Decoherence of a particle in a ring

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References:

D.C. and Baruch Horovitz, arXiv (2007).

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Outline:

The dephasing factor formula

Derivations and Ideology

The power spectrum of the fluctuations

The calculation for a particle in a ring

Implications on mass renormalization

The dephasing factor formula

$$P_\varphi = \exp(-F(t))$$

$$F(t) \;\; = \;\; t \int d\boldsymbol{q} \int \frac{d\omega}{2\pi} \, \tilde{S}(\boldsymbol{q},\omega) \, \tilde{P}(-\boldsymbol{q},-\omega)$$

[Cohen (1998), Cohen and Imry (1999)]

$$F(t) = t \int d\boldsymbol{q} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \, \tilde{S}(\boldsymbol{q},\omega) \, \tilde{P}_\infty(-\boldsymbol{q},-\omega') \left[\frac{(2/t)}{(1/t)^2 + (\omega - \omega')^2} \right]$$

One should use *non*-symmetrized spectral functions.

For finite temperatures:

$$F(t) \;\; \sim \;\; \Gamma_\varphi t$$

At low temperatures: $\log(t)$ spreading?

Derivations

Semiclassical FV approach [Cohen (1997)]:

Given $\tilde{S}(q,\omega)$ the environment is modeled

as a bath of harmonic oscillators

$$P_{\varphi}(t) \;=\; \left| \left\langle \, U[r^A]\chi \, \middle| \, U[r^B]\chi \, \right\rangle \right| \;=\; \mathrm{e}^{-S_N[r^A,r^B]}$$

Scattering point of view [Cohen and Imry (1999)]:

Use of non-symmetrized spectral functions is conjectured on the basis of Fermi-Golden-Rule as a way to overcome an identified problem with the semiclassical approximation.

Purity based decoherence point of view:

Leads to a variation of the Fermi-Golden-Rule

$$P_{\varphi}(t) \;=\; \sqrt{\mathrm{trace}(\rho_{\mathrm{sys}}^2)} \;=\; \sqrt{\mathrm{trace}(\rho_{\mathrm{env}}^2)}$$

Ideology

The notion of “dephasing factor” is problematic.

One should study equilibrium properties:

- Current-Current correlation function
- Ground state energy vs flux and temperature

The theory of the “dephasing factor” should be used in order shed light on the physics involved.

The particle in a ring problem [See Refs (2001-2007)]:

$$\left.\frac{\partial^2 \mathsf{F}}{\partial \Phi^2}\right|_{\Phi=0} \quad = \quad \frac{e^2}{M^* L^2} f(M^* L^2 T)$$

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The characterization of a fluctuating field

$$\tilde{S}(q,\omega) = \mathrm{FT}_{q,\omega}\left[\langle \hat{\mathcal{U}}(x_2,t_2)\hat{\mathcal{U}}(x_1,t_1)\rangle\right]$$

$$\mathcal{F} = -\mathcal{U}'(x,t)$$

$$\tilde{S}(\omega) = \int \frac{dq}{2\pi} q^2 \tilde{S}(q,\omega)$$

$$\tilde{S}_{\mathrm{ohmic}}(\omega) = \frac{2\hbar\eta\omega}{1-\mathrm{e}^{-\hbar\omega/T}} \qquad \text{for} \quad |\omega| < \omega_c$$

$$\eta \quad = \quad \frac{1}{2\hbar\omega}\left[\tilde{S}(\omega) - \tilde{S}(-\omega)\right] \quad = \quad \frac{\tilde{S}(\omega{=}0)}{2T}$$

$$\tilde{S}_{\mathrm{CaldeiraLeggett}}(q,\omega) \quad = \quad \tilde{S}_{\mathrm{ohmic}}(\omega) \times 3\frac{(2\pi)^3\delta^3(\boldsymbol{q})}{\boldsymbol{q}^2}$$

$$\tilde{S}_{\mathrm{DirtyMetal}}(q,\omega) \quad \approx \quad \tilde{S}_{\mathrm{ohmic}}(\omega) \times \frac{4\pi\ell^3}{\boldsymbol{q}^2} \qquad \text{for} \quad |\boldsymbol{q}| \lesssim \frac{1}{\ell}$$

