Quantum stirring of particles

Doron Cohen

Ben-Gurion University

Maya Chuchem (BGU)

Tsampikos Kottos (Wesleyan)

Moritz Hiller (Gottingen)

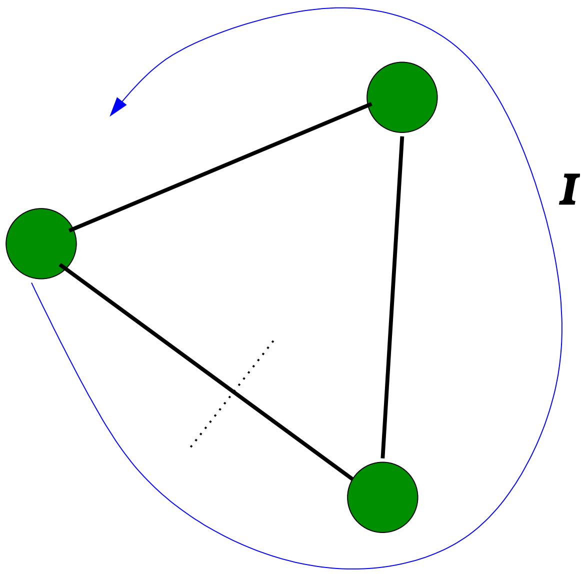
Thanks:

Itamar Sela (BGU)

Oded Agam (HUJI)

Yuli Nazarov (Delft)

$DIP, $BSF



Refs:

Counting statistics for a coherent transition, MC and DC (PRA 2008)

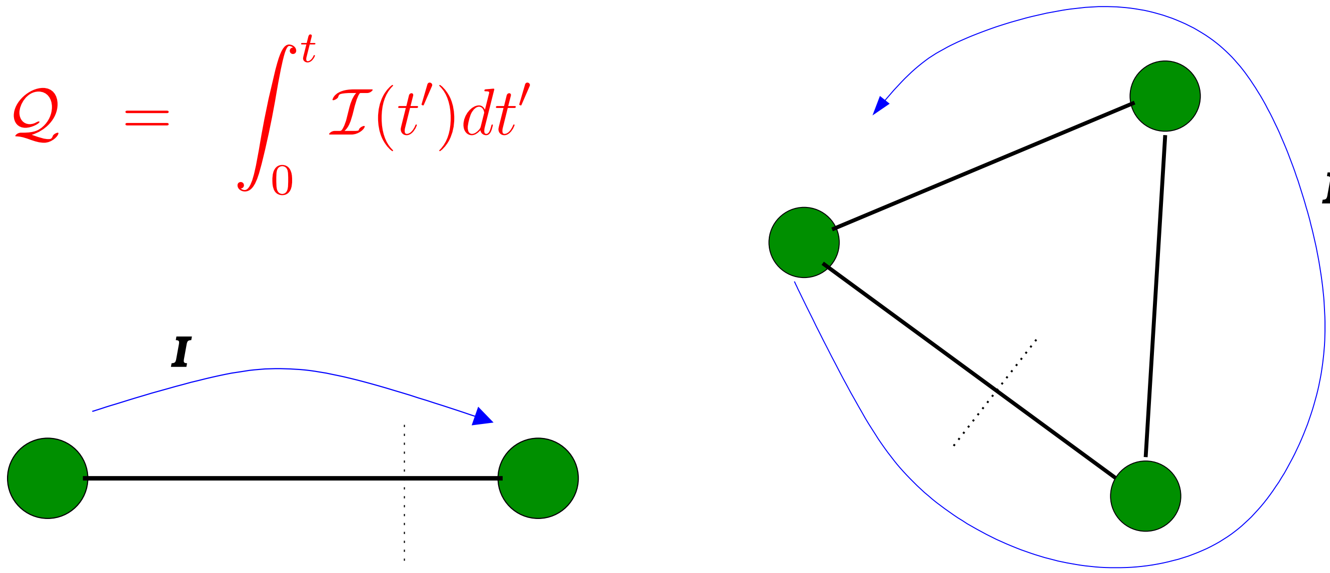
Counting statistics in multiple path geometries, MC and DC (JPA 2008)

