Diffractive energy spreading
and its semiclassical limit

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Reference:
A. Stotland and D. Cohen,
Driven Systems

\[ \mathcal{H} = \mathcal{H}(Q, P; X(t)) \]

Energy is not a constant of motion!

Moments of the energy distribution:

\[ \delta^r E = \int \rho_t(E)E^r dE \]

\( r = 1 \) expectation value
\( r = 2 \) variance

Bohr quantum-classical correspondence (QCC):

- Gaussian wavepacket
- Smooth potentials

\[ \Rightarrow \text{The same moments} \]

Restricted versus detailed QCC:

- \( r = 1, 2 \) restricted QCC (robust)
- \( r > 2 \) detailed QCC (fragile)
Motivation -
the theory of response

“Response” has to do with the $t \to \infty$ dynamics.

Can we expect QCC?

In Linear Response Theory - YES.

- for short times - Restricted QCC
- for long times - Central limit theorem

The long time behavior is determined by the short time behavior.

Explanation:
Response is expressed using correlation functions.
Restricted QCC is extended to correlation functions.
Stochastic-like behavior is established before QCC breakdown.
Extrapolation using central limit theorem.
Three problems

1. particles pulsed by a step potential - the worst case for QCC
2. particles in a box with a moving wall
3. particles in a ring driven by an EMF.

Driving ⇒ jumps in energy space.
The route towards QCC is highly non-trivial.

(1) → Solved analytically.
(2),(3) → Bloch electrons in a tight binding model:
   • The energy levels are like sites in a lattice.
   • The mean level spacing is like a constant electric field
     long range hopping $\propto 1/(n - m)$.
Solved both analytically and numerically.
Phase space picture

- **Step potential**

\[ V(x) \]

\[ t > 0 \]

\[ t = 0 \]

\[ V_{\text{step}} \]

\[ x = 0 \]

\[ \delta E_{\text{cl}} = -V_{\text{step}} \]

- **Moving wall**

\[ V(x) \]

\[ E \]

\[ (Q,P) \]

\[ \text{Velocity} \]

\[ \delta E_{\text{cl}} = 2mv_{E}V_{\text{wall}} \]

\( \text{semiclassical regime: } \delta E_{\text{cl}} \gg \Delta \)
Phase space picture (cont.)

- Ring

\[ \text{EMF} = V \text{oltage} \]

\[ \delta E_{\text{cl}} = eV_{\text{EMF}} \]

semiclassical regime: \( \delta E_{\text{cl}} \gg \Delta \)

Can we gauge away the EMF non-uniformity?

\[ A(x; t) = \Phi(t) \delta(x - x_0) = \text{the vector potential.} \]

\[ \mathcal{E}(x) = -\frac{1}{c} \dot{\Phi} \delta(x - x_0) = \text{the electric field.} \]

Gauge would imply a step potential!

\[ A' = A - \frac{\partial \Lambda}{\partial x}, \quad U' = U + \frac{1}{c} \frac{\partial \Lambda}{\partial t} \]
Wavepacket dynamics with the step potential

Classical energy distribution

QM energy distribution

\[ V_{\text{step}} = -1 \]

\[ m = \hbar = v_E = 1 \]
Step problem - analysis

**Classical moments:**
\[ \delta p_{cl} = -\frac{V_{step}}{v_E} = \text{jump in the momentum} \]
\[ \langle (p - p_0)^r \rangle = \delta p_{cl}^r \times v_E t \]

**Quantum moments:**
\[ |\langle p_2 | \mathcal{U} | p_1 \rangle|^2 = \left[ \frac{\delta p_{cl} v_E t}{(p_2 - p_1)} \text{sinc} \left( \frac{(p_2 - p_1 - \delta p_{cl})v_E t}{2} \right) \right]^2 \]
- \( r = 1 \)
  \[ \langle (p_2 - p_1) \rangle = \delta p_{cl} \times v_E t - \sin(\delta p_{cl} \times v_E t) \]
- \( r = 2 \)
  \[ \langle (p_2 - p_1)^2 \rangle = \delta p_{cl}^2 \times v_E t \]
- \( r > 2 \)
  \[ \langle (p_2 - p_1)^r \rangle = \infty \]

