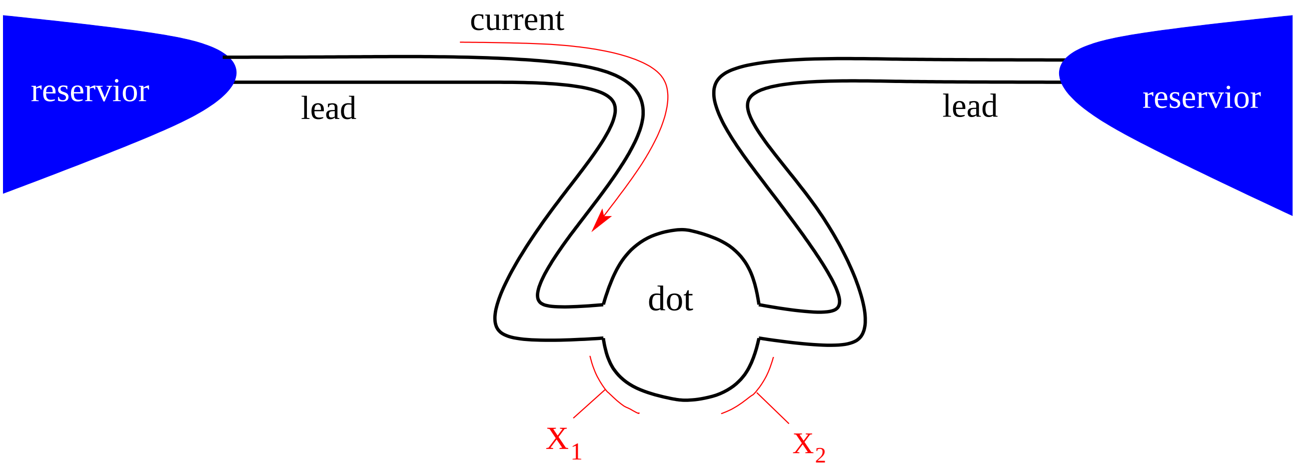
Quantum stirring in low dimensional devices

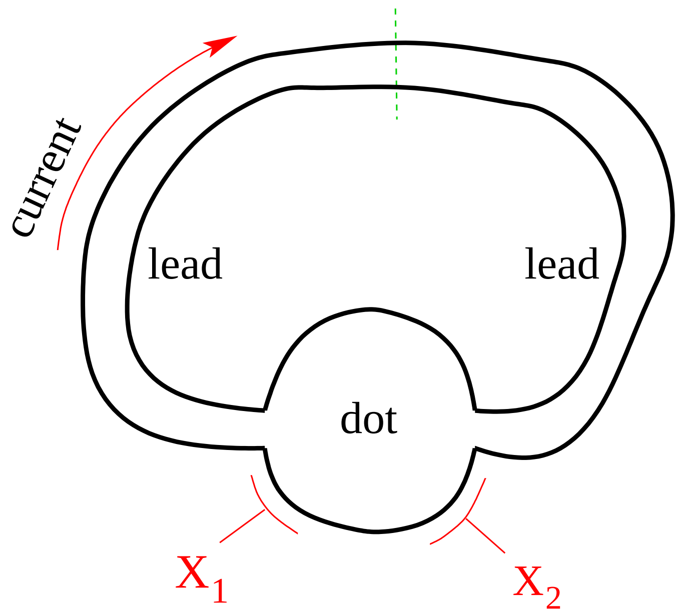
Itamar Sela
Ben-Gurion University

I. Sela and D. Cohen, Phys. Rev. B 77, 245440 (2008)
I. Sela and D. Cohen, arXiv (2008)

Quantum pumping



Quantum stirring

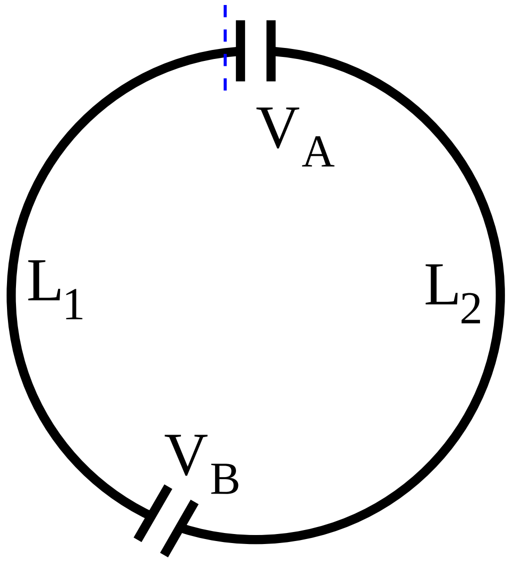


$$\mathcal{H} = \mathcal{H}(x, p;\ X_1(t), X_2(t), X_3(t))$$

$$X_1,\ X_2 = \text{shape parameters}$$

$$X_3 = \Phi = (\hbar/e)\,\phi = \text{magnetic flux}$$

Double well model



$$\mathcal{H} = \frac{\hat{p}^2}{2\mathsf{m}} + V_A\left(\hat{x} - x_A\right) + V_B\left(\hat{x} - x_B\right)$$

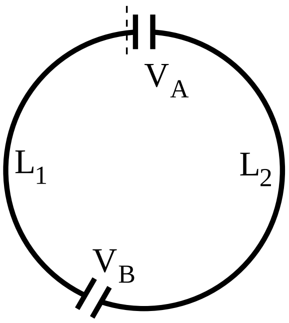
$$\mathcal{I} = \frac{1}{2\mathsf{m}}\left(\hat{p}\,\delta\left(\hat{x} - x_0\right) + \delta\left(\hat{x} - x_0\right)\hat{p}\right)$$

X = barriers height, position etc.