Quantum Stirring of condensed particles, MH, TK and DC (EPL & PRA 2008)

http://www.bgu.ac.il/~dcohen

Outline

$$\mathcal{Q} \;=\; \int_0^t \mathcal{I}(t')dt'$$

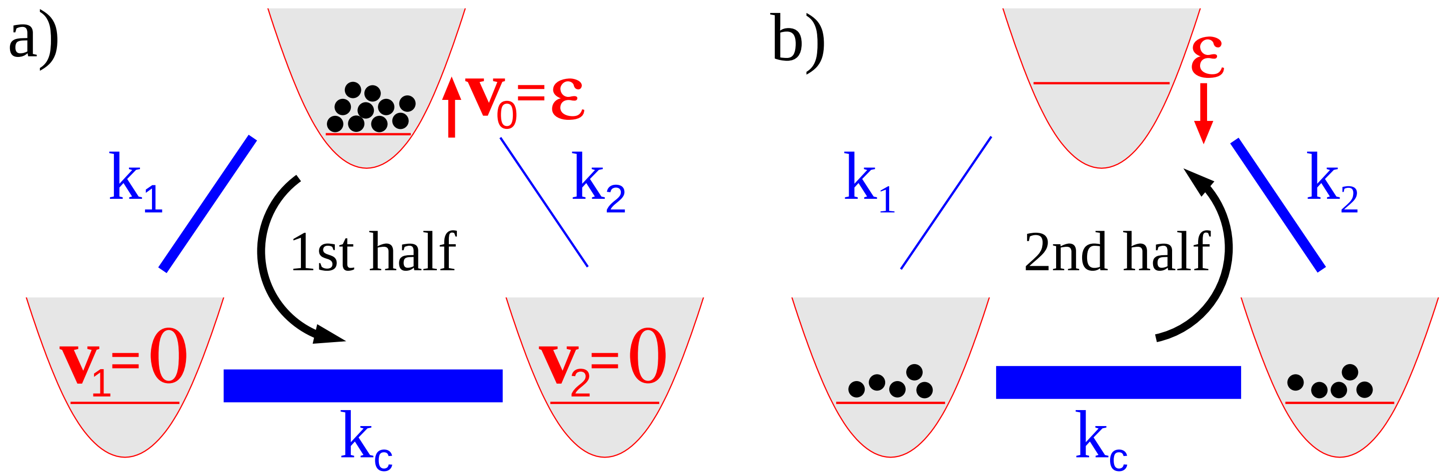


– Single path coherent transition

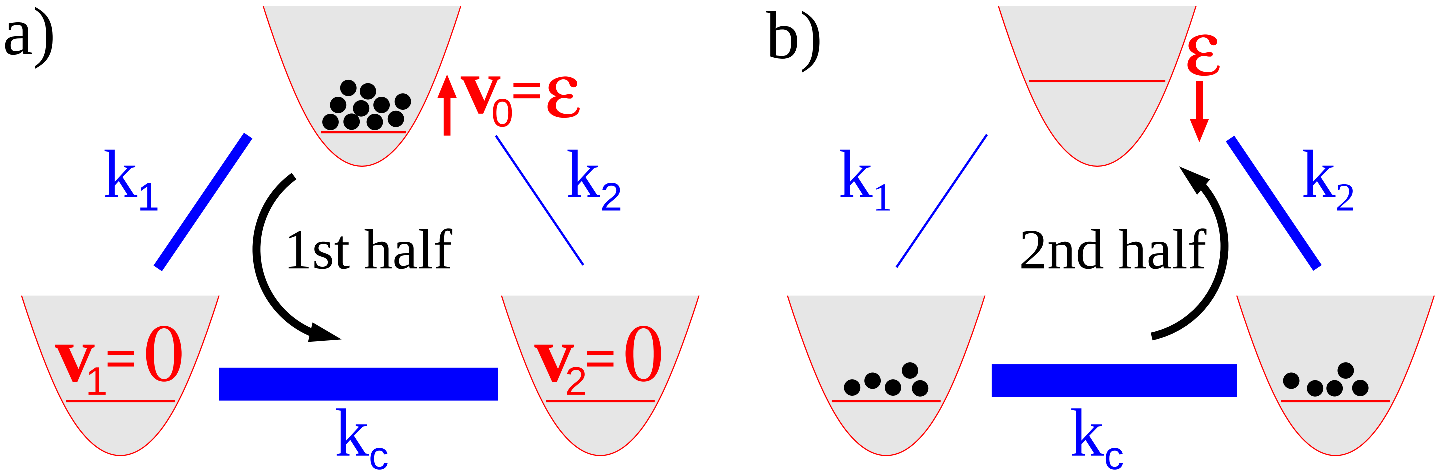
– Double path coherent transition

– quantum stirring (full cycle)

