Quantum vs. stochastic non-equilibrium steady states of sparse or frustrated systems

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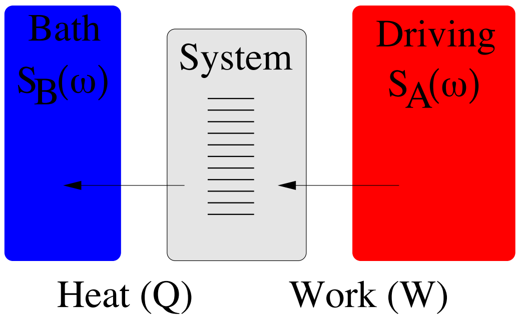
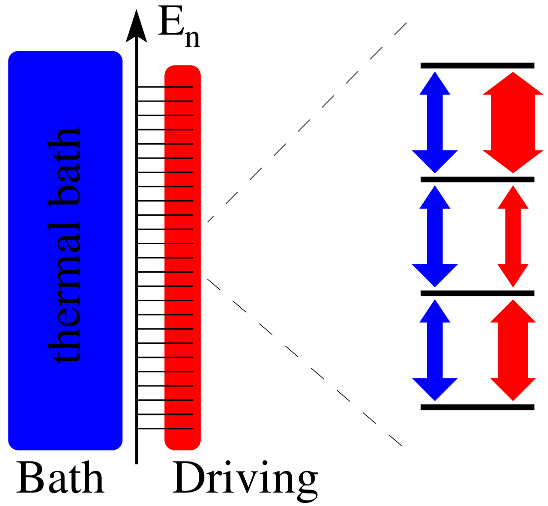
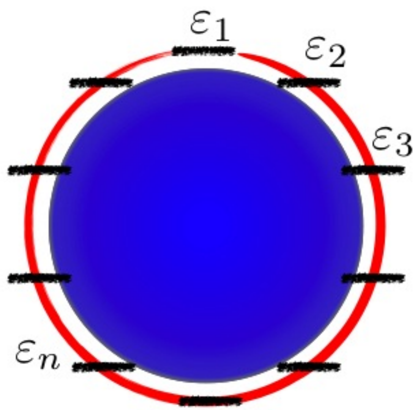
NESS Paradigm: Driven system + Bath

$$\mathcal{H}_{\mathrm{total}} \;=\; E_n\delta_{nm} \;-\; \color{red}{f(t)}\color{black}{V_{nm}} \;+\; \color{blue}{\mathcal{H}_{Bath}}$$

$\color{red}{\varepsilon^2}$ $= \langle f(t)f(t')\rangle \equiv$ Driving intensity $\qquad$ $\color{blue}{T_B}$ $\equiv$ Bath temperature







Master equation description

$$\mathcal{H}_{\text{total}} \;=\; E_n \delta_{nm} \;-\; f(t) V_{nm} \;+\; \mathcal{H}_{Bath}$$

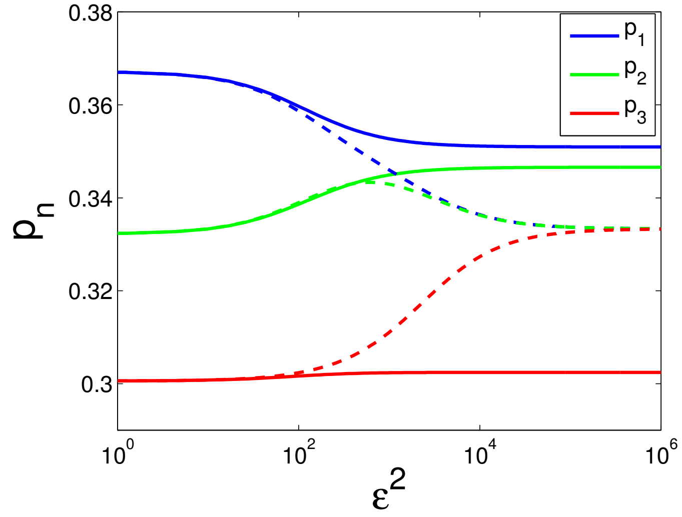
$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] - \frac{\varepsilon^2}{2}[V, [V, \rho]] + \boldsymbol{\mathcal{W}}^{\beta} \rho$$ Quantum master equaton