g_A = barrier A transmission

g_B = barrier B transmission

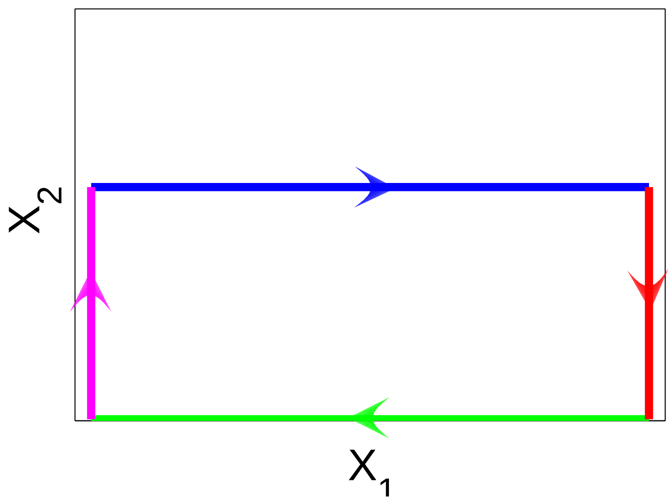
Stirring cycle

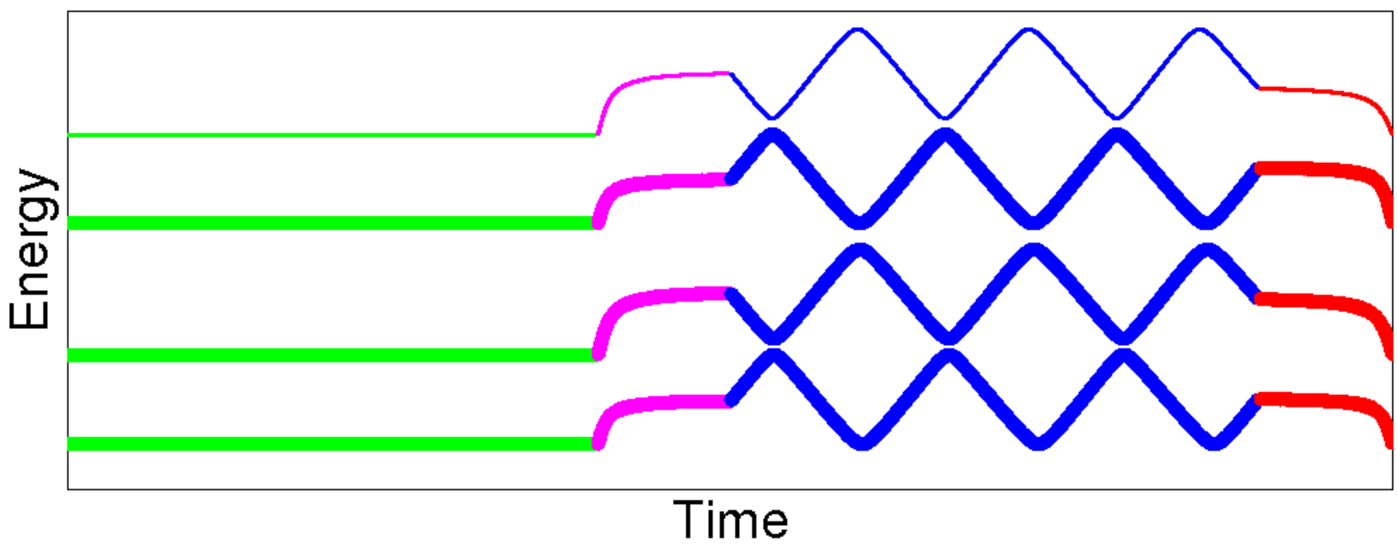


X_1 = Barrier B position

X_2 = Barrier B height

$$Q = -\oint_{\text{cycle}} \boldsymbol{G} \cdot \mathrm{d}\boldsymbol{X}$$





The Kubo formula for G

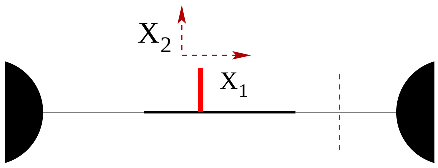
$$
\begin{aligned}
G^1 &= -\sum_n f_n \sum_{m(\neq n)} \frac{2\,\mathrm{Im}\left[\mathcal{I}_{nm}\right]\left[\frac{\partial \mathcal{H}}{\partial X_1}\right]_{mn}}{(E_m - E_n)^2} \\
&= \sum_n f_n B_2^{(n)} \\
G^2 &= -\sum_n f_n \sum_{m(\neq n)} \frac{2\,\mathrm{Im}\left[\mathcal{I}_{nm}\right]\left[\frac{\partial \mathcal{H}}{\partial X_2}\right]_{mn}}{(E_m - E_n)^2} \\
&= -\sum_n f_n B_1^{(n)}
\end{aligned}
$$

$$
\boldsymbol{B} = \nabla \times \boldsymbol{A}^{(n)} = \left(B_1^{(n)},\, B_2^{(n)},\, B_3^{(n)}\right)
$$

$$
\mathrm{BerryPhase} = \oint \boldsymbol{A}^{(n)} \cdot d\boldsymbol{X}
$$

Past results

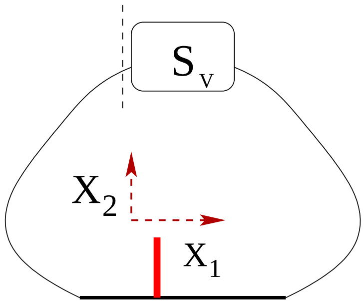
Open geometry



$$Q = (1 - g_{\mathrm{X}}) \frac{e}{\pi} k_{\mathrm{F}} \times \Delta X_1$$

Avron *et al* 2000

Closed geometry - "classical" result



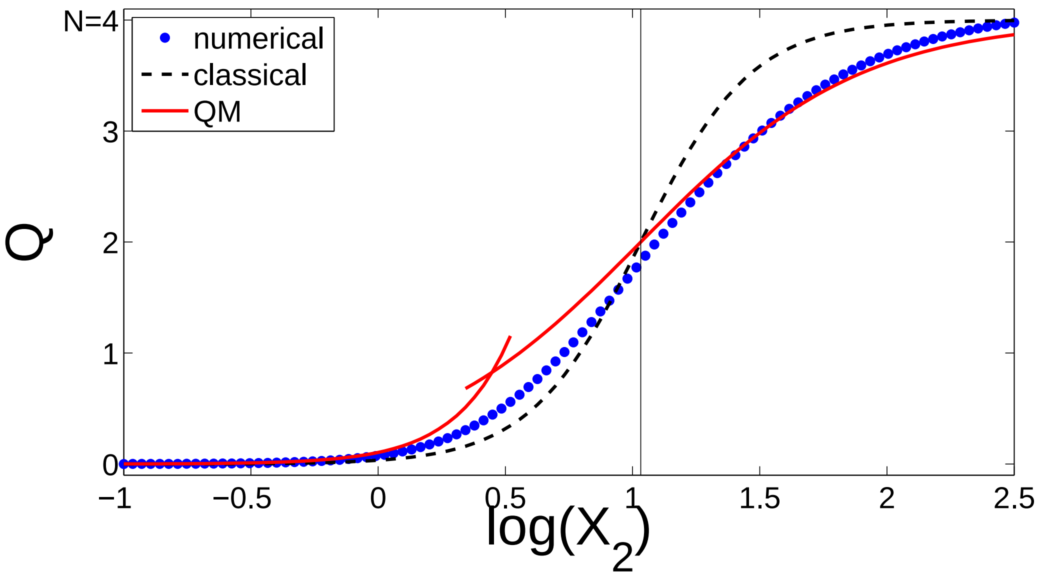
$$Q = \left[\frac{(1 - g_{\mathrm{X}})\, g_{\mathrm{V}}}{g_{\mathrm{X}} + g_{\mathrm{V}} - 2 g_{\mathrm{X}} g_{\mathrm{V}}} \right] \frac{e}{\pi} k_{\mathrm{F}} \times \Delta X_1$$

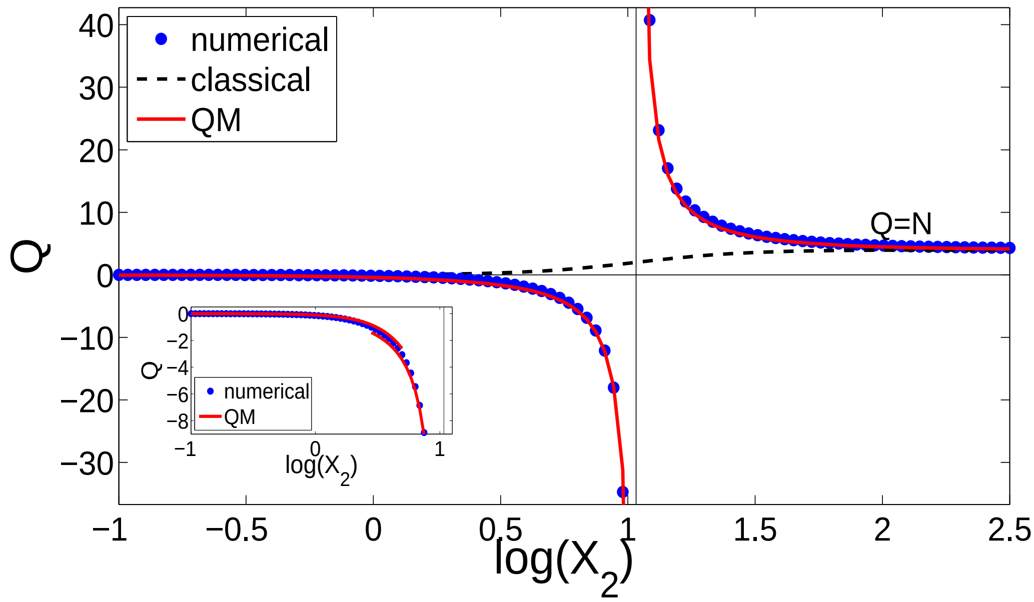
Cohen *et al* 2006

Current study

Closed geometry - QM result

Results for Q (zero temperature Fermi occupation)





