From classical pumps of water to quantum pumping of electrons in closed devices

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$GIF, \ $ISF
Driven Systems

Non interacting “spinless” electrons.

Held by a potential (e.g. AB ring geometry).

\[ x_1, x_2 = \text{shape parameters} \]

\[ x_3 = \Phi = (\hbar/e)\phi = \text{magnetic flux} \]

![Diagram](image-url)
“Ohm law”

For one parameter driving by EMF

\[ I = G^{33} \times (-\dot{x}_3) \]
\[ dQ = -G^{33} \, dx_3 \]

For driving by changing another parameter

\[ I = -G^{31} \, \dot{x}_1 \]
\[ dQ = -G^{31} \, dx_1 \]

For two parameter driving

\[ I = -G^{31} \dot{x}_1 - G^{32} \dot{x}_2 \]
\[ dQ = -G^{31} \, dx_1 - G^{32} \, dx_2 \]
\[ Q = -\oint G \cdot dx \]

and in general

\[ \langle F^k \rangle = -\sum_j G^{k:j} \dot{x}_j \]
Linear response theory

\[ \mathcal{H} = \mathcal{H}(r, p; x_1(t), x_2(t), x_3(t)) \]

\[ F^k = -\frac{\partial \mathcal{H}}{\partial x_k} \]

\[ \langle F \rangle_t = \int \alpha(t - t') \delta x(t') \, dt' \]

\[ \chi^{kj}(\omega) \]

\[ \text{Re}[\chi^{kj}(\omega)] \quad (1/\omega) \times \text{Im}[\chi^{kj}(\omega)] \]

\[ G^{kj} \]

\[ \eta^{kj} \quad B^{kj} \]

(dissipative) (non-dissipative)

\[ \langle F^k \rangle = -\sum_j G^{kj} \dot{x}_j \]
What is the problem?

From Kubo formula we get a formal expression for $G^{kj}$.

Can we trust this expression? Conditions? Quantum chaos!

How to use this expression? The bare Kubo formula gives no dissipation!

To define an energy scale $\Gamma$
Beyond first order perturbation theory!

$\Gamma$ in case of isolated system is due to non-adiabaticity.

$\Gamma$ affects both the dissipative and the non-dissipative (geometric) part of the response.
Some references

Adiabatic transport
Thouless (PRL 1983) - Periodic arrays
Avron, Sadun, Raveh, Zur (1988) - Networks with fluxes
Berry, Robbins (JPA 1993) - Geometric magnetism

Linear response theory and Mesoscopics
Imry, Shiren (PRB 1986) - Using Kubo for a closed ring
Wilkinson, Austin (JPA 1995) - Challenging the validity
DC (PRL 1999) - The “quantum chaos” identification of regimes
DC and Kottos (PRL 2000) - The \((A, \Omega)\) regimes diagram
DC (PRB+Rapid 2003) - the Kubo approach to pumping
DC, Kottos, Schanz (cond-mat 2004) - pumping on networks
Sela, DC (in preparation) - pumping on a ring

Open systems, \(S\) matrix formalism
The Buttiker Pretre Thomas [BPT] formula (1994)
Brouwer (1998)
Avron, Elgart, Graf, Sadun
Buttiker, Texier, Moskalets
Marcus - experiments
Shutenko, Aleiner, Altshuler (PRB 2000) - quantization?
Entin-Wohlman, Aharony, Levinson (2002) - two delta functions
Driven Systems - classification

\[ \mathcal{H} = \mathcal{H}(r, p; x_1(t), x_2(t), x_3(t)) \]

- closed isolated systems
- periodic arrays
- open systems (with reservoirs)
- one of the above interacting with a bath

Questions: Transport? Dissipation?
Simple pumping devices

How can we drive current?
- by changing the height of the potential
- by translating the potential

Specific question:
What is the current which is created by translating a scatterer?
The moving scatterer model

- There is a stationary solution.
- There is no dissipation.
- Pumping: \(dQ \propto 1 \times dX\)

- There is no stationary solution.
- There is dissipation.
- Pumping: \(dQ \propto (1 - g_0) \times dX\)
Simple model systems - networks

\[
dQ = (1 - g_0) \times \frac{e}{\pi} k_F \times dX
\]

\[
dQ = 1 \times \frac{e}{\pi} k_F \times dX
\]

\[
dQ = \left[ \frac{g_T}{1 - g_T} \right] \left[ \frac{1 - g_0}{g_0} \right] \times \frac{e}{\pi} k_F \times dX
\]

Adiabatic versus non-adiabatic result

DC, Kottos, Schanz, cond-mat 2004
Assume periodic ("AC") driving.
Does the current have a "DC" component?
Define charge transported per cycle:

\[ Q = \oint I \, dt = ??? \]

Linear response assumption \( \Rightarrow \) for one parameter driving \( Q = 0 \).

