Comment on "Exact Quantum Dynamics of a Bosonic Josephson Junction"

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Sakmann *et al.* challenge the two-mode Bose-Hubbard (TMBH) and Gross-Pitaevskii (GP) approximations for double-well Bose-Einstein condensates [1]. They find interesting deviations from the predictions of the TMBH model for dimensionless interaction parameter values as low as $\Lambda \approx 1.5$. Moreover, deviations from the GP approximation are obtained despite the validity of the 1D weak-interaction (1DWI) criterion.

In this comment we clarify that the value of Λ by itself is not necesaarily relavant to the TMBH modelling validity question. The relevant dimensionless parameters of this problem are both ν and Λ (see definitions below). In the specific setup of [1], if the interaction strength is increased, both ν and Λ become larger, hance larger deviations are observed. Below we illuminate that Λ can be experimentally controlled *independently* of ν. Hence we conclude that the relevance of Λ to the validity of the TMBH model (for a fixed value of ν) has not been established, and remains an interesting *open question.*

Parameters and approximations.– The experimental parameters of the 1D double-well system are the axial trap-frequency ω, the barrier transmission coefficient T, and the atom number N. The 1D interaction strength is $\lambda_0 = 2\hbar\omega_\perp a_s$, where a_s is the s-wave scattering length, and $\omega_\perp$ is the transverse trap frequency. The atoms mass is m. These parameters define three characteristic length-scales: the axial trap size $L = \sqrt{\hbar/\mathsf{m}\omega}$, the healing length $l_c = \sqrt{\hbar/2\mathsf{m}\lambda_0 n}$, and the mean distance between atoms $d = 1/n$, where $n = N/2L$ is the average atom density.

The conditions for TMBH validity [2–5] and for 1DWI [6] are $l_c \gg L$ and $l_c \gg d$ respectively:

$$\nu \equiv (L/l_c)^2 = \lambda_0 n/\hbar\omega \ll 1, \quad \text{[TMBH]} \quad (1)$$
$$\gamma \equiv (d/l_c)^2 = \mathsf{m}\lambda_0/\hbar^2 n \ll 1, \quad \text{[1DWI]} \quad (2)$$

If the TMBH approximation is valid, its effective parameters are the tunnel-splitting $J \approx \hbar\omega\sqrt{T}$, and the interaction-strength $UN \approx \lambda_0 n$. With these we have

$$\Lambda = NU/(2J), \qquad \nu = NU/(\hbar\omega), \qquad \gamma = \nu/N^2 \quad (3)$$

The interaction parameter Λ distinguishes between three *interaction regimes* [2, 3]: Rabi ($\Lambda<1$); Josephson ($1<\Lambda<N^2$); and Fock ($\Lambda>N^2$). For $\Lambda > 2$ preparations with all particles in one well exhibit self-trapping. We rewrite Eq.(1) and Eq.(2) as follows:

$$\nu \ll 1 \quad \Longleftrightarrow \quad \Lambda \ll [\hbar\omega/J] \;, \quad (4)$$
$$\gamma \ll 1 \quad \Longleftrightarrow \quad \Lambda \ll N^2[\hbar\omega/J] \;. \quad (5)$$

Attainability of the Josephson regime.- It is now clear that the numerical results in [1] do not preclude the experimental realization of the strong-interaction regimes of the TMBH model. Ref.[1] only considers approaching these regimes via increasing $\lambda \approx \lambda_0 N$, keeping ω and J fixed. Hence both Λ and ν become larger. However, Λ can be increased without damaging Eq.(4), by either of the following strategies: **(i) Decreasing J by increasing trap separation -** resulting in a higher barrier between the harmonic traps while keeping fixed UN, with minor variation of the band-gap and the density in each trap; **(ii) Increasing ω by tightening the traps -** increases n as $\sqrt{\omega}$ as well as raises the barrier height, thus increasing Λ. Since n scales as $\sqrt{\omega}$, the TMBH small parameter ν will *decrease* as $1/\sqrt{\omega}$, indicating *improvement* of TMBH validity. The $\Lambda \gg 1$ TMBH regime can thus be realized [8].

The GP condition.- While $\gamma \ll 1$ indicates phase-coherence *in each well* it does not guarantee its extension *across both wells.* Pushing this criterion ad-absurdum, it is clear that for $N > 1$, condition (4) automatically guarantees condition (5), leading to the false conclusion that the GP approximation is valid whenever the TMBH model is. Yet the TMBH model clearly has a quantum domain where GP fails [1–5, 7].

The classical GP limit of the TMBH model is only properly attained by taking the limit $N \to \infty$ while maintaining fixed Λ [5, 7]. Quantum-classical correspondence is thus obtained over a timescale t_b, the quantum break time, which grows with increasing N. It might be argued that substantial deviations from GP are shown in Fig. 1d of [1] despite large N, but a closer inspection reveals they are typical: If all particles are initially prepared in one well and $\Lambda \approx 2$, then t_b only grows as $\log(N)$ [5, 7], precisely as demonstrated in Fig. 1c,d.

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[8] Note this procedure does not apply to 3D traps where the density scales as L^{-3} and the two mode limit is approached at large L. In this case, a variation of N will also be required.