– Condensed particles: the effect of interactions



Quantum stirring of condensed particles



$$\hat{\mathcal{H}} = \sum_{i=0}^{2} v_i n_i + \frac{U}{2}\sum_{i=0}^{2}\hat{n}_i(\hat{n}_i-1) - k_c(\hat{b}_1^\dagger\hat{b}_2 + \hat{b}_2^\dagger\hat{b}_1)$$
$$-k_1(\hat{b}_0^\dagger\hat{b}_1 + \hat{b}_1^\dagger\hat{b}_0) - k_2(\hat{b}_0^\dagger\hat{b}_2 + \hat{b}_2^\dagger\hat{b}_0)$$

Trimer system [Hiller, Kottos, Geisel, Bodyfelt, Stickney, Anderson, Zozulya]

U = the inter-atomic interaction

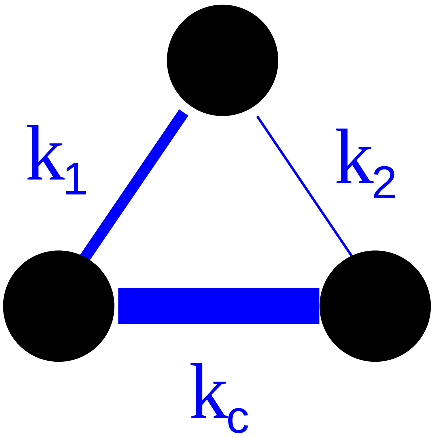
Control parameters:

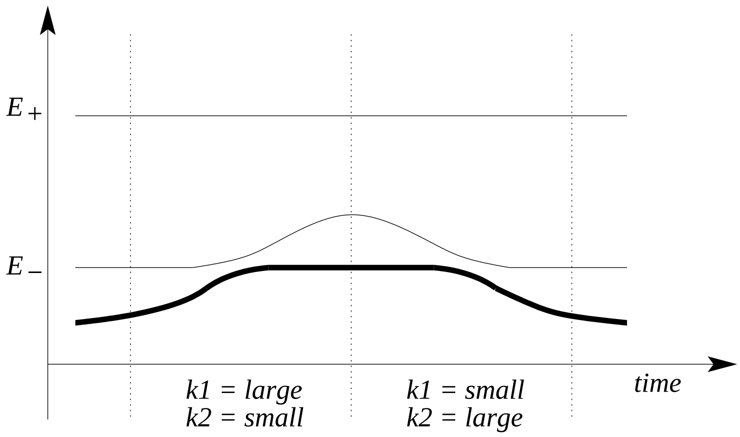
$$X_1 = \left(\frac{1}{k_2} - \frac{1}{k_1}\right)$$
$$X_2 = \varepsilon$$