$$\frac{dp_n}{dt} = \sum_m \mathcal{W}_{nm} p_m(t) - \mathcal{W}_{mn} p_n(t)$$ Stochastic rate equaton

$$\mathcal{W}_{nm} = w^{\varepsilon}_{nm} + \frac{2 w^{\beta}_{nm}}{1 + \mathrm{e}^{(E_n - E_m)/T_B}} \qquad w^{\varepsilon}_{nm} \;=\; \varepsilon^2 |V_{nm}|^2$$

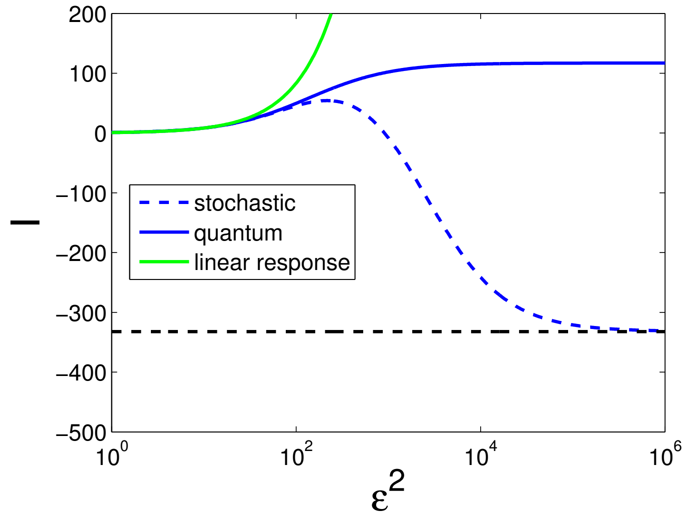
NESS current in a ring

$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] - \frac{\varepsilon^2}{2}[V, [V, \rho]] + \boldsymbol{\mathcal{W}}^{\beta}\rho$$



Current in the stochastic model

$$\begin{aligned} I^{n\rightarrow m} &= \mathcal{W}_{mn}p_n - \mathcal{W}_{nm}p_m = \\ &= I^{\varepsilon} + I^{\beta} \end{aligned}$$



Current in the quantum model”

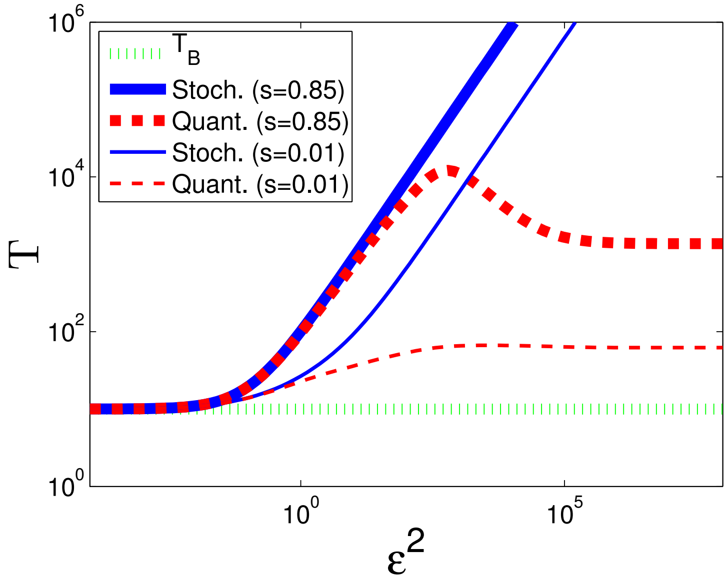
$$\begin{aligned} I^{\varepsilon}_{n\rightarrow m} &= tr\left(\hat{I}^{\varepsilon}_{n\rightarrow m}\rho\right) \\ \hat{I}^{\varepsilon}_{n\rightarrow m} &= i\varepsilon^2\left[\hat{J}^{nm}, \hat{V}\right] \\ \hat{J}^{nm} &= i\Big(|m\rangle V_{mn}\langle n| - |n\rangle V_{nm}\langle m|\Big) \end{aligned}$$

