Quantum pumping and dissipation in closed systems

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$GIF, ISF$
Driven Systems

Non interacting “spinless” electrons.

Held by a potential (e.g. AB ring geometry).

\[ x_1, x_2 = \text{shape parameters} \]

\[ x_3 = \Phi = (\hbar/e)\phi = \text{magnetic flux} \]
“Ohm law”

For one parameter driving by EMF

\[
I = G^{33} \times (-\dot{x}_3) \\
\quad dQ = -G^{33} \, dx_3
\]

For driving by changing another parameter

\[
I = -G^{31} \, \dot{x}_1 \\
\quad dQ = -G^{31} \, dx_1
\]

For two parameter driving

\[
I = -G^{31} \, \dot{x}_1 - G^{32} \, \dot{x}_2 \\
\quad dQ = -G^{31} \, dx_1 - G^{32} \, dx_2 \\
\quad Q = -\oint G \cdot dx
\]

and in general

\[
\langle F^k \rangle = -\sum_j G^{kj} \, \dot{x}_j
\]
What is the problem?

From Kubo formula we get a formal expression for $G^{kj}$.

Can we trust this expression? Conditions? Quantum chaos!

How to use this expression? The bare Kubo formula gives no dissipation!

To define an energy scale $\Gamma$
Beyond first order perturbation theory!

$\Gamma$ in case of isolated system is due to non-adiabaticity.

$\Gamma$ affects both the dissipative and the non-dissipative (geometric) part of the response.
Some references

Adiabatic transport
Thouless (PRL 1983) - Periodic arrays
Avron, Sadun, Raveh, Zur (1988) - Networks with fluxes
Berry, Robbins (JPA 1993) - Geometric magnetism

Linear response theory and Mesoscopics
Imry, Shiren (PRB 1986) - Using Kubo for a closed ring
Wilkinson, Austin (JPA 1995) - Challenging the validity
DC (PRL 1999) - The “quantum chaos” identification of regimes
DC and Kottos (PRL 2000) - The \((A, \Omega)\) regimes diagram
DC (PRB+Rapid 2003) - the Kubo approach to pumping
DC, Kottos, Schanz (cond-mat) - pumping on networks
Sela, DC (in preperation) - pumping on a ring

Open systems, \(S\) matrix formalism
The Buttiker Pretre Thomas [BPT] formula (1994)
Brouwer (1998)
Avron, Elgart, Graf, Sadun
Buttiker, Texier, Moskalets
Marcus - experiments
Shutenko, Aleiner, Altshuler (PRB 2000) - quantization?
Entin-Wohlman, Aharony, Levinson (2002) - two delta functions
Linear response theory

\[ \mathcal{H} = \mathcal{H}(r, p; x_1(t), x_2(t), x_3(t)) \]

\[ \mathcal{F}^k = -\frac{\partial \mathcal{H}}{\partial x_k} \]

\[ \langle \mathcal{F} \rangle_t = \int \alpha(t - t') \delta x(t') \, dt' \]

\[ \alpha^{kj}(t - t') \]

\[ \chi^{kj}(\omega) \]

\[ \text{Re}[\chi^{kj}(\omega)] \quad (1/\omega) \times \text{Im}[\chi^{kj}(\omega)] \]

\[ G^{kj} \]

\[ \eta^{kj} \quad B^{kj} \]

(dissipative) \quad (non-dissipative)

\[ \langle \mathcal{F}^k \rangle = -\sum_j G^{kj} \dot{x}_j \]
From Kubo to a “FD relation”

\[ \mathcal{H} = \mathcal{H}(r, p; x_1(t), x_2(t), x_3(t)) \]

\[ \mathcal{F}^k = - \frac{\partial \mathcal{H}}{\partial x_k} \]