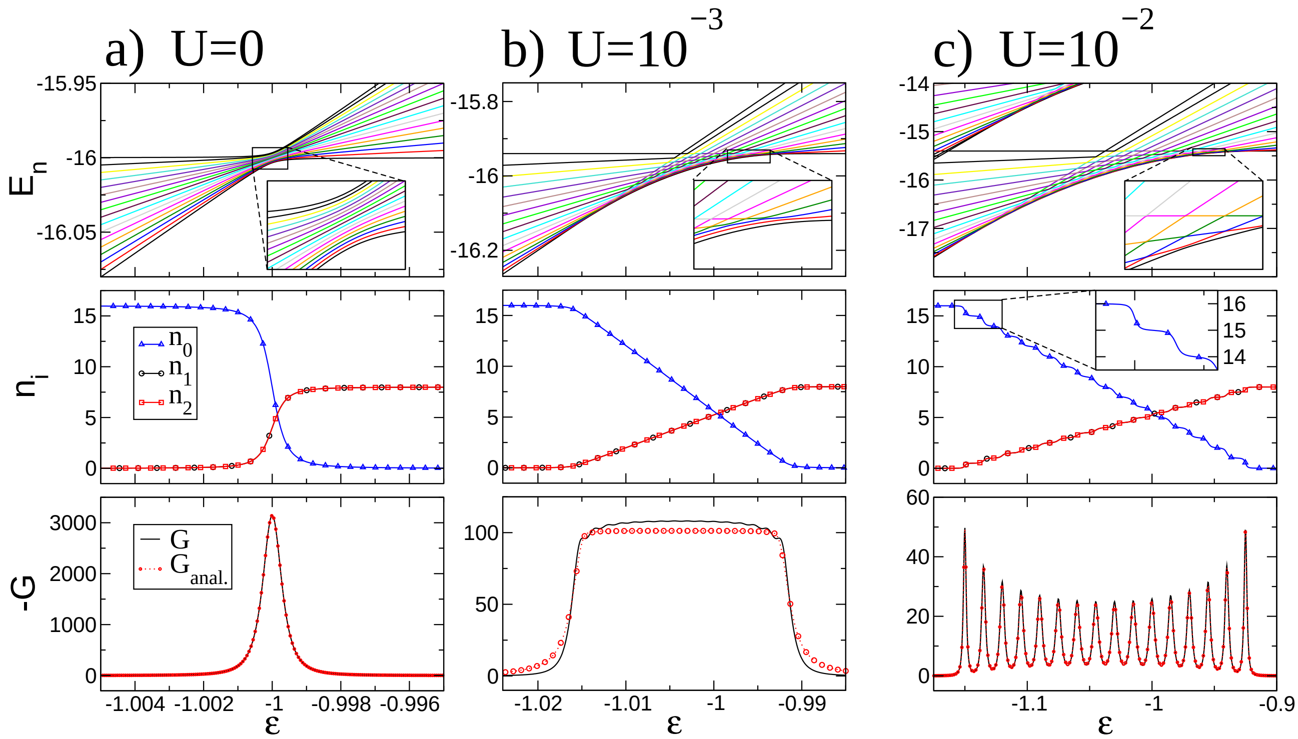
Induced Current:

$$I = -G_2\,\dot{X}_2$$

Dynamical scenarios







$$Q = -\oint G \cdot \mathrm{d}X$$

strong attractive interaction: classical ball dynamics

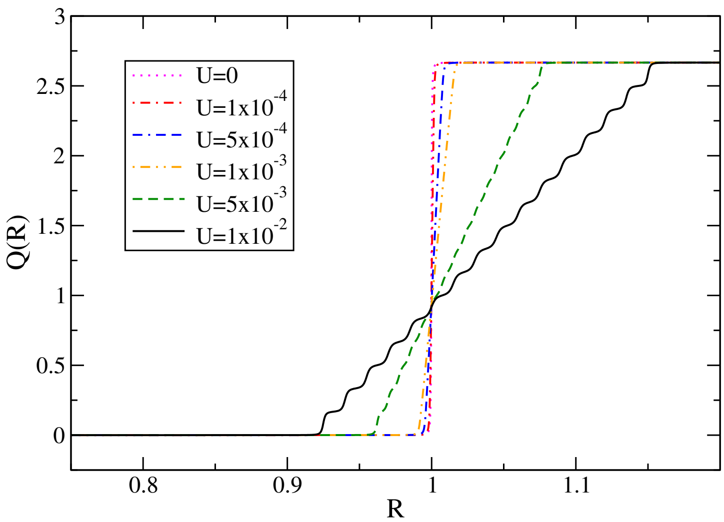
negligible interaction ($|U| \ll \kappa/N$): mega-crossing

weak repulsive interaction: gradual crossing

strong repulsive interaction ($U \gg N\kappa$): sequential crossing

Conclusions

If we have N condensed particles the sign and the strength of the interaction determine the nature of the dynamics: mega crossing or gradual crossing or sequential crossing, or classical ball dynamics.