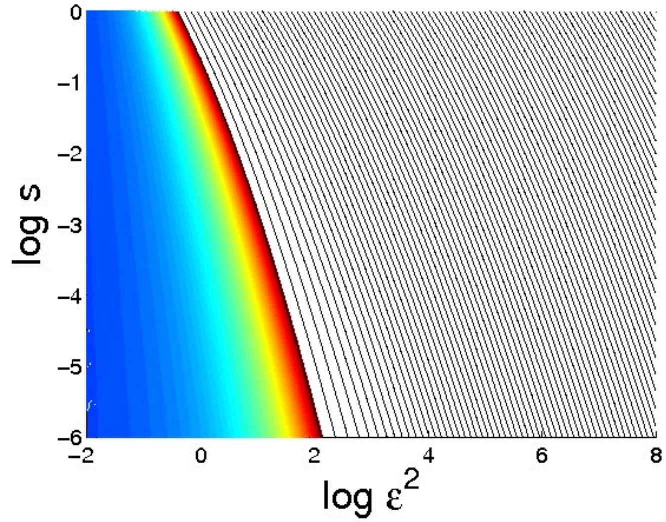
NESS temperature in the chain model

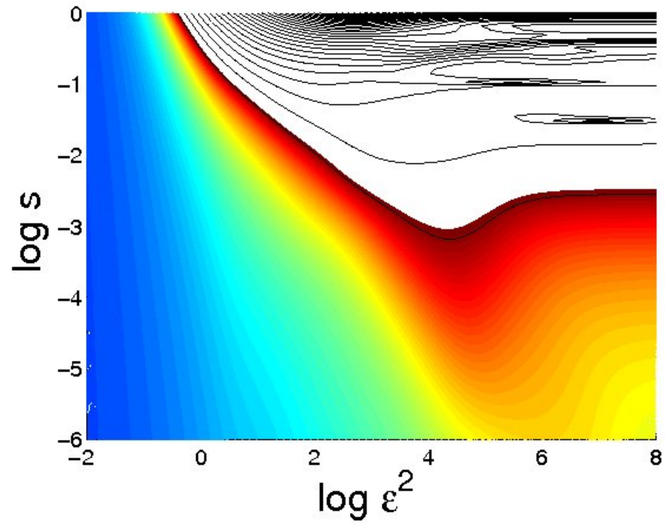
$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] - \frac{\varepsilon^2}{2}[V, [V, \rho]] + \mathcal{W}^{\beta}\rho$$

