The Quantum Chaos Group 2016

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[1] Geva Arwas (PhD) - Chaos in low dimensional Bose-Hubbard circuits.
[2] Isaac Weinberg (MSc) - Localization, heat transport.
[3] Daniel Hurowitz (PhD) - Non-equilibrium Stat-Mech.
[4] Dekel Shapira (PhD) - Stochastic spreading.

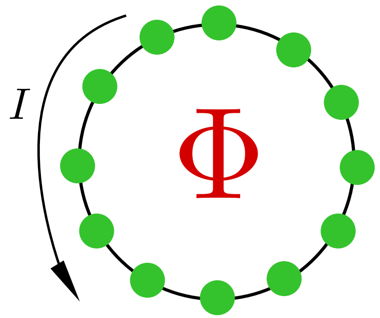
Metastability and Thermalization in Bose-Hubbard circuits

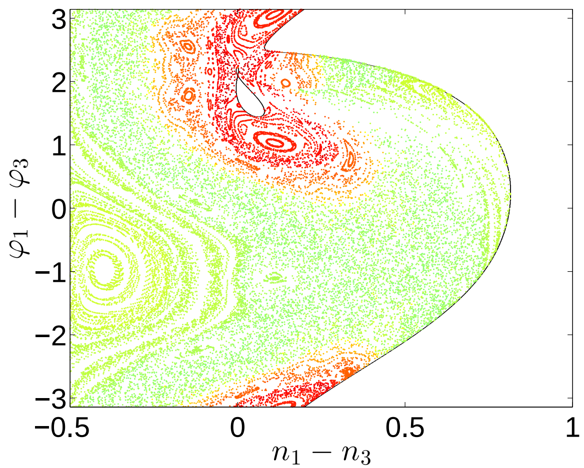
The system consists of N bosons in M sites. Later we add a gauge-field Φ.

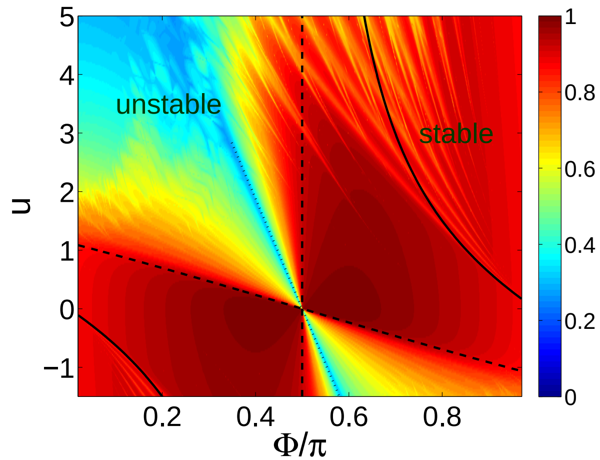
$$\mathcal{H} = \sum_{j=1}^{M} \left[\frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2} \left(a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1} \right) \right]$$

This is like M coupled oscillators with

$$H = \sum_{j=1}^{M} \left[\frac{U}{2} \boldsymbol{n}_j^2 - K \sqrt{\boldsymbol{n}_{j+1} \boldsymbol{n}_j} \cos\left(\boldsymbol{\varphi}_{j+1} - \boldsymbol{\varphi}_j \right) \right]$$







[1] Geva Arwas, Doron Cohen [**New Journal of Physics 2016**]

[2] Geva Arwas, Amichay Vardi, Doron Cohen [**Scientific Reports 2015**]

Localization

Stochastic rate equation	$\frac{d}{dt}\mathbf{p} = \mathbf{W}\mathbf{p},$	$\mathbf{p} \equiv \text{vector}\{p_n\}$
Resistor network problem	$\frac{d}{dt}\mathbf{Q} = \mathbf{G}\mathbf{V},$	$\mathbf{Q} \equiv \text{vector}\{Q_n\}, \quad Q_n = CV_n$
Schrödinger equation	$\frac{d}{dt}\boldsymbol{\psi} = -i\boldsymbol{\mathcal{H}}\boldsymbol{\psi},$	$\boldsymbol{\psi} \equiv \text{vector}\{\psi_n\}$
Newton eq for balls+springs	$\frac{d^2}{dt^2}\mathbf{u} = -\mathbf{K}\mathbf{u},$	$\mathbf{u} \equiv \text{vector}\{u_n\}$

Real symmetric matrices:

$$\mathbf{W} \equiv \text{diag}\{-\gamma_n\} + \text{offdiag}\{w_{nm}\}$$

Conservative if:

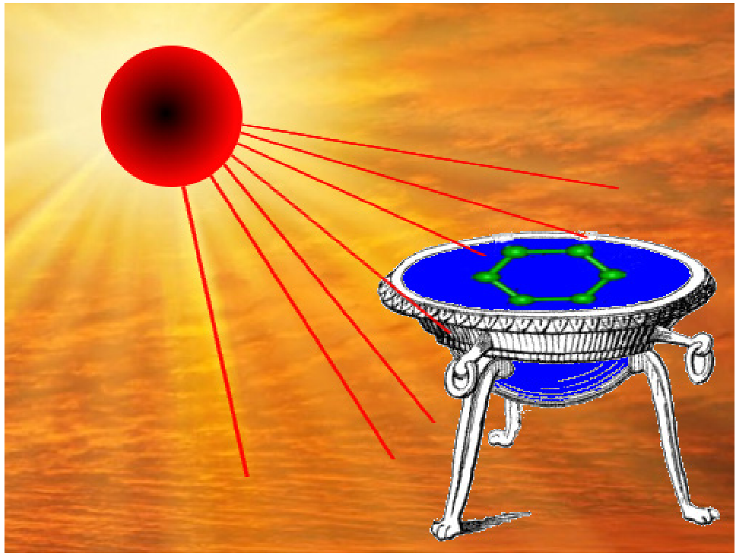
$$\gamma_n = \sum_{m \neq n} w_{mn}$$

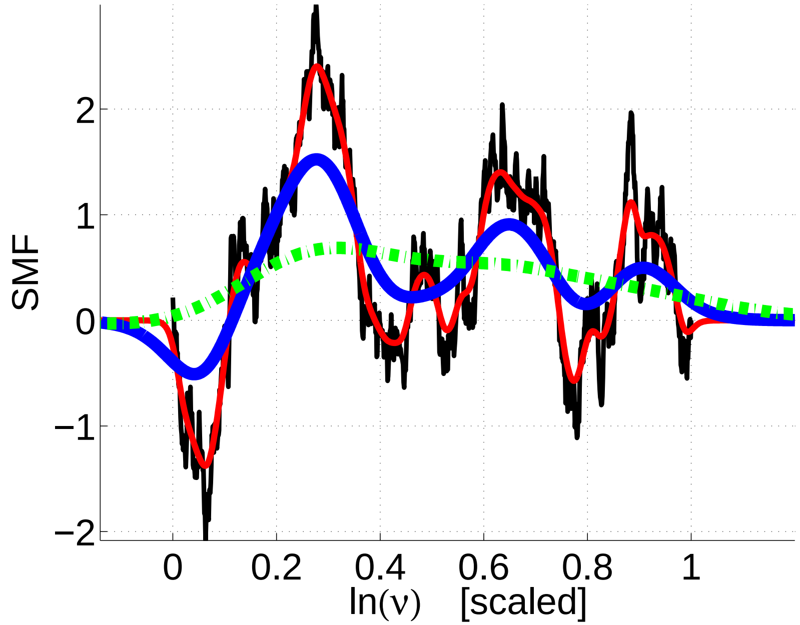
- Balls-connected-by-springs $\rightsquigarrow$ heat-transport problem
- Debye localization versus Anderson localization
- The "resistor network" aspect: glassy disorder, percolation
- Duality: Glassy off-diagonal disorder $\mapsto$ weak diagonal disorder.

[**1**] Yaron de Leeuw, Doron Cohen [**PRE 2012**].

[**2**] Isaac Weinberg, Yaron de Leeuw, Tsampikos Kottos, Doron Cohen [**arXiv 2016**].

Nonequilibrium Steady State of Closed Circuits





$$w_{n,m} \;=\; w^{\beta}_{n,m} + \nu g_{n,m}$$

$$I(\nu) \;\sim\; \frac{1}{N}\, w_{\varepsilon}\, \mathrm{e}^{-B}\, 2\sinh\left(\frac{\mathcal{S}_{\circlearrowleft}}{2}\right)$$