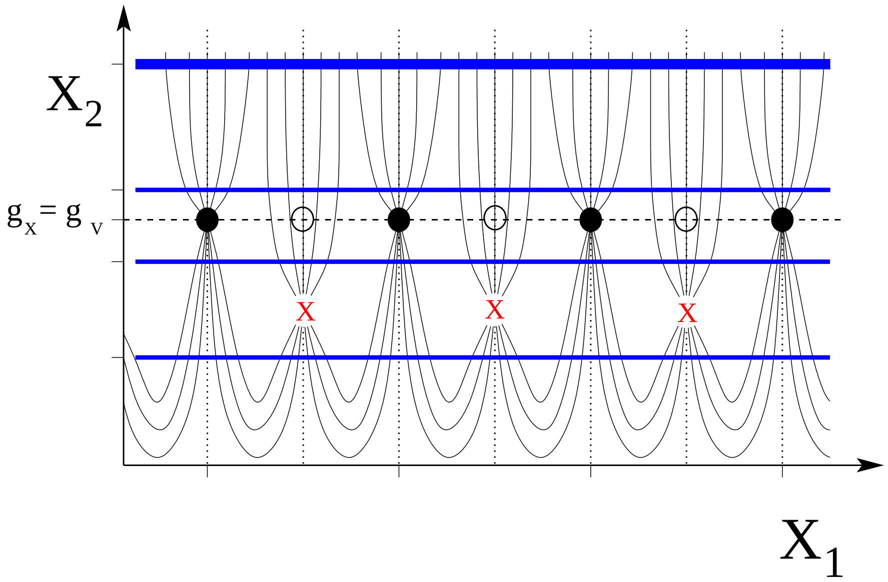
$$G\left(X_1\right) = \begin{cases} k_{\rm F} \sum_r \frac{1/\lambda}{\left((2k_{\rm F}(X_1 - X_1^r))^2 + (\sqrt{g_{\rm V}}/\lambda)^2\right)^{3/2}} & g_{\rm X} \ll 1 \\ \frac{k_{\rm F}}{\pi} \sum_{\nu=0}^{\infty} \mathcal{G}_\nu \cos\left(\nu \; 2k_{\rm F} X_1\right) & g_{\rm X} \sim 1 \end{cases}$$

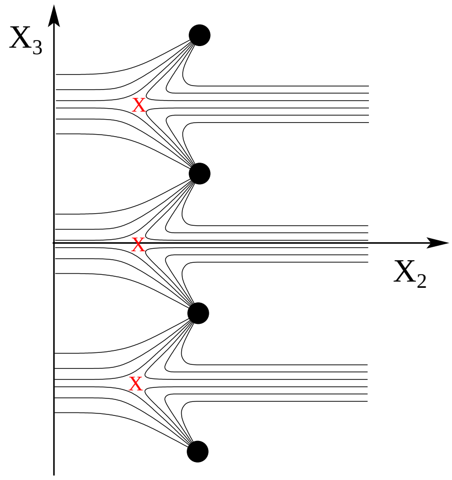
$$\mathcal{G}_0 = \pm 2\sqrt{g_{\rm V}} \left(\frac{1}{k_{\rm F} L} \sqrt{\frac{1-g_{\rm X}}{g_{\rm X}}} + \frac{4}{\pi^2} \; \frac{1-g_{\rm X}}{g_{\rm X}} \right), \quad \mathcal{G}_1 = \ldots$$

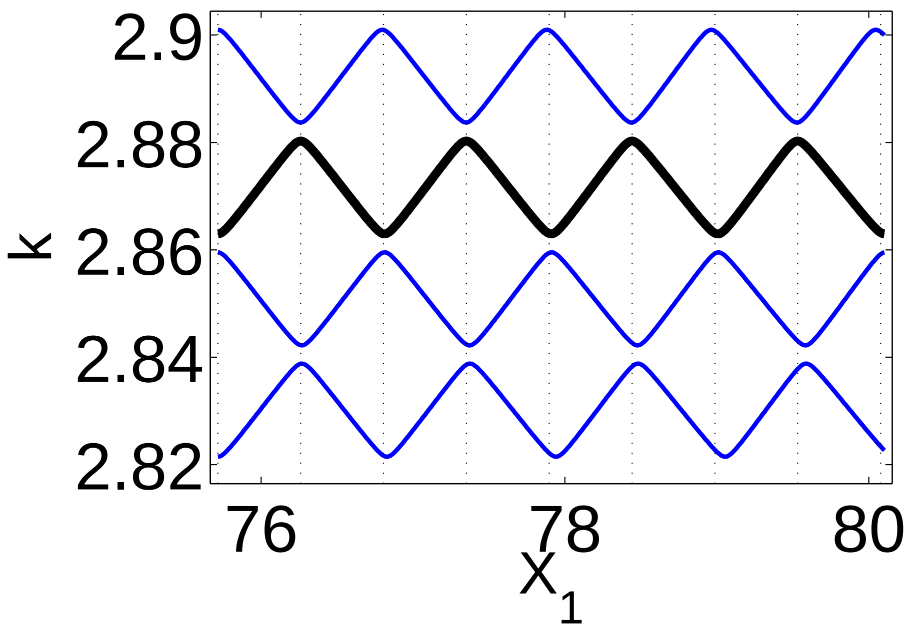
$$\lambda = \frac{\sqrt{g_{\rm V}}}{\sqrt{g_{\rm V}} \pm \sqrt{g_{\rm X}}}$$