- FCS for a coherent transition: restricted QCC

- Multiple path geometry: splitting ratio concept

- Full stirring cycle: interference of LZ transitions

- Long time counting statistics: Floque picture

References

[1] D.J. Thouless, Phys. Rev. B **27** 6083 (1983).

[2] Q. Niu, and D.J. Thouless, J. Phys. A **17**, 2453 (1984).

[3] M.V. Berry, Proc. R. Soc. Lond. A **392**, 45 (1984).

[4] J.E. Avron, A. Raveh and B. Zur, Rev. Mod. Phys. **60**, 873 (1988).

[5] M.V. Berry and J.M. Robbins, Proc. R. Soc. Lond. A **442**, 659 (1993).

[6] M. Buttiker, H. Thomas and A Pretre (1994).

[7] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998)

[8] D. Cohen, Phys. Rev. B **68**, 155303 (2003).

[9] D. Cohen, Phys. Rev. B **68**, 201303(R) (2003).

[10] D. Cohen, T. Kottos and H. Schanz, Phys. Rev. E 71, 035202(R) (2005).

[11] G. Rosenberg and D. Cohen, J. Phys. A **39**, 2287 (2006).

[12] I. Sela and D. Cohen, J. Phys. A **39**, 3575 (2006).

[13] L.S. Levitov and G.B. Lesovik, JETP Letters **55**, 555 (1992).

[14] L.S. Levitov and G.B. Lesovik, JETP Letters **58**, 230 (1993).

[15] Y.V. Nazarov and M. Kindermann, European Physical Journal B (2003).

[16] Ya.M. Blanter and M. Buttiker, Physics Reports **336**, 1 (2000)

[17] C.W.J. Beenakker and H. van Houten, Phys. Rev. B **43**, 12066 (1991)

[18] C.W.J. Beenakker and M. Buttiker, Phys. Rev. B **46**, 1889 (1992).

[19] K.E. Nagaev, Phys. Lett. A **169**, 103 (1992).

[20] B.L. Altshuler, L. S. Levitov, and A. Yu. Yakovets (1994).

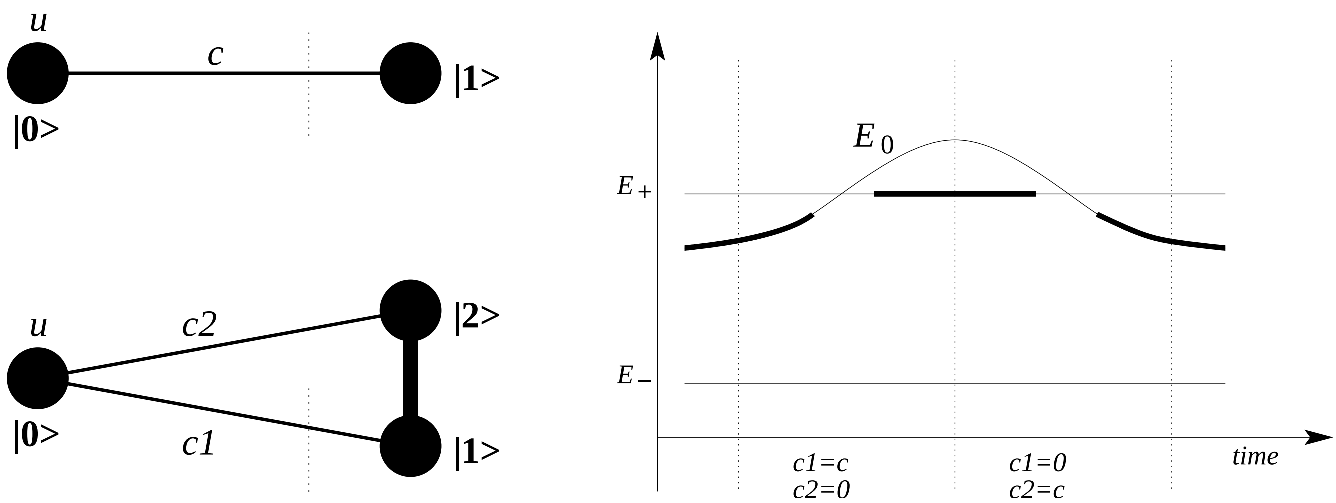
[21] H.U. Baranger and P. Mello, Phys. Rev. Lett. **73**, 142 (1994).

