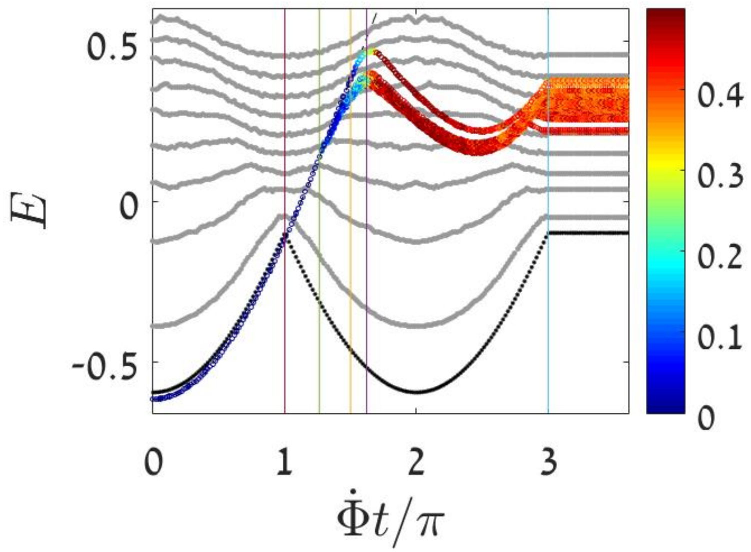


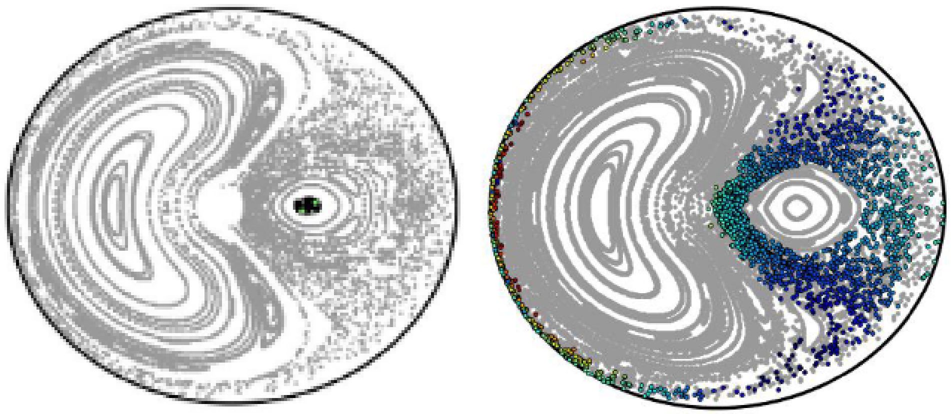

Thresholds: $(\Phi_{\mathrm{mts}}, \Phi_{\mathrm{stb}}, \Phi_{\mathrm{dyn}}, \Phi_{\mathrm{swp}})$

