Sakmann et al. challenge the two-mode Bose-Hubbard (TMBH) and Gross-Pitaevskii (GP) approximations for double-well Bose-Einstein condensates [1]. They find interesting deviations from the predictions of the TMBH model for dimensionless interaction parameter values as low as $\Lambda \approx 1.5$. Moreover, deviations from the GP approximation are obtained despite the validity of the 1D weak-interaction (1DWI) criterion.

In this comment we clarify that the value of $\Lambda$ by itself is not necessarily relevant to the TMBH modelling validity question. The relevant dimensionless parameters of this problem are both $\nu$ and $\Lambda$ (see definitions below). In the specific setup of [1], if the interaction strength is increased, both $\nu$ and $\Lambda$ become larger, hence larger deviations are observed. Below we illuminate that $\Lambda$ can be experimentally controlled independently of $\nu$. Hence we conclude that the relevance of $\Lambda$ to the validity of the TMBH model (for a fixed value of $\nu$) has not been established, and remains an interesting open question.

**Parameters and approximations.**—The experimental parameters of the 1D double-well system are the axial trap-frequency $\omega$, the barrier transmission coefficient $T$, and the atom number $N$. The 1D interaction strength is $\lambda_0 = 2\hbar \omega_\perp a_s$, where $a_s$ is the s-wave scattering length, and $\omega_\perp$ is the transverse trap frequency. The atoms mass is $m$. These parameters define three characteristic length-scales: the axial trap size $L = \sqrt{\hbar/m\omega}$, the healing length $l_\perp = \sqrt{\hbar/2m\lambda_0 n}$, and the mean distance between atoms $d = 1/n$, where $n = N/2L$ is the average atom density.

The conditions for TMBH validity [2-5] and for 1DWI [6] are $l_\perp \gg L$ and $l_\perp \gg d$ respectively:

$$\nu \equiv (L/l_\perp)^2 = \lambda_0 n/\hbar \omega \ll 1, \quad [\text{TMBH}] \quad (1)$$

$$\gamma \equiv (d/l_\perp)^2 = m\lambda_0/\hbar^2 n \ll 1, \quad [\text{1DWI}] \quad (2)$$

If the TMBH approximation is valid, its effective parameters are the tunnel-splitting $J \approx \hbar \sqrt{F}$, and the interaction-strength $UN \approx \lambda_0 n$. With these we have

$$\Lambda = NU/(2J), \quad \nu = NU/(\hbar \omega), \quad \gamma = \nu/N^2 \quad (3)$$

The interaction parameter $\Lambda$ distinguishes between three interaction regimes [2, 3]: Rabi ($\Lambda < 1$); Josephson ($1 < \Lambda < N^2$); and Fock ($\Lambda > N^2$). For $\Lambda > 2$ preparations with all particles in one well exhibit self-trapping. We rewrite Eq.(1) and Eq.(2) as follows:

$$\nu \ll 1 \iff \Lambda \ll [\hbar \omega/J], \quad (4)$$

$$\gamma \ll 1 \iff \Lambda \ll N^2[\hbar \omega/J]. \quad (5)$$

**Attainability of the Josephson regime.**—It is now clear that the numerical results in [1] do not preclude the experimental realization of the strong-interaction regimes of the TMBH model. Ref.[1] only considers approaching these regimes via increasing $\Lambda \approx \lambda_0 N$, keeping $\omega$ and $J$ fixed. Hence both $\Lambda$ and $\nu$ become larger. However, $\Lambda$ can be increased without damaging Eq.(4), by either of the following strategies: (i) *Decreasing $J$ by increasing trap separation* - resulting in a higher barrier between the harmonic traps while keeping fixed $UN$, with minor variation of the band-gap and the density in each trap; (ii) *Increasing $\omega$ by tightening the traps* - increases $n$ as $\sqrt{\omega}$ as well as raises the barrier height, thus increasing $\Lambda$. Since $n$ scales as $\sqrt{\omega}$, the TMBH small parameter $\nu$ will decrease as $1/\sqrt{\omega}$, indicating improvement of TMBH validity. The $\Lambda \gg 1$ TMBH regime can thus be realized [8].

**The GP condition.**—While $\gamma \ll 1$ indicates phase-coherence in each well it does not guarantee its extension across both wells. Pushing this criterion ad-absurdum, it is clear that for $N > 1$, condition (4) automatically guarantees condition (5), leading to the false conclusion that the GP approximation is valid whenever the TMBH model is. Yet the TMBH model clearly has a quantum domain where GP fails [1–5, 7].

The classical GP limit of the TMBH model is only properly attained by taking the limit $N \to \infty$ while maintaining fixed $\Lambda$ [5, 7]. Quantum-classical correspondence is thus obtained over a timescale $t_b$, the quantum break time, which grows with increasing $N$. It might be argued that substantial deviations from GP are shown in Fig. 1d of [1] despite large $N$, but a closer inspection reveals they are typical: If all particles are initially prepared in one well and $\Lambda \approx 2$, then $t_b$ only grows as $\log(N)$ [5, 7], precisely as demonstrated in Fig. 1c,d.

---

[8] Note this procedure does not apply to 3D traps where the density scales as $L^{-3}$ and the two mode limit is approached at large $L$. In this case, a variation of $N$ will also be required.