$$\alpha \quad = \quad \frac{1}{2\pi}\eta\ell^2 \quad = \quad \frac{e^2}{8\pi^2\sigma\ell} \quad = \quad \frac{3}{8(k_F\ell)^2}$$

$$\gamma \quad = \quad \frac{\eta}{M} \quad = \quad \frac{2\pi\alpha}{M\ell^2}$$

The fluctuations within a ring

$$\tilde{S}(q,\omega) \quad = \quad \mathrm{FT}_{q,\omega}\left[\langle \hat{\mathcal{U}}(x(\theta_2),t_2)\hat{\mathcal{U}}(x(\theta_1),t_1)\rangle\right]$$

$$\quad = \quad \tilde{S}_{\mathrm{ohmic}}(\omega) \times \sum_m w_m \, 2\pi\delta(q-q_m)$$

$$q_m \quad = \quad \frac{2\pi}{L} m$$

$$\sum_{m=-\infty}^{\infty} w_m q_m^2 = 1$$

$$w_{\pm 1} = \frac{1}{2}\left(\frac{L}{2\pi}\right)^2 \qquad \text{for } \ell \gg L$$

$$w_m = \frac{\ell^2}{2\pi} \times \frac{1}{\mathcal{M}} \ln\left(\frac{\mathcal{M}}{|m|}\right) \qquad \text{for } \ell \ll L$$

$$\mathcal{M} = \mathrm{maximum}\{\,1,\ L/(2\pi\ell)\,\}$$

Digression: Noise induced dephasing

$$\text{phase} = \int_0^t \mathcal{U}(r(t'), t') dt'$$

$$\begin{aligned}
F(t) &= \langle \text{phase}^2 \rangle = \iint dt' dt'' \, \langle \mathcal{U}(r(t'), t') \mathcal{U}(r(t''), t'') \rangle \\
&= \iint dx' dx'' \iint dt' dt'' \, \langle \mathcal{U}(x', t') \mathcal{U}(x'', t'') \rangle \, \delta(x' - r(t')) \delta(x'' - r(t'')) \\
&= \iint dq d\omega \, \tilde{S}(q, \omega) \iint \iint \mathrm{e}^{iq(x' - x'') - i\omega(t' - t'')} \, \delta(x' - r(t')) \delta(x'' - r(t'')) \\
&= \iint dq d\omega \, \tilde{S}(q, \omega) \iint dt' dt'' \mathrm{e}^{-i\omega(t' - t'')} \exp(iqr(t')) \exp(iqr(t'')) \\
&= t \iint dq d\omega \, \tilde{S}(q, \omega) \, \tilde{P}(-q, -\omega)
\end{aligned}$$

$$F(t) = t \int d\boldsymbol{q} \int \frac{d\omega}{2\pi} \, \tilde{S}(\boldsymbol{q}, \omega) \, \tilde{P}(-\boldsymbol{q}, -\omega')$$

If we treat properly the end points:

$$\tilde{P}(q, \omega) \quad \longmapsto \quad t \left[\frac{\sin(\omega t/2)}{\omega t/2} \right]^2 (*) \, \tilde{P}(q, \omega)$$

Digression: Noise induced spreading

$$\text{displacement} = \int_0^t f(t')dt'$$

$$F(t) = \int_0^t \int_0^t C(t_1 - t_2)\, dt_1 dt_2 \quad = \int \frac{d\omega}{2\pi} \tilde{C}(\omega) \left[\frac{\sin(\omega t/2)}{\omega/2} \right]^2$$