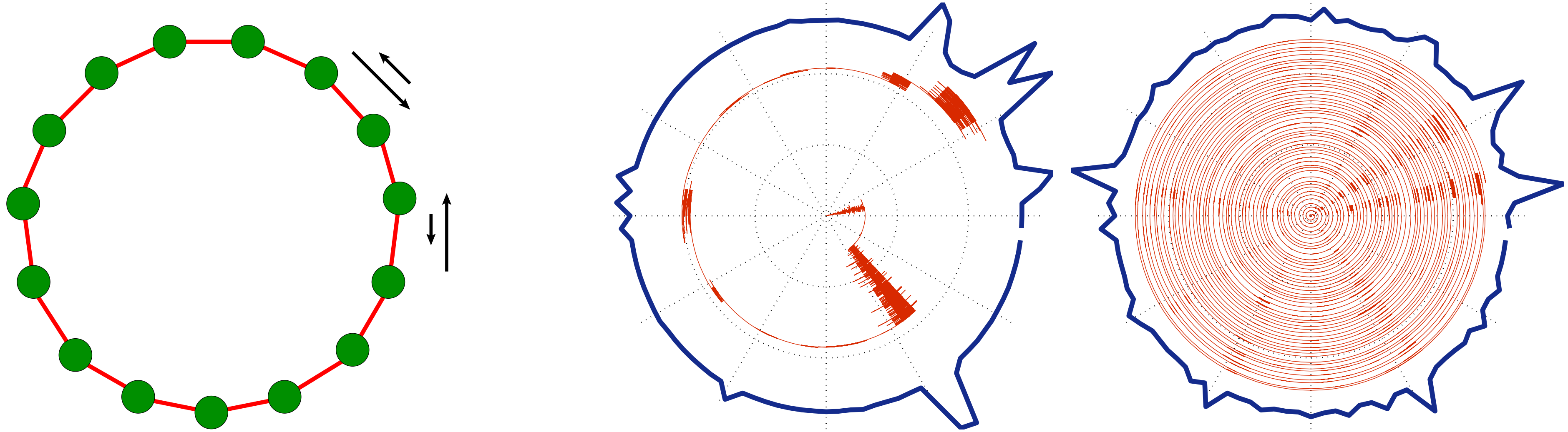
$\mathcal{S}_{\circlearrowleft}$ - Stochastic Motive Force (aka “affinity”)

B - Effective Activation Barrier

[**1**] Daniel Hurowitz, Saar Rahav, Doron Cohen [**PRE 2013**]

[**2**] Daniel Hurowitz, Doron Cohen [**PRE 2014**]

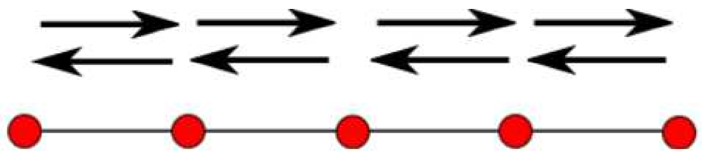
Percolation, sliding, localization and relaxation in topologically closed circuits



[**1**] Daniel Hurowitz, Doron Cohen [**Scientific Reports 2016**]

Types of random walk

Simple random walk, aka Brownian motion [Einstein]



Strictly periodic lattice ($a = 1$). All rates are equal (w)

$$D \quad = \quad w \qquad \text{(near-neighbor hopping)}$$

Random walk on a disordered lattice [1]

Random lattice. Symmetric transition rates w_n

$$P(w) \quad \propto \quad w^{\alpha-1} \quad \text{(for small } w\text{)}$$

$$D \quad = \quad \left\langle \frac{1}{w} \right\rangle^{-1}$$

Non-percolating for $\alpha < 1$

Percolation-like transition

$\alpha \sim$ sparsity parameter

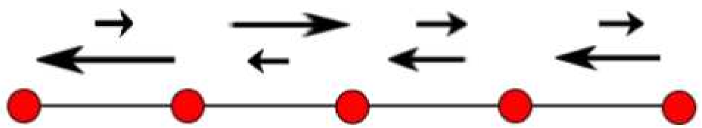
(resistor network calculation)

Random walk in random environment [2]

Rates allowed to be asymmetric: $\overleftarrow{w}_n \neq \overrightarrow{w}_n$

Sub-diffusion for low bias [Sinai, Derrida,...]

Sliding transition



[1] Alexander, Bernasconi, Schneider, Orbach, Rev. Mod. Phys. (1981).

[2] Bouchaud, Comtet, Georges, Doussal, Annals Phys. (1990).

Definition of the model

Conservative rate equation

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{W}\boldsymbol{p}$$

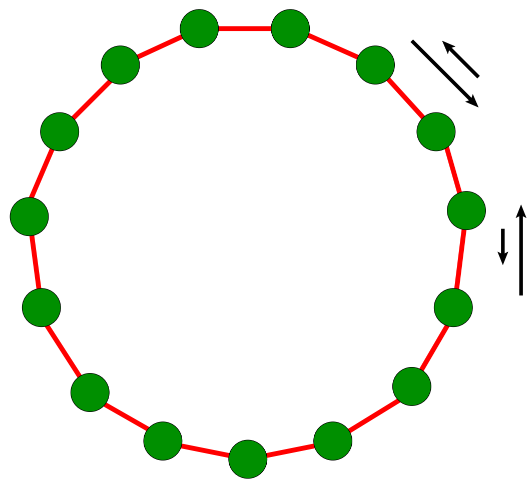
Rates allowed to be asymmetric $\overrightarrow{w}_n / \overleftarrow{w}_n = e^{\mathcal{E}_n}$