$$\frac{p_n}{p_m} = \exp\left(-\frac{E_n - E_m}{T_{nm}}\right)$$

$$T_{\text{system}} = \text{average}[T_{nm}]$$



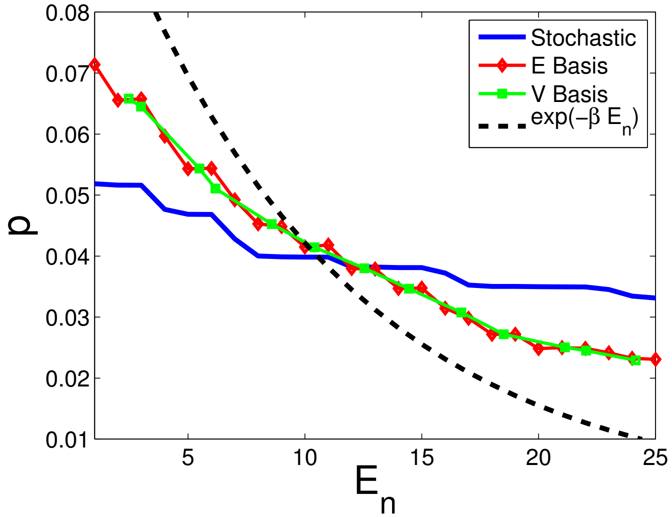




Quantum NESS for toy chain model with n.n. transitions

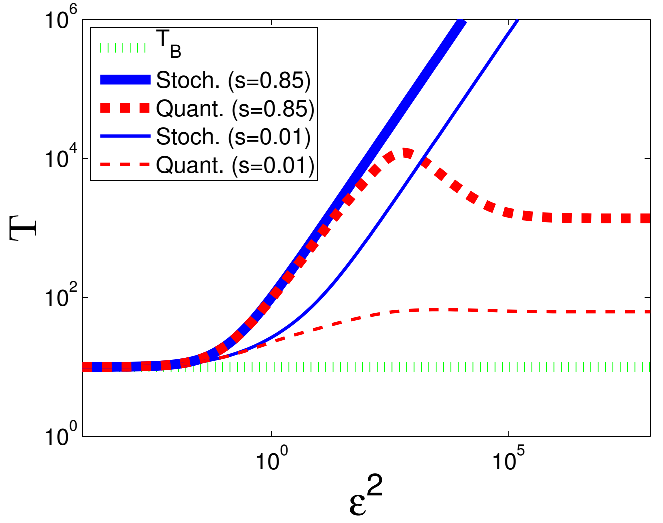
$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] - \frac{\varepsilon^2}{2}[V, [V, \rho]] + \mathcal{W}^{\beta}\rho$$

For very strong driving,
the NESS is a mixture of V eigenstates:



$$p_r \sim \exp(-\langle E\rangle_r / T_B)$$

leading to:



$$p_n \sim \exp(-E_n / T_\infty)$$

$T_B < T_\infty < \infty$ [depends on the sparsity]

How the temperature is defined

$$\mathcal{H}_{\text{total}} = \{E_n\} - f(t)\{V_{nm}\} + \{W_{nm}\} \cdot Bath$$

The sources temperature: $T_A = \infty$

$$\tilde{S}_A(\omega) \ \equiv \ \text{FT} \ \langle \dot{f}(t)\dot{f}(0)\rangle$$

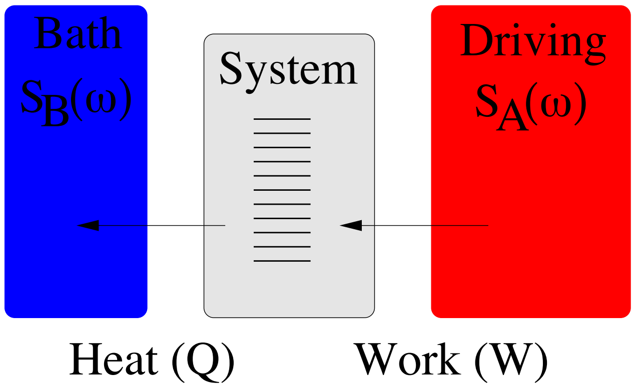
The bath temperature: T_B

$$\tilde{S}_B(\omega)/\tilde{S}_B(-\omega) = \exp(-\omega/T_B)$$

Temperature of the system?

$$\dot{\mathsf{W}} = \text{rate of heating} = \frac{D(\varepsilon)}{T_{\text{system}}}$$

$$\dot{\mathsf{Q}} = \text{rate of cooling} = \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}}$$



$$\frac{p_n}{p_m} = \exp\left(-\frac{E_n - E_m}{T_{nm}}\right)$$

$$T_{\text{system}} = \text{average}[T_{nm}]$$

$$= \left(1 + \frac{D(\varepsilon)}{D_B}\right) T_B$$

Conclusions

1. In the chain model the stochastic NESS resembles that of a glassy phase (wide distribution of microscopic temperatures).

2. Definition of effective NESS temperature, and extension of the FDT phenomenology.

3. For very strong driving - quantum saturation of the NESS temperature ($T \to T_{\infty}$).

4. An expression for the current operator in the reduced description has been derived.

5. The dependence of the current on ε^2 exhibits a non trivial crossover from LRT to saturation. QM saturation is different from the stochastic saturation.