$$\Gamma(t) = \frac{d}{dt} F(t) = \int_{-t}^{t} C(\tau)\, d\tau$$

High temperatures: $\tilde{C}(\omega) = 2\eta T$

implies $C(t-t') = 2\eta T \delta(t-t')$

$$F(t) = 2\eta T\, t$$

Zero temperature: $\tilde{C}(\omega) = \eta|\omega|$

implies $C(t-t') \sim -(\eta/\pi)/(t-t')^2$

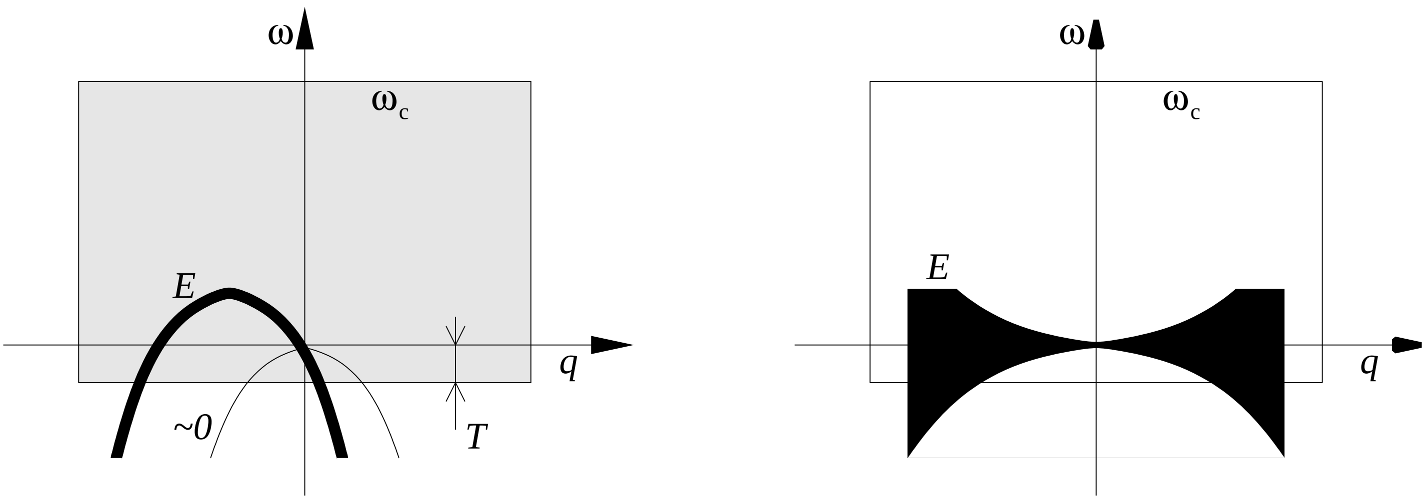
$$F(t) = \frac{2}{\pi}\eta\, \ln(\omega_c t) + \text{const}$$

The calculation of the dephasing

$$\tilde{P}(q,\omega) = \int \left[\langle \mathrm{e}^{-iqx(\tau)} \mathrm{e}^{iqx(0)} \rangle - \langle \mathrm{e}^{iqx} \rangle^2 \right] \mathrm{e}^{i\omega\tau}\, d\tau$$

$$\tilde{P}(q,\omega) \;=\; 2\pi\delta(\omega - \omega(q))$$ Ballistic case

$$\tilde{P}(q,\omega) \;=\; \frac{2Dq^2}{\omega^2 + (Dq^2)^2}$$ Diffusive case



$$\int_{\omega\neq 0} \tilde{P}(q,\omega) \frac{d\omega}{2\pi} \;=\; 1$$ for any q

If it is narrow compared with ω_c and T then

$$\int \tilde{S}(q_m,\omega) \tilde{P}(-q_m,-\omega) \frac{d\omega}{2\pi} \;\approx\; 2\eta T w_m$$

Some results

$$\Gamma_\varphi \;=\; 2\eta T \times \sum_{\text{effective}} w_m \;\approx\; 2\eta T\, \bar{w} \mathcal{M}_{\text{eff}}$$

$$\Gamma_\varphi \;=\; 2\eta T \times \left(\frac{L}{2\pi}\right)^2 \qquad \left[\text{Caldeira-Leggett}\right]$$