Generalized Ohm law:

\[ \langle \mathcal{F}^k \rangle = - \sum_j G^{kj} \dot{x}_j \]

\[ K^{kj}(\tau) = \frac{i}{\hbar} \langle [\mathcal{F}^k(\tau), \mathcal{F}^j(0)] \rangle \]

\[ C^{kj}(\tau) = \frac{1}{2} \left( \langle \mathcal{F}^k(\tau) \mathcal{F}^j(0) \rangle + cc \right) \]

\[ G^{kj} = \lim_{\omega \to 0} \frac{\text{Im}[\chi^{kj}(\omega)]}{\omega} = \int_0^\infty K_{F}^{kj}(\tau) \tau d\tau \]

\[ = g(E_F) \int_0^\infty C_{E_F}^{kj}(\tau) d\tau \]

DC, PRB(R) 2003
BPT versus Geometric magnetism

\[ G^{kj} = g(E_F) \int_0^\infty C^{kj}_{E_F}(\tau) d\tau \quad \text{from now on } k=3, j=1,2 \]

Due to non-adiabaticity

\[ C_E^{kj}(\tau) \mapsto C_E^{kj}(\tau) e^{-(\Gamma/\hbar)t} \]

\[ \Gamma = \left( \frac{\hbar \sigma}{\Delta^2} |\dot{x}| \right)^{2/3} \times \Delta \sim \left( L |\dot{x}| \right)^{2/3} \frac{1}{L} \]

In the absence of magnetic field one obtains

\[ G^{3j} = 2\hbar \sum_n f(E_n) \sum_{m(\neq n)} \frac{\text{Im} [\mathcal{I}_{nm}\mathcal{F}_{mn}]}{(E_m - E_n)^2 + (\Gamma/2)^2} \]

For dot-wire system in the \( L \to \infty \) limit we have \( \Delta \ll \Gamma \to 0 \) leading to

\[ G^{3j} = \frac{e}{2\pi i} \text{trace} \left( P_A \frac{\partial S}{\partial x_j} S^\dagger \right) \quad \text{[BPT]} \]
Simple model systems - networks

\[ dQ = (1-g_0) \times \frac{e}{\pi} k_F \times dX \]

\[ dQ = 1 \times \frac{e}{\pi} k_F \times dX \]

\[ dQ = \left[ \frac{g_T}{1-g_T} \right] \left[ \frac{1-g_0}{g_0} \right] \times \frac{e}{\pi} k_F \times dX \]

Adiabatic versus non-adiabatic result

DC, Kottos, Schanz, cond-mat 2004
The network Hamiltonian

\[ \mathcal{H} = \text{network} + \lambda \frac{\hbar^2}{2m} \delta(x - X_0) \]

\[ g(E) = \text{density of states} \]

\[ g_0 = \frac{1}{1 + (\lambda/(2k_F))^2} = \text{transmission} \]

\[ \mathcal{I} = \frac{e}{2m} \left( \delta(x - X_1)p + p\delta(x - X_1) \right) \]

\[ \mathcal{F} = -\frac{\partial \mathcal{H}}{\partial X_0} = \lambda \frac{\hbar^2}{2m} \delta'(x - X_0) \]

\[ C(\tau) = \text{cross correlation function of } \mathcal{I} \text{ and } \mathcal{F} \]

\[ \mathcal{I} \leftrightarrow -ie\hbar \frac{e\hbar}{2m}(\overrightarrow{\partial} - \overleftarrow{\partial})_{x=X_1} \]

\[ \mathcal{F} \leftrightarrow -\lambda \hbar \frac{\hbar^2}{2m}(\overrightarrow{\partial} + \overleftarrow{\partial} - \lambda)_{x=X_0+0} \]
Kubo - “classical” calculation

\[ G^{IF} = g(E_F) \int_0^\infty C(\tau) d\tau \]

\[ C(\tau) = e \frac{v_F}{2L} 2m v_F \left[ (1 - g_0) \sum_{\pm} \pm \delta(\tau \pm \tau_1) \right] + \ldots \]

\[ \tau_1 = \frac{(X_1 - X_0)}{v_F} \]

\[ \int_{short}^\infty C(\tau) d\tau = -e \frac{mv_F^2}{L} [1 - g_0] \]