*Ratchests* are non-linear devices that use mixed or damped dynamics in order to pump with only one parameter.
For the open system, using BPT:

\[ Q \approx 1 - g \]

- Is it due to non-adiabaticity?
- Is it a dissipative contribution?

What about strict adiabatic cycle in a closed system?

Shutenko, Aleiner, Altshuler (2000):

“If the system were closed [and strictly adiabatic], the charge distribution after each period of the perturbation would return to the original distribution, and therefore, the pumped charge would be exactly quantized.”

Not correct!
Questions

• Take the “two barrier model” as an example.
• Assume Fermi occupation.
• Adiabatic limit: Find the current of each level.
• Is there one level that carries most of the current?
• What is the effect of non-adiabaticity?
• What is the role of dissipation?
• Can we get in a closed system $Q > 1$ or even $Q \gg 1$?
• Why in an open system always $Q = 1 - g$?

Formalism:

• Start with Kubo formula.
• Derive from it the adiabatic limit.
• Explain the implications of non-adiabaticity.
• Explain the emergence/role of dissipation.
• Take the limit of large $L$.
• Express the results using the $S$ matrix.
• Does the result coincides with BPT?
The two barrier model

\[ X_1 + X_2 \sim \text{dot potential floor} \]
The dot-wire ring system (I)

\[ \mathcal{H} = \mathcal{H}(r, p; x_1, x_2, x_3) \]

\[ x_3 = \Phi = (\hbar/e)\phi = \text{magnetic flux} \]

\[ -\dot{x}_3 = \text{electro motive force [Volt]} \]

\[ \oint \vec{A} \cdot d\vec{r} = \Phi \]

There is more than one way to put \( \Phi \) into \( \mathcal{H} \)...

Conjugate current operator?  Continuity?
The dot-wire ring system (II)

\[ \mathcal{H} = \mathcal{H}(r, p; x_1, x_2, x_3) \]

\[ x_1, x_2 = \text{shape parameters} \]

Possibilities:

- \( x = \) dot potential floor
- \( x = \) position of a wall element
- \( x = \) position of a scatterer inside the dot
- \( x = \) height of a barrier
Generalized forces / currents (I)

\[ \mathcal{H} = \mathcal{H}(r, p; x) \]
\[ F = -\frac{\partial \mathcal{H}}{\partial x} \]

Example 1:

\( x \) = position of a wall element (or scatterer)
\( \dot{x} \) = wall (or scatterer) velocity
\( F \) = Newtonian force

\[ \langle F \rangle = -\eta \dot{x} \quad \text{[friction law]} \]
\[ \dot{\mathcal{W}} = \eta \dot{x}^2 \quad \text{[rate of heating]} \]

\[ x(t) \]

chaos!
Generalized forces / currents (II)

\[ H = H(r, p; x) \]
\[ F = -\frac{\partial H}{\partial x} \]

Example 2:

\( x = \) magnetic flux through the ring
\[ \dot{x} = -\text{EMF} \]
\( F = \) electrical current
\[ \langle F \rangle = -G\dot{x} \quad \text{[Ohm law]} \]
\[ \dot{\mathcal{V}} = G\dot{x}^2 \quad \text{[Joule law]} \]

chaos!
Linear response theory

\[ \langle F \rangle_t = \int \alpha(t - t') \delta x(t') \, dt' \]

\[ \chi^{kj}(\omega) \]

\[ \text{Re}[\chi^{kj}(\omega)] \quad (1/\omega) \times \text{Im}[\chi^{kj}(\omega)] \]

\[ \Gamma^{kj} \]

\[ \eta^{kj} \quad B^{kj} \]