# Chaos-assisted depletion





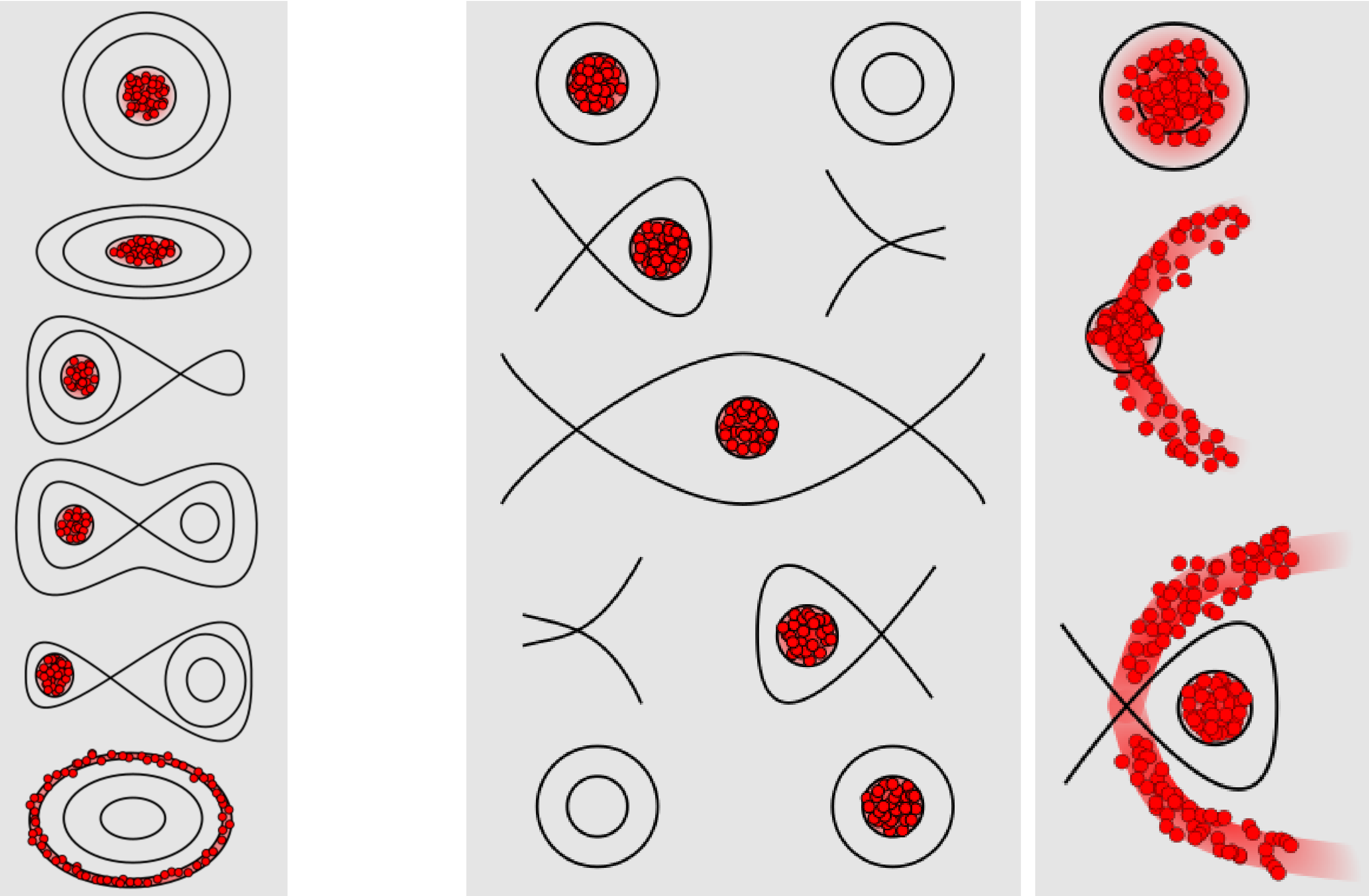



Optimal sweep: The cloud is shuttled by a fixed-point (PO) that has bifurcated from the center.

Slow sweep: The cloud spreads through the corridor, and after that shuttled by an outer torus.

# Dynamics in Phase space



Problem of interest:
Depletion of an orbital.

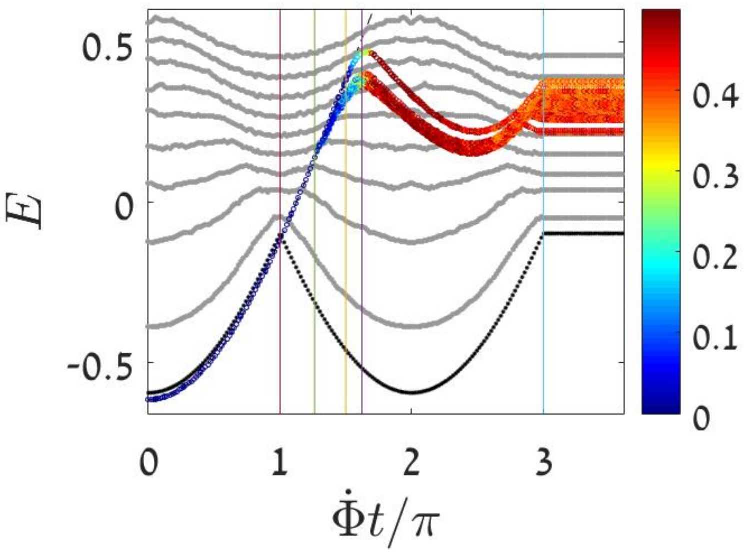
Competing mechanisms:
Relay shuttling;
Chaos-assisted depletion.