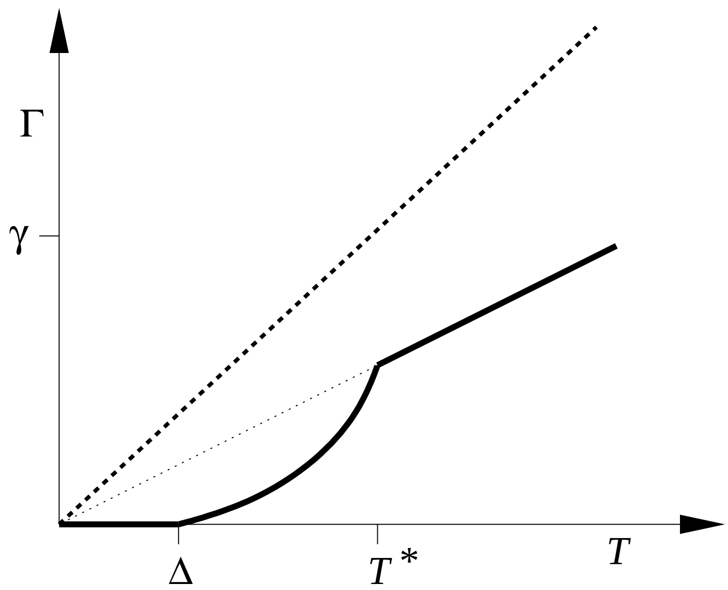


Figure assumes $\eta\ell^2 \ll 1$ and

$$\omega_c[\text{effective}] \;=\; \max\{\,\gamma,\; \Delta\,\} \;=\; \gamma$$

$$T^* \;=\; \gamma \;=\; \eta/M$$

$$\Gamma_\varphi \;\approx\; \begin{cases} (2\eta\ell^2)^{3/2}\, T & \text{for } T > T^* \\ \eta\ell^3 M^{1/2}\, T^{3/2} & \text{for } T < T^* \end{cases}$$

Dirty metal environment

Low temperatures:

$$\Gamma_\varphi \quad = \quad \frac{e^2}{4\pi^2\sigma} T\, k_T \ln\left(\frac{1}{k_T \ell}\right)$$

Zero temperatures:

$$p_\varphi(t) \quad = \quad \frac{\eta}{\pi}\sum_m w_m \ln\left(\frac{\omega_c}{(1/t)+\omega(q_m)}\right)$$

Caldeira-Leggett limit:

$$p_\varphi(t) = \frac{\eta}{\pi}\left(\frac{L}{2\pi}\right)^2 \ln\left(\omega_c t\right)$$

The dirty limit:

$$p_\varphi \quad \sim \quad \int_0^{1/\ell} dq\, \eta \ell^3 \ \ln\left[\frac{M\omega_c}{q^2}\right] \quad \sim \quad \eta\ell^2 \ \ln\left(\frac{\omega_c \ell^2}{M}\right)$$

The criterion for not having dephasing

$$\lim_{L\to\infty} p_\varphi(t=\infty) \quad \ll \quad 1$$

Mass renormalization

Measure of coherence:

$$x(T,L) \quad = \quad p_\varphi\left(t{=}\frac{1}{\Delta_{\text{eff}}}\right) \quad = \quad \frac{\Gamma_\varphi}{\Delta_{\text{eff}}}$$

For a dirty metal:

$$x(T,L) = \eta \bar{w} M L^2 T = \begin{cases} \eta M \ell^3 LT, & L \gg \ell \\ \eta M L^4 T, & L \ll \ell \end{cases}$$

$$y(T,L) \quad \equiv \quad \frac{M^*}{M}$$

extracted from:

$$\left.\frac{\partial^2 \mathsf{F}}{\partial \Phi^2}\right|_{\Phi=0} \quad = \quad \frac{e^2}{M^* L^2} f(M^* L^2 T)$$

Conclusions

The dephasing factor as defined by purity is related to FGR transitions, which justifies the non-symmetrization conjecture.

Results for the dephasing factor are obtained in the case of a dirty metal environment for both $\ell \ll L$ and $\ell \gg L$.

The results for the dephasing rate, and hence for the measure of coherence, suggest LT and L^4T scaling respectively, in agreement with MC.

The dephasing factor formula gives power law decay of coherence at zero temperature, only in the Caldeira-Leggett limit.