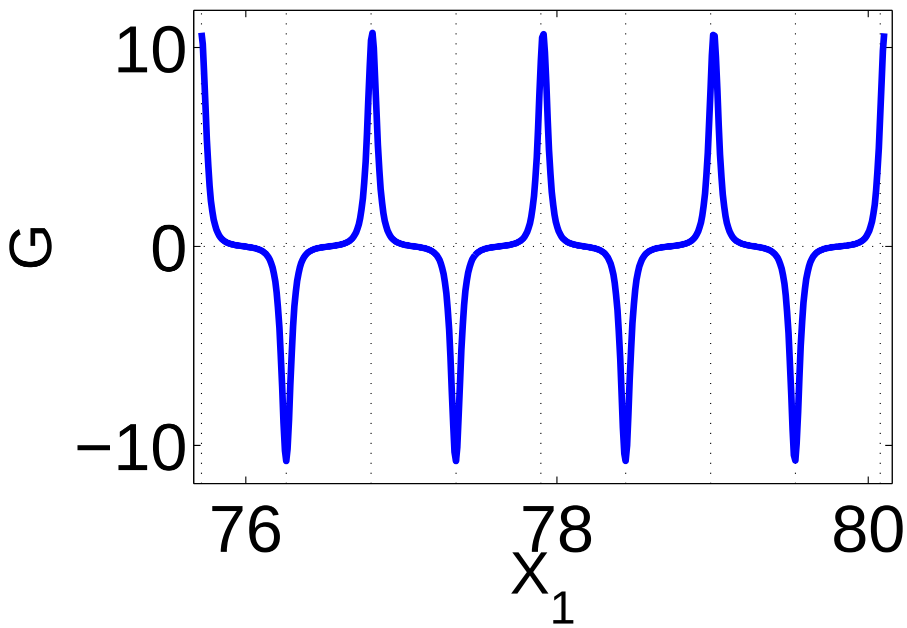
Results for B

One level occupation



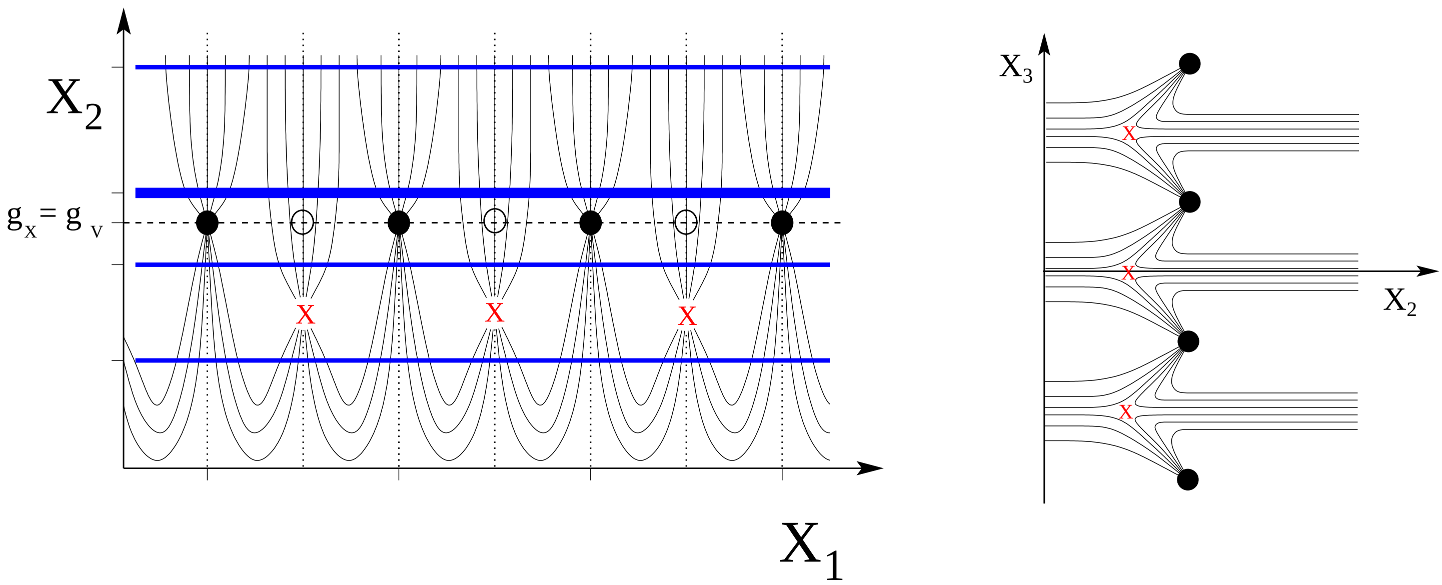


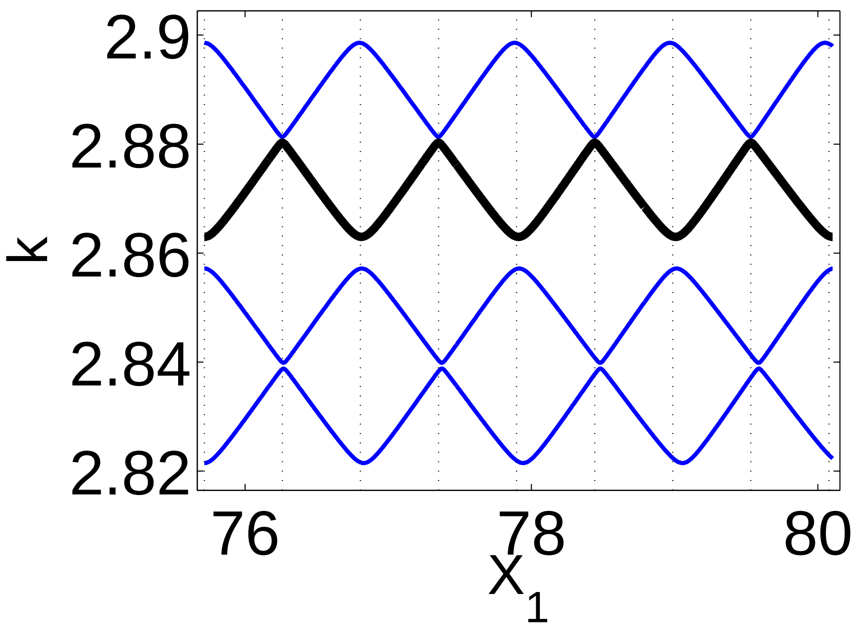


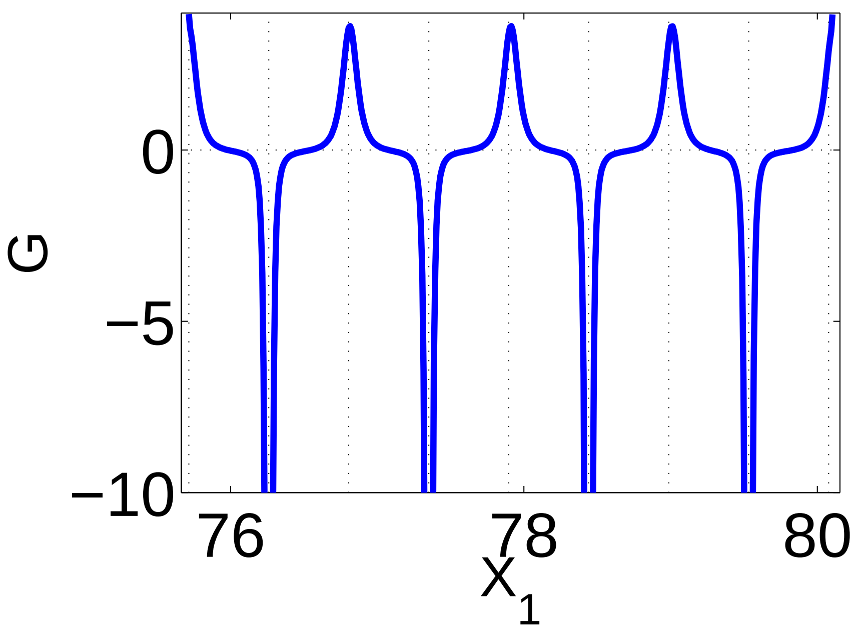


Results for B

One level occupation

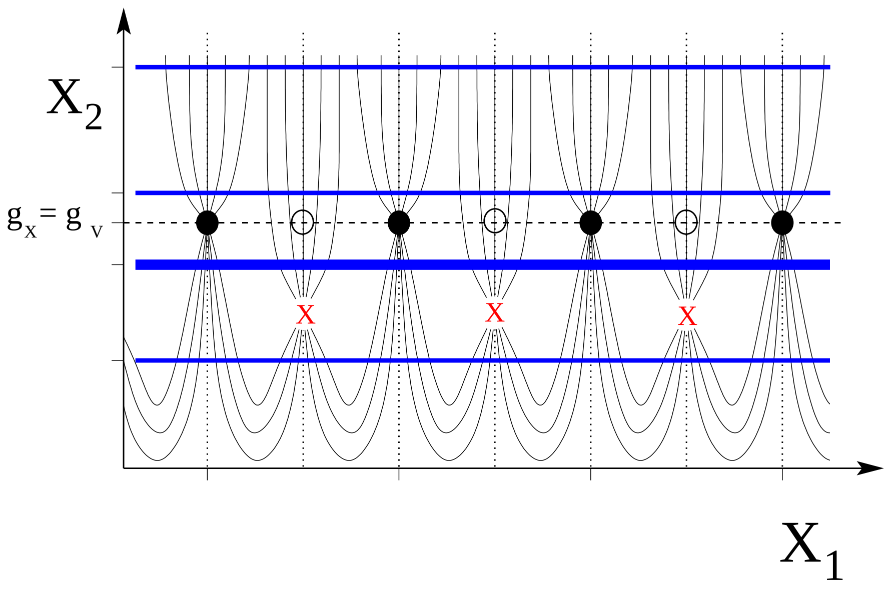


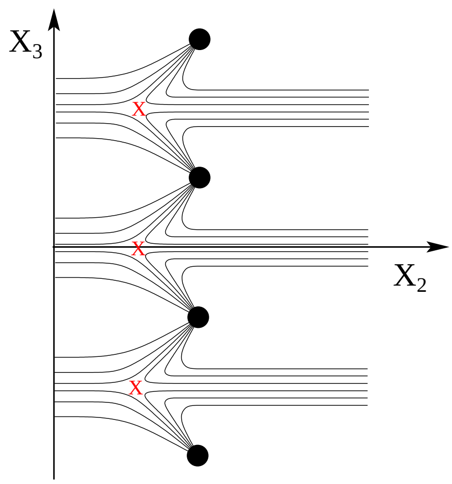


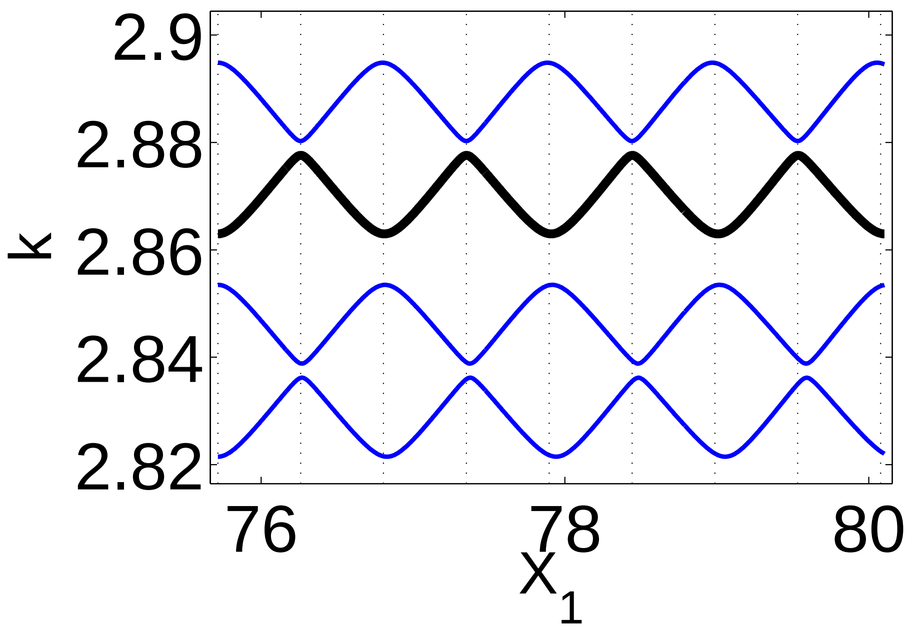


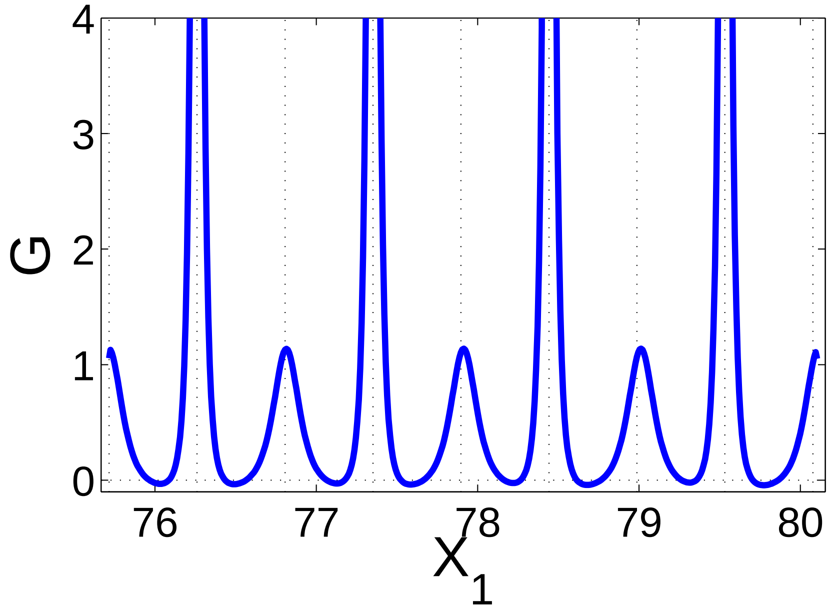
Results for B

One level occupation



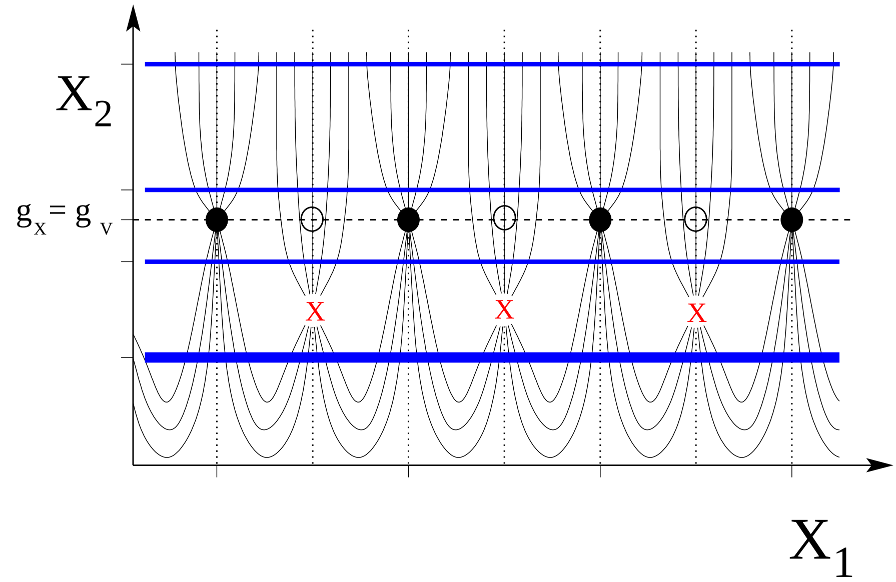


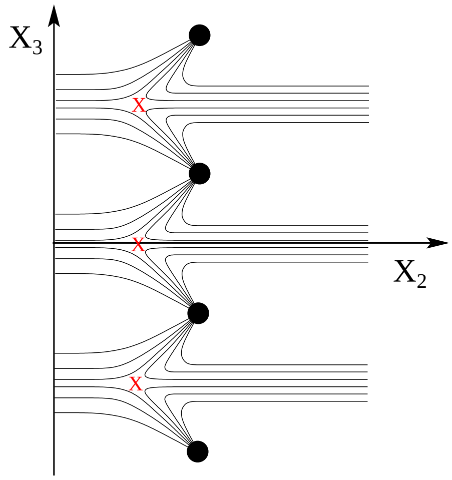


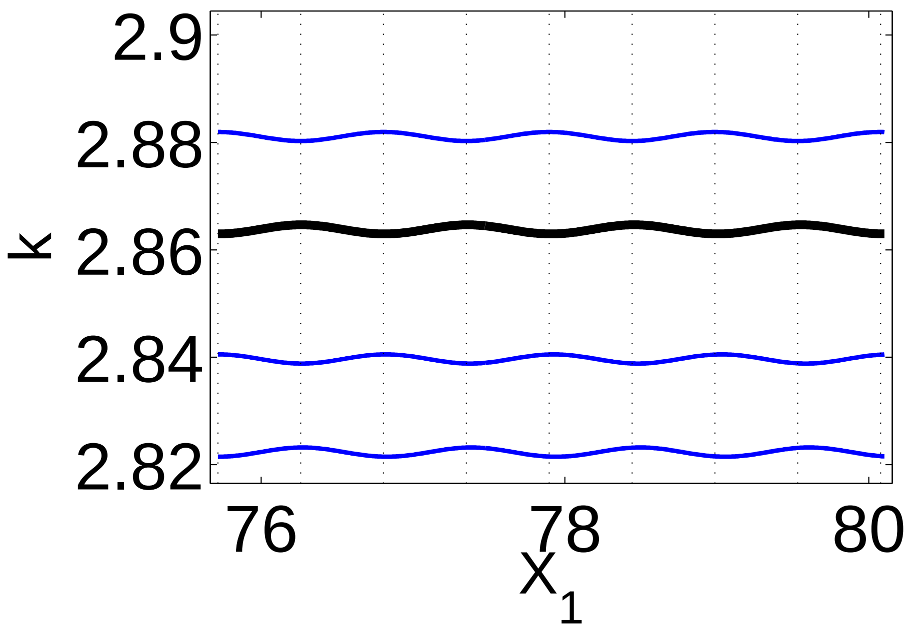


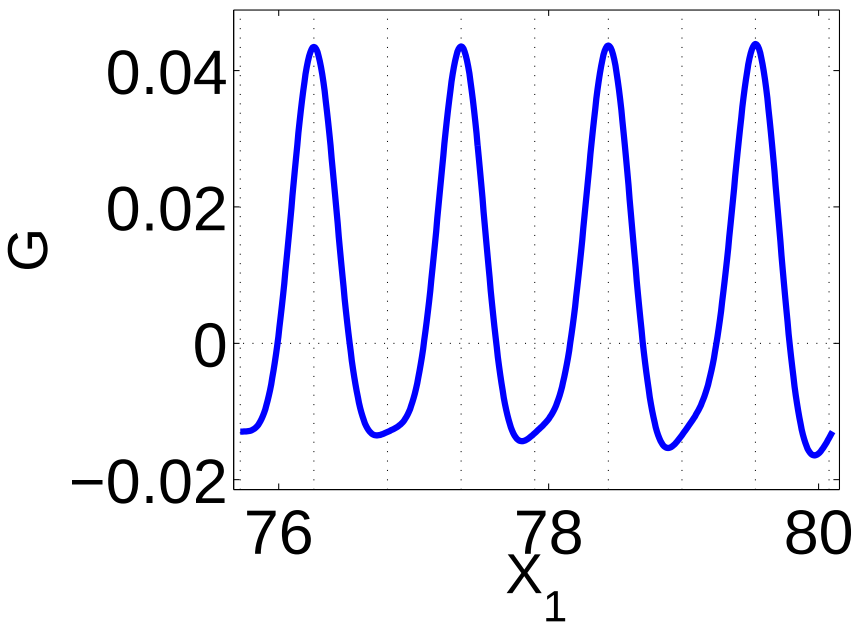
Results for B

One level occupation