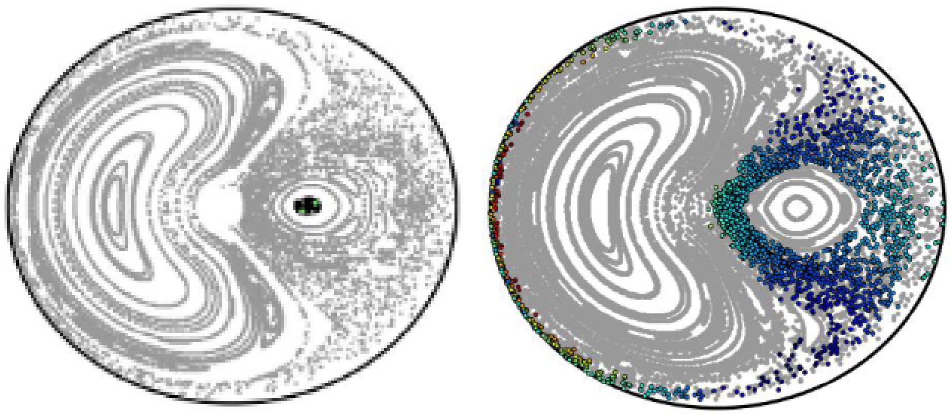
Observations:
Irreversibility;
Breakdown of QCC.

- Adiabatic shuttling (if no bifurcations are involved)
- Diabatic ejection (saddle-node bifurcation) - The nonlinear Landau-Zener transition
- Relay shuttling (pitchfork bifurcation)
- Chaos-assisted depletion (if we have a surrounding chaotic region)

# Main messages I

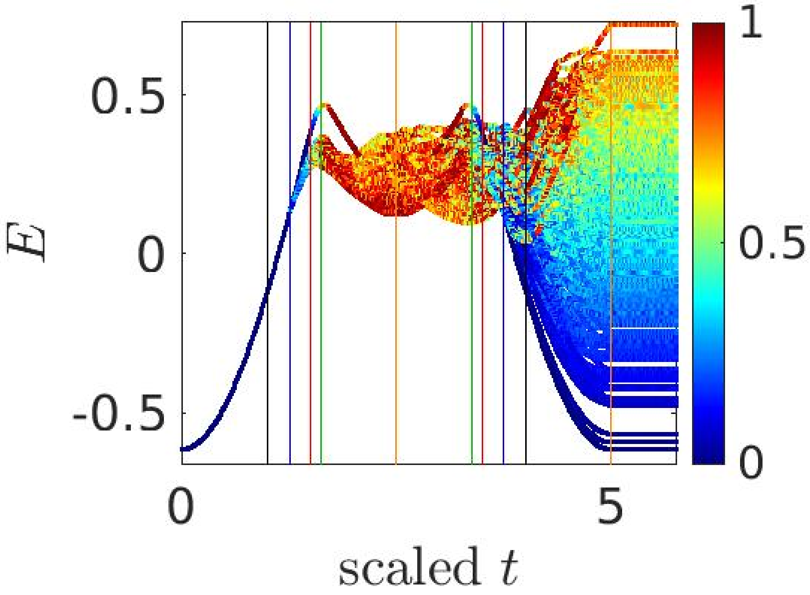
- A quasi-static protocol is in general not adiabatic,
  and hence not reversible, due to mixed-chaotic dynamics.
- It is implied that slowness is bad for adiabaticity.
- We have considered a protocol whose aim is to transfer condensed particles
  from a source orbital to a target orbital.
- Two competing mechanisms: adiabatic shuttling versus chaos-assisted depletion.
- The irreversible chaos-assisted depletion mechanism dominates in the quasi-static limit.
- An implied optimal sweep rate for the performance of the transfer protocol.