\[ \int_0^\infty C(\tau) d\tau = -e \frac{mv_F^2}{L} \left[ \frac{1 - g_0}{g_0} \right] \left[ \frac{g_T}{1 - g_T} \right] \]

\[ G^{IF} = -(1 - g_0) \times \frac{e}{\pi} k_F \]

\[ G^{IF} = -\frac{e}{\pi} k_F \left[ \frac{1 - g_0}{g_0} \right] \left[ \frac{g_T}{1 - g_T} \right] \]
Kubo - "diagonal" quantum calculation

\[
G^{IF} = g(E_F) \int_0^\infty C(\tau) d\tau
\]

\[
C(\tau) \mapsto C(\tau) e^{-\left(\Gamma/\hbar\right)t}
\]

\[
G^{IF} = \hbar g(E_F) \int_{-\infty}^{\infty} \frac{-i\tilde{C}(\omega)}{\hbar\omega - i(\Gamma/2)} \frac{d\omega}{2\pi}
\]

\[
= -\frac{i\hbar}{4\pi} \int_{-\infty}^{\infty} \frac{C(E_F + \hbar \omega, E_F) + C(E_F - \hbar \omega, E_F)}{\omega + i(\Gamma/2)} d\omega
\]

\[
C(E', E) = \frac{2}{\pi} \text{trace}[\mathcal{I} \text{Im}[G(E')]] \mathcal{F} \text{Im}[G(E)]
\]

\[
\langle x|G(E)|x_0\rangle = -\frac{i}{\hbar v_F} \sum_p A_p e^{i k_E L_p}
\]

\[
G^{IF} = -\frac{e}{\pi} k_F \frac{1 - g_0}{g_0} \sum_p s_p^+ s_p^- |A_p|^2 e^{L_p/L\Gamma}
\]
Kubo - numerical quantum calculation

\[ G^{IF} = g(E_F) \int_0^\infty C(\tau) d\tau \]

same steps as before...

\[ C'(E', E) = 2\pi \sum_{nm} I_{nm} \delta(E' - E_m) F_{mn} \delta(E - E_n) \]

\[ G^{IF} = 2\hbar \sum_n f(E_n) \sum_{m(\neq n)} \frac{\text{Im}[I_{nm}F_{mn}]}{(E_m - E_n)^2 + (\Gamma/2)^2} \]

\[ I_{nm} = -i \frac{e\hbar}{2m} (\psi^n \partial\psi^m - \partial\psi^n \psi^m)_{x=X_1} \]

\[ F_{nm} = -\lambda \frac{\hbar^2}{2m} (\psi^n \partial\psi^m + \partial\psi^n \psi^m - \lambda \psi^n \psi^m)_{x=X_0+0} \]
numerics
Summary

• LRT gives a unified framework for the theory of quantum pumping.
• Quantum chaos considerations are essential.
• Distinction between adiabatic, non-adiabatic and non-prt regimes.
• We have a generalized FD relation for pumping calculation.

\[ G^{kj} = g(E_F) \int_0^\infty C^{kj}(\tau) \, e^{-\left(\Gamma/2\hbar\right)\tau} \, d\tau \]

• It reduces to geometric magnetism for \( \Gamma = 0 \).
  \( \Gamma = 0 \) means strict adiabaticity and coherence

• It reduces to the BPT formula in case of a dot-wire geometry.
  This assumes the non-adiabatic limit \( L \to \infty \)

\[ G^{3j} = \frac{e}{2\pi i} \text{trace} \left( P_A \frac{\partial S}{\partial x_j} S^\dagger \right) \quad \text{[BPT]} \]

• We have analyzed pumping on networks using Green function expressions.
• The diagonal approximation agrees with classical stochastic modeling.
• Our expression for \( dQ \) depends on both \( g_0 \) and \( g_T \).
• The dispersion (“UCF”) of \( G \) decreases with \( \Gamma \).

\[ G^{3j} = \left[ \frac{g_T}{1-g_T} \right] \left[ \frac{1-g_0}{g_0} \right] \times \frac{e}{\pi k_F} \]