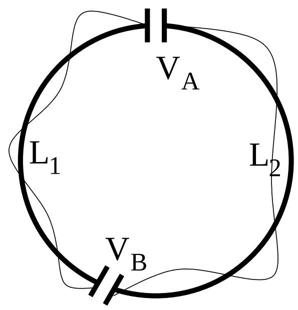


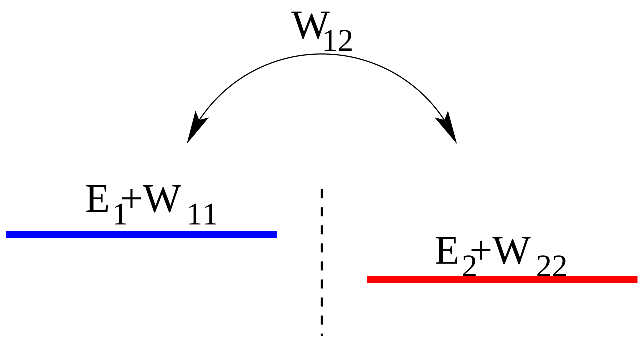


Strategy

- Getting Q without Kubo - TLS modeling
- Getting Q using Kubo - TLS modeling
- Beyond the TLS modeling

Getting Q without Kubo - TLS modeling





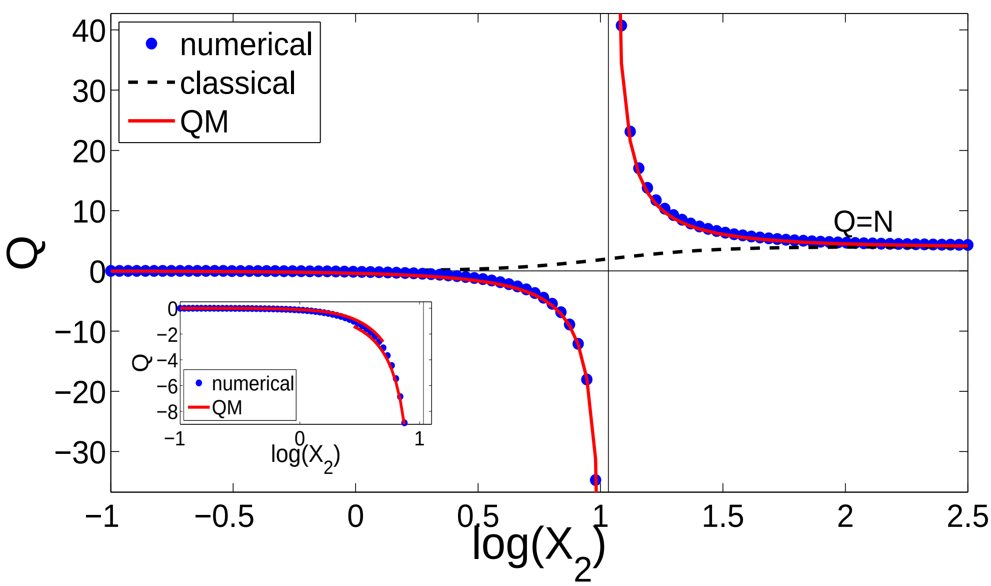
$$\mathcal{H} = \mathcal{H}_\infty + W = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}$$

$$W_{12} = W_{12}^A + W_{12}^B = -\frac{v_E}{2\sqrt{L_1 L_2}}\left(\sqrt{g_A} \pm \sqrt{g_B}\right)$$

$$\lambda = \frac{W_{12}^A}{W_{12}^A + W_{12}^B} = \frac{\sqrt{g_A}}{\sqrt{g_A} \pm \sqrt{g_B}}$$

$Q = \lambda$

per crossing



Getting Q using Kubo - TLS modeling

TLS Hamiltonian

$$\mathcal{H} \;=\; \frac{\varepsilon}{2}\boldsymbol{\sigma}_z + \frac{\kappa}{2}\boldsymbol{\sigma}_x \;\equiv\; \frac{\boldsymbol{\Omega}}{2}\cdot\sigma$$

with eigenstates

$$|n_0\rangle = \begin{pmatrix} \mp\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}, \qquad |m_0\rangle = \begin{pmatrix} \cos(\theta/2) \\ \pm\sin(\theta/2) \end{pmatrix}$$

$$\Longrightarrow \qquad \mathcal{I}^A_{n_0 m_0} \qquad\qquad \left[\frac{\partial\mathcal{H}}{\partial X}\right]_{n_0 m_0}$$

TLS geometric conductance

$$G = \frac{\lambda\kappa}{2\Omega^3}\left[\kappa\frac{\partial\varepsilon}{\partial X} - \varepsilon\frac{\partial\kappa}{\partial X}\right]$$