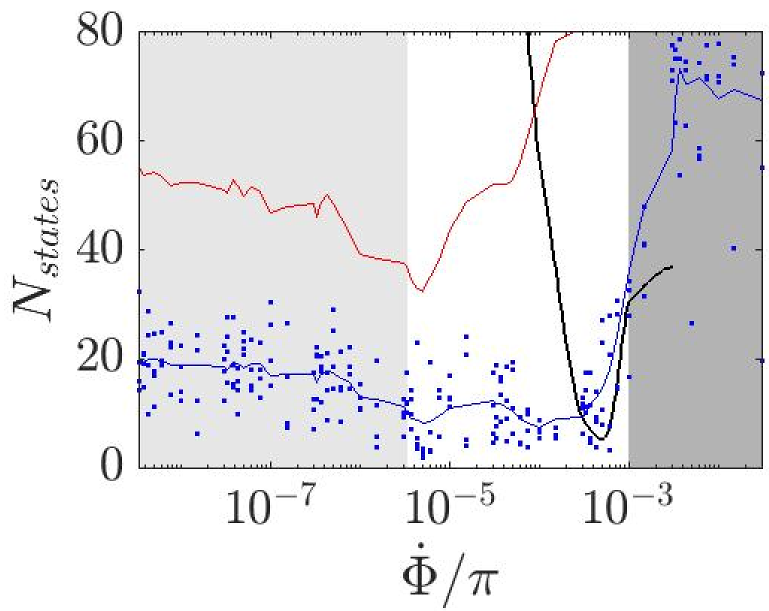


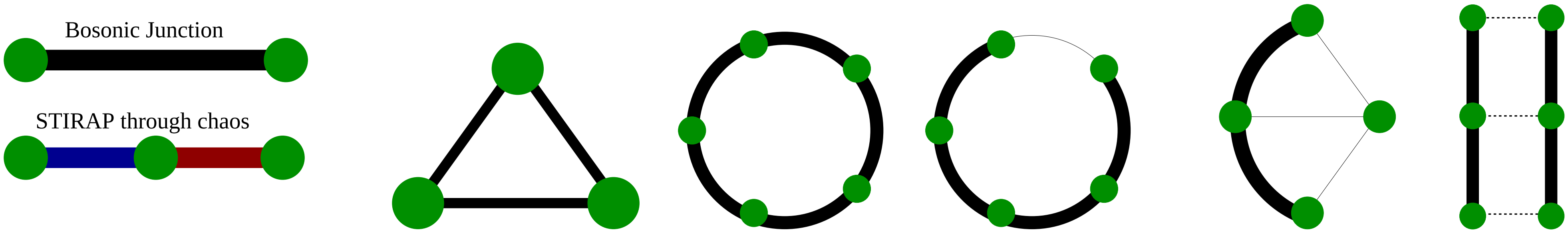
## Main messages II

- Semiclassical fingerprints in the Quantum dynamics.
- Universal Quantum Fluctuations (UQF) in chaos-assisted-depletion regime.
- Breakdown of Quantum to Classical Correspondence (QCC).
- Limitations of the two-orbital approximation.
- Limitations of the Bogolyubov approximation.
- Limitations of the Semiclassical approximation.



# Credits / related studies




Dimer ($L$=2): Bosonic Josephson junction; Pendulum physics [1a].
Driven dimer: Landau-Zener dynamics [1b]; Kapitza effect [1c]; Zeno effect [1d]; Scars [1e].
Rings ($L$>2): Superfluidity [2a]; SF-Mott transition [2b].
Driven trimer: Many body STIRAP [3a]; Hamiltonian Hysteresis [3b]; Quasistatic protocols [3c].
Coupled subsystems ($L$>3): Minimal model for Thermalization [4a,4b].

[1a] Chuchem, Smith-Mannschott, Hiller, Kottos, Vardi, DC (PRA 2010).
[1b] Smith-Mannschott, Chuchem, Hiller, Kottos, DC (PRL 2009).
[1c] Boukobza, Moore, DC, Vardi (PRL 2010).
[1d] Khripkov, Vardi, DC (PRA 2012)
[1e] Khripkov, DC, Vardi (JPA 2013, PRE 2013).
[2a] Arwas, DC (SREP 2015, NJP 2016, PRB 2017, PRA 2019).
[2b] Arwas, DC, Hekking, Minguzzi (PRA 2017).
[3a] Dey, DC, Vardi (PRL 2018, PRA 2019).
[3b] Burkle, Vardi, DC, Anglin (PRL 2019).
[3c] Winsten, DC (SREP 2021, PRA 2023).
[4a] Tikhonenkov, Vardi, Anglin, DC, (PRL 2013).
[4b] Khripkov, Vardi, DC (NJP 2015, PRE 2018, PRA 2020).