Quantum stirring of particles

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Outline:
Quantum Stirring of BEC in a 3 site system;
The linear response / Kubo analysis;
Counting Statistics in closed geometries;
Analysis of a double path adiabatic passage;
Analysis of a double path Bloch transition.

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Quantum Stirring in a 3 site system

\[ \hat{H} = \sum_{i=0}^{2} \varepsilon_i n_i + \frac{U}{2} \sum_{i=0}^{2} \hat{n}_i (\hat{n}_i - 1) - k_c (\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) \\
- k_1 (\hat{b}_0^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{b}_0) - k_2 (\hat{b}_0^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_0) \]

Trimer system [Hiller, Kottos, Geisel, Bodyfelt, Stickney, Anderson, Zozulya]

\[ U = \text{the inter-atomic interaction} \]

Control parameters:

\[ X_1 = \left( \frac{1}{k_2} - \frac{1}{k_1} \right) \]
\[ X_2 = \varepsilon \]
Dynamical scenarios

\[ I = -G \dot{X} \]
\[ dQ = -G d\varepsilon \]

strong attractive interaction: classical ball dynamics

negligible interaction (\(|U| \ll \kappa\)): mega-crossing

weak repulsive interaction: gradual crossing

strong repulsive interaction (\(U \gg N\kappa\)): sequential crossing
Results for the geometric conductance

For $U = 0$, mega crossing

$$G = -N \frac{(k_1^2 - k_2^2)/2}{[(\varepsilon - \varepsilon_-)^2 + 2(k_1+k_2)^2]^{3/2}}$$

For $\kappa \ll U \ll N\kappa$, gradual crossing

$$G \approx - \left[ \frac{k_1 - k_2}{k_1 + k_2} \right] \frac{1}{3U}$$

For $U \gg N\kappa$, sequential crossing

$$G = - \left( \frac{k_1 - k_2}{k_1 + k_2} \right) \sum_{n=1}^{N} \frac{(\delta\varepsilon_n)^2}{[(\varepsilon - \varepsilon_n)^2 + (2\delta\varepsilon_n)^2]^{3/2}}.$$
References


Linear response theory

For one parameter driving by EMF

\[ I = G^{33} \times (-\dot{X}_3) \]
\[ dQ = -G^{33} \, dX_3 \]

For driving by changing another parameter

\[ I = -G^{31} \, \dot{X}_1 \]
\[ dQ = -G^{31} \, dX_1 \]

For two parameter driving

\[ I = -G^{31} \, \dot{X}_1 - G^{32} \, \dot{X}_2 \]
\[ dQ = -G^{31} \, dX_1 - G^{32} \, dX_2 \]
\[ Q = -\oint G \cdot dX \]

(*) In general

\[ \langle F^k \rangle = - \sum_j G^{kj} \, \dot{X}_j \]
The Kubo formula approach

\[ Q = - \oint (G^{31} dX_1 + G^{32} dX_2) = \oint \mathbf{B} \cdot d\mathbf{s} \]

\[ \mathbf{B} = (-G^{32}, G^{31}) \quad d\mathbf{s} = (dX_2, -dX_1) \]

\[ B_j = \sum_{n \neq n_0} \frac{2 \text{Im} [\mathcal{I}_{n0n}] \mathcal{F}_{jnn_0}}{(E_n - E_{n_0})^2}. \]

\[ X_1 = \left( \frac{1}{k_2} - \frac{1}{k_1} \right) \]

\[ X_2 = \varepsilon \]
Counting Statistics

\[ Q = \int_0^t \mathcal{I}(t') dt' \]

\[ \langle Q \rangle = ??? \quad \delta Q = ??? \quad P(Q) = ??? \]

Simple example: adiabatic passage with \( c_1 = c_2 \).
We say “\( \langle Q \rangle = \frac{1}{2} \)” . What do we mean by that?

• maybe \( P(1/2) = 100\% \) and hence \( \delta Q = 0 \)
• maybe: \( P(0) = P(1) = 50\% \) and hence \( \delta Q = 1/2 \)
How $Q$ is measured

Naive answer:
The distribution $P(Q)$ can be determined using a continuous measurement scheme.

In such setup the current induces (so to say) a "translation" of a Von-Neumann pointer. After time $t$ the position of the pointer is measured.