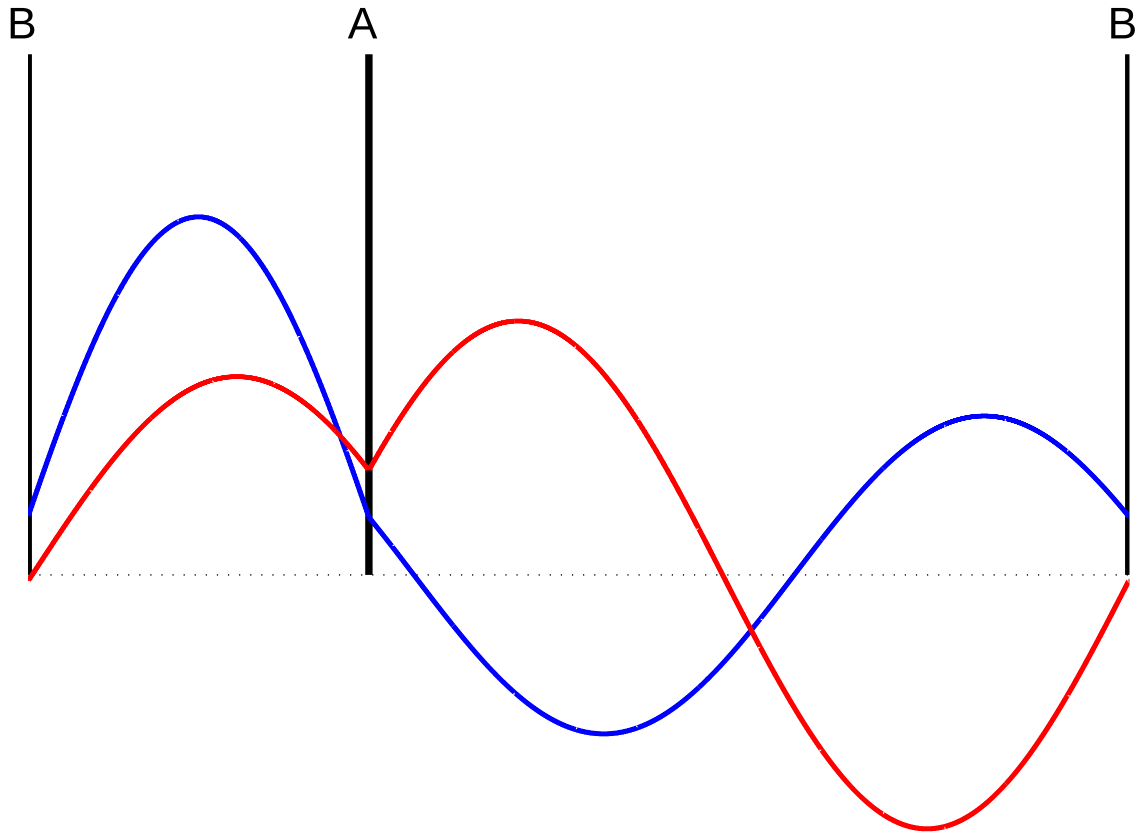
Total coupling

$$\frac{\kappa}{2} = W^A_{12} + W^B_{12}$$

Energy difference

$$\varepsilon = (E_1 + W^A_{11} + W^B_{11}) - (E_2 + W^A_{22} + W^B_{22})$$

Beyond the TLS modeling



The Neighboring Level approximation

$$\Theta \quad \equiv \quad 2\arctan\left(\sqrt{\frac{\mathrm{Prob}(x\in 2)}{\mathrm{Prob}(x\in 1)}}\right)$$

$$\Longrightarrow \quad \psi_n(x) \;\Longrightarrow\; \mathcal{I}^{A}_{n_0 m_0}, \qquad \left[\frac{\partial\mathcal{H}}{\partial X}\right]_{n_0 m_0}$$

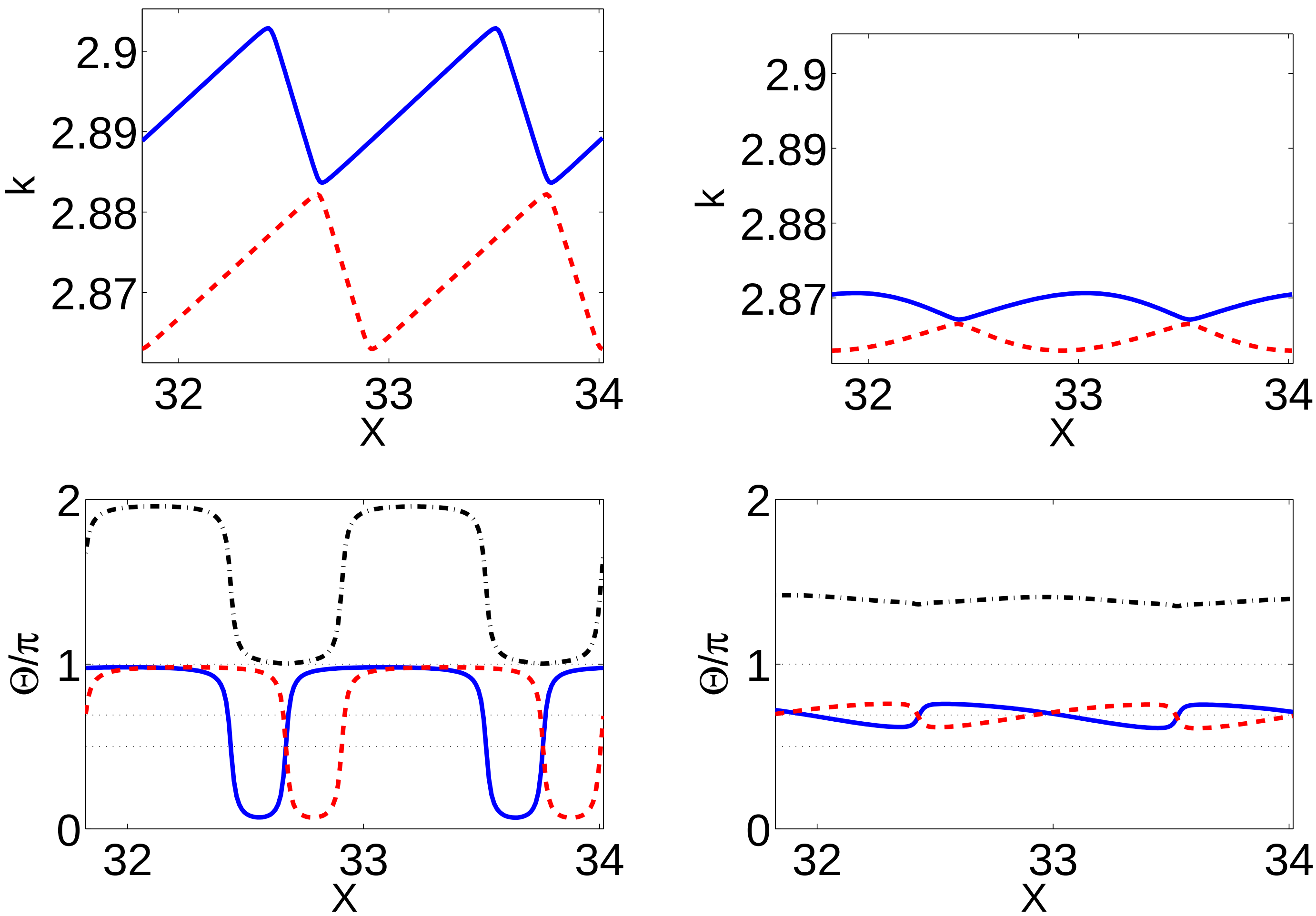
When TLS approximation applies

$$\Theta^{(m_0)} \quad \approx \quad \theta$$

$$\Theta^{(n_0)} \quad \approx \quad \pi - \theta$$

TLS approximation applies

TLS approximation does not apply



Thank you

The TLS current operator

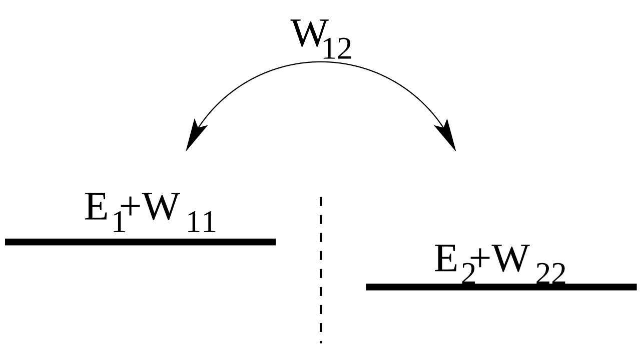
Occupation operator

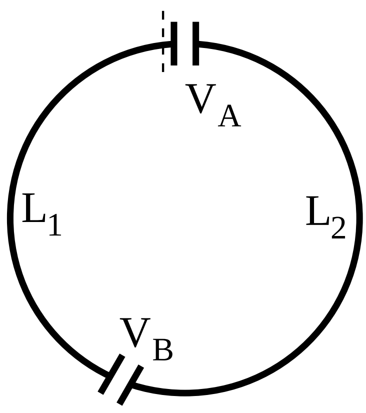
$$\mathcal{N} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Current operator

$$\mathcal{I} \equiv \frac{d}{dt}\mathcal{N} = i[\mathcal{H}, \mathcal{N}]$$

$$\mathcal{I} = \begin{pmatrix} 0 & -i\frac{\kappa}{2} \\ i\frac{\kappa}{2} & 0 \end{pmatrix}$$