(dissipative) \quad (non-dissipative)

\[ \langle F^k \rangle = - \sum_j G^{kj} \dot{x}_j \]

\[ \langle F \rangle = -G \cdot \dot{x} = -\eta \cdot \dot{x} - B \wedge \dot{x} \]

\[ \dot{\mathcal{W}} = -\langle F \rangle \cdot \dot{x} = \sum_{kj} \eta^{kj} \dot{x}_k \dot{x}_j \]
The Kubo Formula

\[
\alpha^{kj}(\tau) = \Theta(\tau) \times \frac{i}{\hbar} \langle [F^k(\tau), F^j(0)] \rangle
\]

\[
\chi^{kj}(\omega) = \sum_{n,m} f(E_n) \left( \frac{-F_{nm}^k F_{mn}^j}{\hbar \omega - (E_m - E_n) + i0} + \frac{F_{nm}^j F_{mn}^k}{\hbar \omega + (E_m - E_n) + i0} \right)
\]

\[
G^{kj} = \frac{1}{\omega} \times \text{Im}[\chi^{kj}(\omega)] \bigg|_{\omega \sim 0}
\]

\[
\eta^{kj} = \pi \hbar \sum_{n,m} F_{nm}^k F_{mn}^j \delta(E_n - E_F) \delta(E_m - E_n)
\]

\[
B^{kj} = 2\hbar \sum_n f(E_n) \sum_{m(\neq n)} \frac{\text{Im}[F_{nm}^k F_{mn}^j]}{(E_m - E_n)^2 + (\Gamma/2)^2}
\]
The Kubo Formula and "quantum chaos"

\[ \tau_{cl} = \text{classical correlation time} \]

\[ \Delta \propto \hbar^d / L = \text{mean level spacing} \]

\[ \Delta_b \sim \hbar / \tau_{cl} = \text{bandwidth} \]

Effective width of the energy levels:

\[ \Gamma = \left( \frac{\hbar \sigma}{\Delta^2 |\dot{x}|} \right)^{2/3} \Delta \sim \left( \frac{L |\dot{x}|}{\Delta} \right)^{2/3} \frac{1}{L} \]

- \( \Gamma \ll \Delta \) \quad \text{adiabatic regime}
- \( \Delta < \Gamma < \Delta_b \) \quad \text{non-adiabatic regime}
- \( \Delta_b < \Gamma \) \quad \text{non-perturbative regime}

\( L \to \infty \) is not the semiclassical limit!
The generalized FD relation

\[ K^{kj}(\tau) = \left( i/\hbar \right) \langle [F^k(\tau), F^j(0)] \rangle \]

\[ C^{kj}(\tau) = \frac{1}{2} (\langle F^k(\tau) F^j(0) \rangle + \text{cc}) \]

\[ \alpha^{kj}(\tau) = \Theta(\tau) K^{kj}(\tau) \quad \text{[“Kubo formula”]} \]

\[ G^{kj} = \lim_{\omega \to 0} \frac{\text{Im}[\chi^{kj}(\omega)]}{\omega} = \int_0^\infty K^{kj}(\tau) \tau d\tau \]

\[ G^{kj} = \frac{1}{\Delta} \int_0^\infty C^{kj}(\tau) d\tau \quad \text{[“FD relation”]} \]

\[ B^{kj} = \frac{-i}{\Delta} \int_{-\infty}^\infty \left[ \frac{\tilde{C}^{kj}(\omega)}{\omega} \right] d\omega \frac{1}{2\pi} \]

\[ \eta^{kj} = \frac{1}{2\Delta} \tilde{C}^{kj}(\omega \sim 0) \]
Kubo formula - Green functions - BPT formula

\[ \eta^{3j} = \frac{\hbar}{\pi} \text{trace} \left[ F^3 \text{Im}[G^+] F^j \text{Im}[G^+] \right] \]
\[ = \frac{\hbar}{4\pi} \text{trace} \left[ \frac{\partial S^\dagger}{\partial x_3} \frac{\partial S}{\partial x_j} \right] \]

\[ B^{3j} = \frac{\hbar}{4\pi} \text{trace} \left[ \mathcal{F}^3 G^+ \mathcal{F}^j G^- - \mathcal{F}^3 G^- \mathcal{F}^j G^+ \right] \]
\[ = \frac{e}{4\pi i} \text{trace} \left[ P_A \left( \frac{\partial S}{\partial x_j} S^\dagger - \frac{\partial S^\dagger}{\partial x_j} S \right) \right] + \text{intrf} \]

\[ G^{3j} = \frac{e}{2\pi i} \text{trace} \left( P_A \frac{\partial S}{\partial x_j} S^\dagger \right) \quad \text{[BPT]} \]