Proper answer:
One can measure the quasi distribution $P(Q; x)$

$$\rho_t(q, x) = \int P(q - q'; x) \rho_0(q', x) dq'$$

where

$$P(Q; x=0) = \frac{1}{2\pi} \int \left\langle \left[ T e^{-i(r/2)Q} \right]^\dagger \left[ T e^{i(r/2)Q} \right] \right\rangle e^{-iQr} dr$$

If we ignore time ordering we get:

$$P(Q) = \frac{1}{2\pi} \int \left\langle e^{+irQ} e^{-iQr} dr \right\rangle = \left\langle \delta(Q - Q) \right\rangle$$

H. Everett, Rev. Mod. Phys. 29, 454 (1957).
Hamiltonians for 2 and 3 site systems

\[ \mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix}, \quad \mathcal{I} = \begin{pmatrix} 0 & i c \\ -i c & 0 \end{pmatrix} \]

\[ \mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}, \quad \mathcal{I} = \begin{pmatrix} 0 & i c_1 & 0 \\ -i c_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ \mathcal{H} = \begin{pmatrix} u & \frac{(c_1 + c_2)}{\sqrt{2}} \\ \frac{(c_1 + c_2)}{\sqrt{2}} & 1 \end{pmatrix}, \quad \mathcal{I} = \frac{c_1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \]
Double path adiabatic passage

\[ \mathcal{H} = \begin{pmatrix} u(t) & c \\ c & 1 \end{pmatrix}, \quad \mathcal{I} = \lambda \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix} \]

with effective coupling and splitting ratio

\[ c \equiv \frac{(c_1 + c_2)}{\sqrt{2}}, \quad \lambda \equiv \frac{c_1}{c_1 + c_2} \]

Accordingly:

\[ \langle Q \rangle = \lambda p \]
\[ \delta Q^2 = \lambda^2 (1 - p)p \neq (1 - \lambda p)\lambda p \]

If \( c_1 \) and \( c_2 \) have opposite signs then (say) \( \lambda \) becomes larger than unity, while \( (1 - \lambda) \) is negative. This reflects that the driving induces a circulating current within the ring, and illuminates the fallacy of the classical peristaltic point of view.
Counting statistics for a coherent transition

Naive expectation:

Given the probability \( p \) to make the transition

\[
P(Q) = \begin{cases} 
1-p & \text{for } Q = 0 \\
p & \text{for } Q = 1 
\end{cases}
\]

\[
\langle Q^k \rangle = P(1) \cdot 1^k + P(0) \cdot 0^k = p
\]

\[
\delta Q^2 = (1 - p)p
\]

Quantum result:

\[
P(Q) = \begin{cases} 
p_- & \text{for } Q = Q_- \\
p_+ & \text{for } Q = Q_+ 
\end{cases}
\]

where

\[
Q_\pm = \pm \sqrt{p}
\]

\[
p_\pm = \frac{1}{2}(1 \pm \sqrt{p})
\]

hence

\[
\langle Q^k \rangle = p_+ Q^k_+ + p_- Q^k_- = p \left[ \frac{k+1}{2} \right]
\]
Restricted quantum-classical correspondence

$\mathcal{N} =$ occupation operator (eigenvalues = 0, 1)

$\mathcal{I} =$ current operator

Heisenberg equation of motion:

$$\frac{d}{dt} \mathcal{N}(t) = \mathcal{I}(t)$$

leads to

$$\mathcal{N}(t) - \mathcal{N}(0) = \mathcal{Q}$$

hence

$$\langle \mathcal{Q}^k \rangle = \langle \mathcal{N}^k \rangle_t = p \quad \text{for } k = 1, 2.$$

Restricted QCC is robust

Detailed QCC is fragile

Restricted QCC survives even in the presence of diffraction:

Digression: Analysis of quantum stirring

Assume $|c_1| > |c_2| > 0$ in the 1st half of the cycle and interchange them in the 2nd half of the cycle.

Naive expectation:

$\langle Q \rangle < 1$ \[\text{per cycle}\]

Quantum result:

$\langle Q \rangle = \frac{c_1 - c_2}{c_1 + c_2}$ \[\text{per cycle}\]

An optional way to derive this result is to make a full 3 level calculation using the Kubo formula:

Naive expectation:
Probabilistic point of view implies
\[ \delta Q \propto \sqrt{t} \]

Quantum result:
The eigenvalues \( Q_{\pm} \) of the \( Q \) operator grow linearly with the number of cycles
\[ \delta Q \propto t \]

If we have good control over the preparation we can select it to be a Floque state of the quantum evolution operator. For such preparation the linear growth of \( \delta Q \) is avoided, and it oscillates around a residual value.
Conclusions

Quantum mechanics is a “deterministic” rather than a “probabilistic” theory.

Coherent splitting unlike probabilistic splitting of a wavepacket is “exact”.

In a double path adiabatic passage one may find that (say) 170% of the particle goes via one path, while −70% goes via the second path.

There is at most restricted quantum-classical correspondence for the first and second moments.

If we have $N$ condensed particles the sign and the strength of the interactions determines the nature of the dynamics: mega crossing or gradual crossing or sequential crossing, or classical ball dynamics.