Affinity: $S_{\circlearrowleft} = \sum \mathcal{E}_n = Ns$

Stochastic field: $\mathcal{E}_n = s + \varsigma_n$ where $\varsigma_n \in [-\sigma, \sigma]$

Transition rates across n^{th} bond are $w_n \mathrm{e}^{\pm \mathcal{E}_n/2}$

Resistor network disorder: $P(w) \propto w^{\alpha - 1}$



$$\boldsymbol{W} = \begin{bmatrix} -\gamma_1 & w_{1,2} & 0 & \dots \\ w_{2,1} & -\gamma_2 & w_{2,3} & \dots \\ 0 & w_{3,2} & -\gamma_3 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

Sum of elements in each column is zero

How do spectral properties of $\boldsymbol{W}$ depend on (α, σ, s)?

$\alpha \sim$ sparasity, $\sigma \sim$ field disorder, $s \sim$ affinity

Related models

Vortex depinning in type II superconductors ($s =$ applied transverse magnetic field)

- Hatano, Nelson, PRL (1996), PRB (1997).
- Shnerb, Nelson, PRL (1998).
- Follow ups: Brouwer, Silvestrov, Beenakker, PRB (1997). Goldsheid, Khoruzhenko, PRL (1998). Feinberg, Zee, PRE (1999). Molinari, Linear Algebra and its Applications (2008).

Pulling pinned polymers, DNA denaturation ($s =$ pulling force)

- Lubensky, Nelson, PRL (2000), PRE (2002).

Population biology ($s =$ convective flow of bacteria relative to the nutrients)

- Nelson, Shnerb, PRE (1998).
- Dahmen, Nelson, Shnerb, Springer (1999).

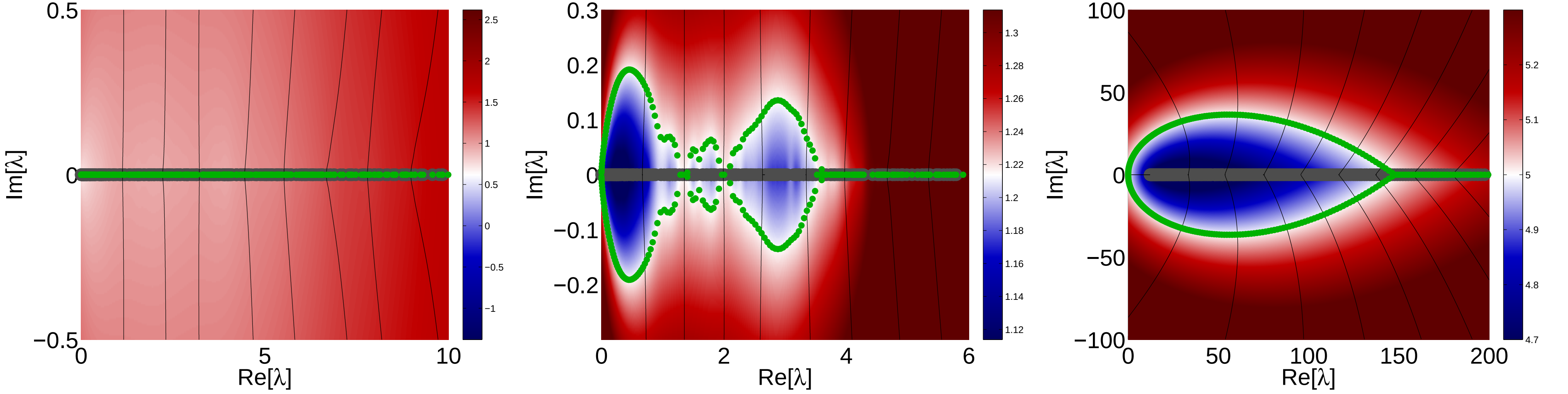
Molecular motors ($s =$ affinity of chemical cycle)

- Fisher, Kolomeisky, PNAS (1999).
- Rief et al, PNAS (2000).
- Kafri, Lubensky, Nelson, Biophysical Journal (2004), PRE (2005).

Non of the above concern relaxation modes of a conservative systems!

Implications of the percolation and sliding transitions on relaxation modes of the ring?

The spectrum of W



- Due to conservativity $\lambda_0 = 0$
- The other eigenvalues are $\{-\lambda_k\}$
- Complex low-laying bubble for $s > s_c$
- Complexity saturation for $s \gg s_\infty$
- Implication of the percolation transition?
- Implication of the sliding transition?