DC, PRB(R) 2003
Pumping within the Kubo formulation

\[ Q = \oint \langle F^3 \rangle dt \]

\[ \langle F \rangle = -\eta \cdot \dot{x} - B \wedge \dot{x} \]

\[ Q = \left[ -\oint \eta \cdot dx - \oint B \wedge dx \right]_{k=3} \]

Consider a planar \((x_1, x_2)\) pumping cycle.

\[ Q = -\oint B \cdot ds \]

No magnetic field. Onsager \(\implies\)

\[ \eta^{31} = \eta^{32} = 0 \quad \text{(no dissipative contribution)} \]

\[ B^{12} = 0 \quad \text{(no vertical component)} \]
The $B$ field

$$Q = \left[- \oint B \wedge dx \right]_{k=3}$$

$$B^{ij} = \sum_{n} f(E_n) B^{ij}_n$$  \quad [\text{geometric magnetism}]$$

$$B^{kj}_n = 2\hbar \sum_{m(\neq n)} \frac{\text{Im} \left[ F^{k}_{nm} F^{j}_{mn} \right]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

This field is divergenceless (for $\Gamma = 0$)

A chain of degeneracies:

$$\left( x_1^{(0)}, \ x_2^{(0)}, \ \Phi^{(0)} + 2\pi \frac{e}{\hbar} \times \text{integer} \right)$$

The degeneracies are like Dirac monopoles

- The issue of bandwidth
- The effect of screening
Summary

- LRT gives a unified framework for the theory of pumping.
- Derivation of $S$ matrix expressions for $\eta$ and $B$.
- Distinction between adiabatic, non-adiabatic and non-prt regimes.
- "Quantum chaos" considerations are essential ($\Gamma$).
- The emergence / relevance of dissipation.
- The $L \to \infty$ limit versus the $\hbar \to 0$ limit.
- Near-field versus far field pumping cycles around "Dirac chains".
- The analysis of deviations from "quantized" pumping.

\[
x(t) = Vt
\]

\[
x(t) = A \sin(\Omega t)
\]
Digression - The simplest pump

Assume that the current is given by Ohm law:

\[ I = -G \frac{d}{dt} \Phi = -G \frac{dx_3}{dt} \]

\[ Q = \int I dt = - \int G dx_3 \]

\[ Q(a) = -G(a) \times (\Phi_2 - \Phi_1) \]

\[ Q(b) = -G(b) \times (\Phi_1 - \Phi_2) \]

The net pumped charge:

\[ Q = (G(b) - G(a)) \times (\Phi_2 - \Phi_1). \]
Digression - The BPT formula

\[ S = \begin{pmatrix} r_B & t_{AB} e^{-i\phi} \\ t_{BA} e^{i\phi} & r_A \end{pmatrix} \]

\[ P_A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ G^{3j} = \frac{1}{2\pi i} \text{trace} \left( P_A \frac{\partial S}{\partial x_j} S^\dagger \right) \]

\[ \langle F^3 \rangle = -\sum_j G^{3j} \dot{x}_j \]

\[ S = e^{i\gamma} \begin{pmatrix} i\sqrt{1 - ge^{i\alpha}} & \sqrt{ge^{-i\phi}} \\ \sqrt{ge^{i\phi}} & i\sqrt{1 - ge^{-i\alpha}} \end{pmatrix} \]

\[ Q = \frac{1}{2\pi} \int (1 - g) \frac{d\alpha}{dx} \cdot d\vec{x} \approx 1 - g \]
Digression - The $S$ matrix

$$S = e^{i\gamma} \begin{pmatrix} i\sqrt{1-g}e^{i\alpha} & \sqrt{g}e^{-i\phi} \\ \sqrt{g}e^{i\phi} & i\sqrt{1-g}e^{-i\alpha} \end{pmatrix}$$

$g$ = transmission

$\gamma$ = global phase shift

$\phi$ = magnetic flux phase

$\alpha$ = displacement phase

\[d\alpha = 2kdX\]
Strictly adiabatic:

\[ dQ_n = \frac{1}{L} dX \]

\[ dQ = \frac{p_F dX}{\pi \hbar} \]

\[ N = \frac{2p_F L}{2\pi \hbar} \]

Non adiabatic:

\[ dQ = (1 - g) \times \frac{p_F dX}{\pi \hbar} \]