$\mathcal{I}^A = ?$

The splitting ratio

$$\mathcal{I}^A = \lambda_A \mathcal{I}$$

$$\mathcal{I}^B = \lambda_B \mathcal{I}$$

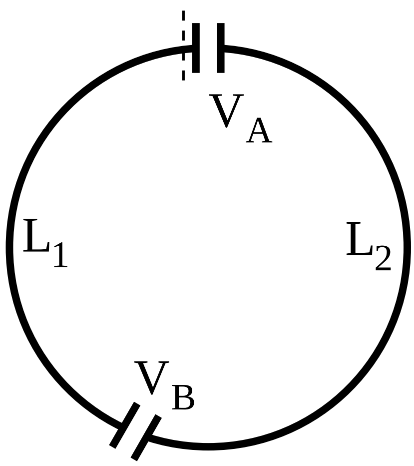
$$\mathcal{I} = \begin{pmatrix} 0 & -i\frac{\kappa}{2} \\ i\frac{\kappa}{2} & 0 \end{pmatrix}$$

$$\lambda_A = \frac{W_{12}^A}{W_{12}^A + W_{12}^B} = \frac{\sqrt{g_A}}{\sqrt{g_A} \pm^c \sqrt{g_B}}$$

$$\lambda_A + \lambda_B = 1$$

We can have $\lambda < 0$ or $\lambda > 1$

$$\frac{\kappa}{2} = W_{12}^A + W_{12}^B$$



Ohm law

One parameter driving by EMF

$$I = G^{33} \times \left(-\dot{\Phi}\right)$$

$$dQ = -G^{33}\,\mathrm{d}\Phi$$

Driving by changing another parameter

$$I = -G^{31}\dot{X}_1$$

$$dQ = -G^{31}\,\mathrm{d}X_1$$

Two parameters driving

$$I = -G^{31}\dot{X}_1 - G^{32}\dot{X}_2$$

$$dQ = -G^{31}\,\mathrm{d}X_1 - G^{32}\,\mathrm{d}X_2$$

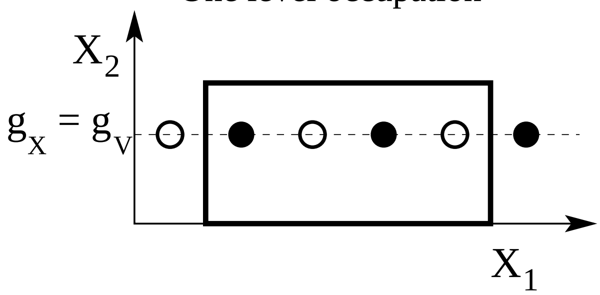
$$Q = \oint_{\text{cycle}} I\mathrm{d}t = -\oint \left(G^{31}\,\mathrm{d}X_1 + G^{32}\,\mathrm{d}X_2\right)$$

The Dirac Monopole picture

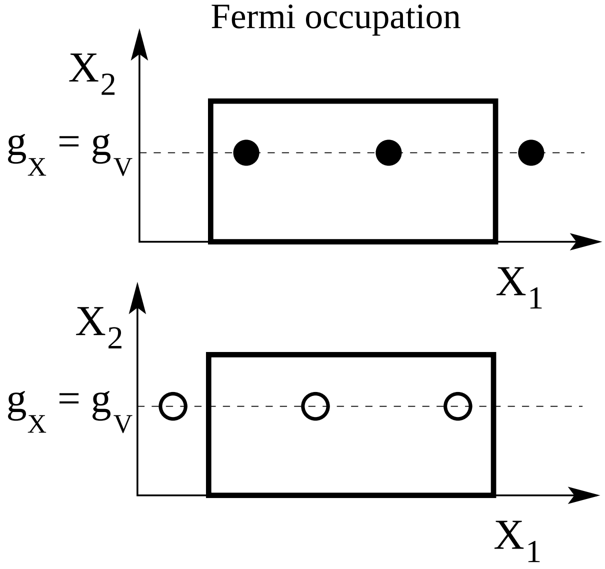
$$\boldsymbol{B} \;=\; \left(-G^{32},\; G^{31},\; 0\right)$$
$$\mathrm{d}\boldsymbol{s} \;=\; \left(\mathrm{d}X_2,\; -\mathrm{d}X_1,\; 0\right)$$

$$Q \;=\; \oint_{\text{cycle}} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{s}$$

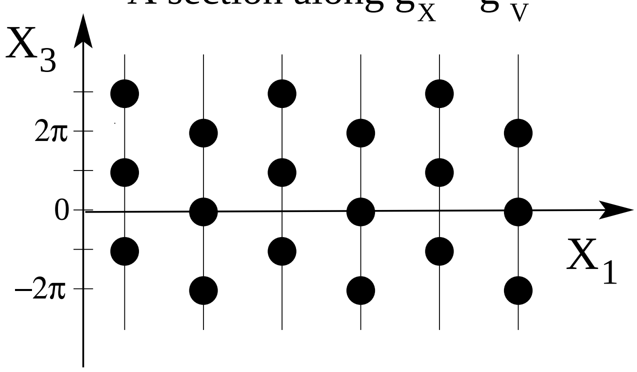
One level occupation



Fermi occupation

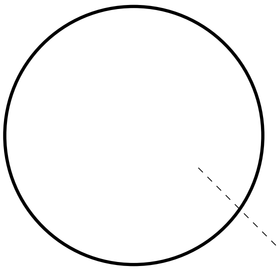


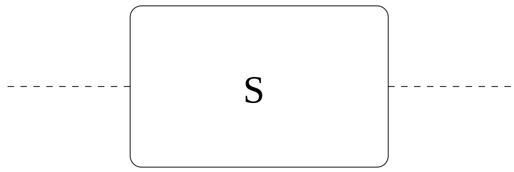
A section along gX = g V



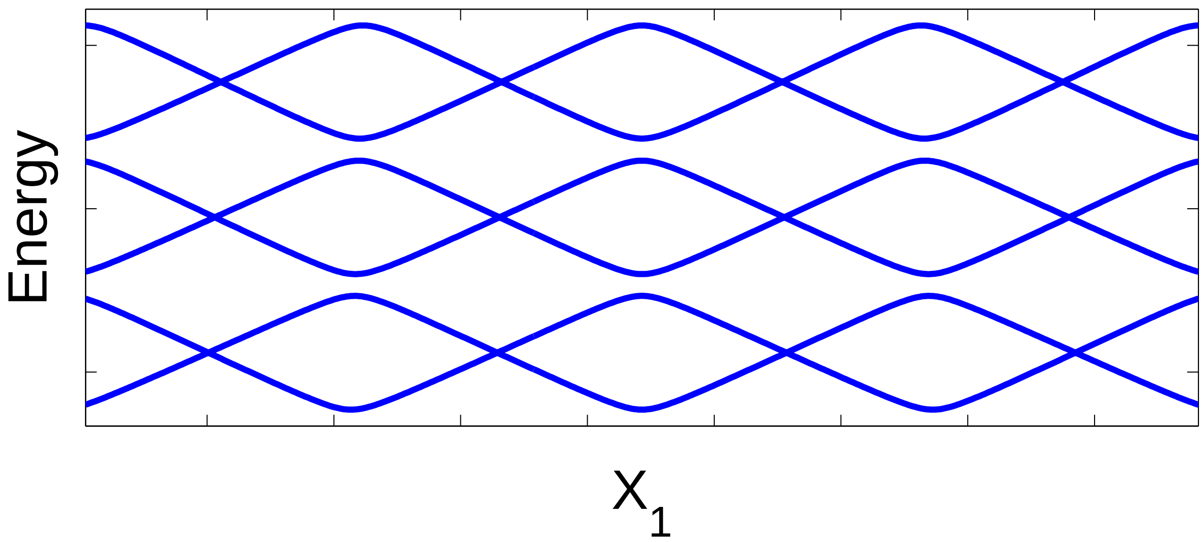
X space

$$E^{(r)},\ \boldsymbol{X}^{(r)} = (X_1^{(r)}, X_2^{(r)})$$





$$\cos(\gamma(E)) = \sqrt{g(E)}\,\cos(\phi)$$



$$X_1^{(r)} \ = \ \frac{\alpha_1 - \alpha_2}{2k_{\mathrm{E}}^{(r)}} + \frac{L}{2} + \left(\left[\frac{1}{2}\right] + \mathrm{integer}\right)\ \frac{\pi}{k_{\mathrm{E}}^{(r)}}$$

$\alpha(E)$ is an S-matrix parameter