**Restricted QCC** (\( r = 2 \)) is preserved.
**Detailed QCC** (\( r > 2 \)) is destroyed.
Moving Wall problem - Analysis

Adiabatic regime: \( \delta E_{\text{cl}} \ll \Delta \iff V \ll \frac{\hbar}{mL} \)

Semiclassical regime: \( \delta E_{\text{cl}} \gg \Delta \)

\[
|E_n - E_m| \approx "\hbar\omega" \quad \text{???} \quad "\hbar\omega" = \delta E_{\text{cl}} = 2mv_E V
\]

In our problem there is no AC driving!

Is there a self-generated \( \omega \) ??? YES!
Moving Wall problem - Numerics

\[ \frac{d a_n}{dt} = -\frac{i}{\hbar} E_n a_n - \frac{V}{L} \sum_{m(\neq n)} \frac{2nm}{n^2 - m^2} a_m \]

Density plots of \( |a_n(t)|^2 \):

**Semiclassical:**

**Adiabatic:**
The Ring problem

\[ \mathcal{H} = \frac{1}{2m} \left( p - \frac{e}{c} A(t) \right)^2 \]

\[ A(x; t) = \Phi(t) \delta(x - x_0) \]

\[ E(x) = -\frac{1}{c} \dot{\Phi} \delta(x - x_0) \]

\[ E_n = \frac{1}{2m} \left( \frac{2\pi \hbar}{L} \right)^2 \left( n - \frac{\Phi(t)}{\Phi_0} \right)^2 \]

\[ \frac{d a_n}{dt} = -\frac{i}{\hbar} E_n a_n - \frac{\dot{\Phi}}{\Phi_0} \sum_{m(\neq n)} \frac{1}{n - m} a_m \]

\[ \delta E_{cl} = eV_{EMF} \]
Ring ⇐⇒ Bloch

\[ \frac{da_n}{dt} = -\frac{i}{\hbar} E_n a_n + \alpha \sum_{m(\neq n)} \frac{1}{n-m} a_m \]

\[ E_n = \varepsilon n \]

Bloch electrons in a tight-binding model:

<table>
<thead>
<tr>
<th>n</th>
<th>site index</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_n )</td>
<td>on site energy</td>
</tr>
<tr>
<td>( \varepsilon ) - mean level spacing</td>
<td>electric field</td>
</tr>
<tr>
<td>[ \frac{\alpha}{n-m} ]</td>
<td>hopping</td>
</tr>
</tbody>
</table>

• \( \varepsilon = 0 \Rightarrow \) ballistic motion

• \( \varepsilon \to \infty \Leftrightarrow \alpha \to 0 \Leftrightarrow \) ”stuck”

• But what happens in between?

Analytical solution:

\[ |a_t(n)|^2 = \left(\frac{2\alpha}{\varepsilon}\right)^2 \frac{\sin^2\left(\frac{1}{2} \varepsilon t \left(n-n_0 + \alpha \left(t - \frac{2\pi}{|\varepsilon|}\right)\right)\right)}{(n-n_0 + \alpha t)^2(n-n_0 + \alpha \left(t - \frac{2\pi}{|\varepsilon|}\right))^2} \]
The ring / bloch problem - Solution

MOVIE
Is the $\propto 1/(n-m)$ hopping significant ???
Bloch electrons - Nearest Neighbors

\[
\frac{da_n}{dt} = -i\varepsilon na_n + \frac{\alpha}{2} [a_{n+1} - a_{n-1}]
\]

Analytical solution:

\[
|a_t(n)|^2 = \left| J_{n-n_0} \left( \frac{2\alpha}{\varepsilon} \sin \left( \frac{1}{2} \varepsilon t \right) \right) \right|^2
\]

\( \propto \frac{1}{n-m} \) hopping

Uni-directional oscillations

n.n hopping

Bi-directional oscillations
Different regimes:

- **adiabatic**
- **diabatic**
- **semi-classical**

**Landau-Zener**

**non-adiabatic**

Semiclassical regime: \( \delta E_{cl} \gg \Delta \iff V_{EMF} \gg \frac{\hbar v_E}{L} \)