[22] R.A. Jalabert, J.L. Pichard, and C.W.J. Beenakker, EPL **27**, 255 (1994)

[23] O. Agam, I.L. Aleiner, and A.I. Larkin, Phys. Rev. Lett. **85**, 3153 (2000).

[24] A. Stotland and D. Cohen, J. Phys. A **39**, 10703 (2006).

Counting Statistics



$$\mathcal{Q} \;=\; \int_0^t \mathcal{I}(t')dt'$$

$\langle \mathcal{Q} \rangle =???$ $\qquad \delta Q =???$ $\qquad P(Q) =???$

Simple example: adiabatic passage with $c_1 = c_2$.
We say "$\langle \mathcal{Q} \rangle = \frac{1}{2}$". What do we mean by that?

- maybe $P(1/2)$=100% and hence $\delta Q = 0$
- maybe: $P(0)$=$P(1)$=50% and hence $\delta Q = 1/2$

Main results

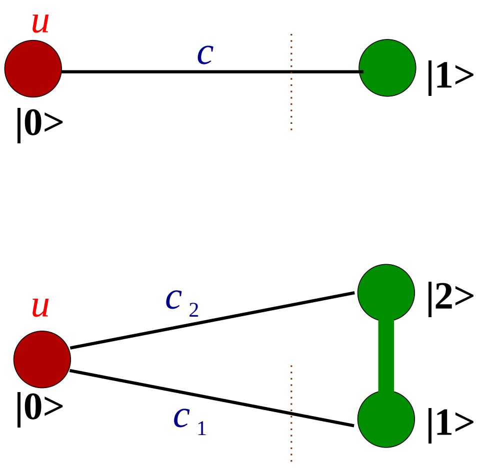
For a half a cycle:

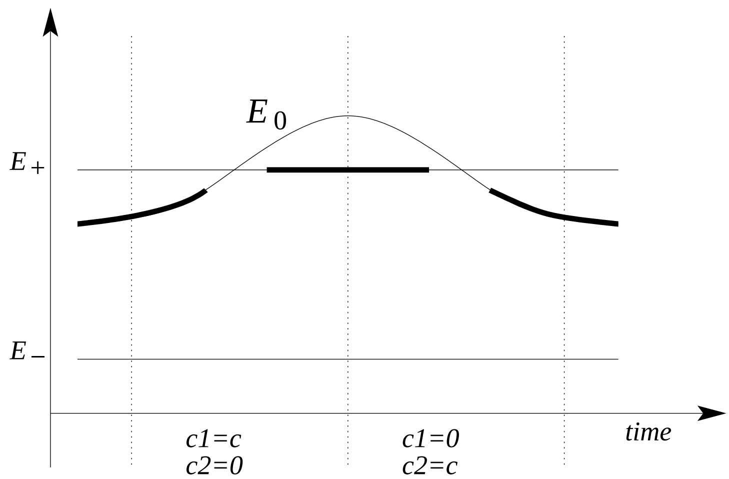
$$p = 1 - P_{\mathrm{LZ}}$$
$$\langle \mathcal{Q} \rangle = \lambda p$$
$$\mathrm{Var}(\mathcal{Q}) = \lambda^2 \underbrace{(1-p)p} \neq (1-\lambda p)\lambda p$$

$$P_{\mathrm{LZ}} = \mathrm{e}^{-\frac{\pi (c_1+c_2)^2}{\dot{u}}} \quad , \quad \lambda = \frac{c_1}{c_1+c_2} = \text{splitting ratio}$$

For a full stirring cycle:

$$p \approx \left| \mathrm{e}^{i\varphi_1} - \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}$$
$$\langle \mathcal{Q} \rangle \approx \lambda_{\circlearrowleft} - \lambda_{\circlearrowright}$$
$$\mathrm{Var}(\mathcal{Q}) \approx \left| \lambda_{\circlearrowleft} \mathrm{e}^{i\varphi_1} + \lambda_{\circlearrowright} \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}$$





How Q is measured

The counting statistics can be determined using a continuous measurement scheme.

$$\mathcal{H}_{\text{total}} \quad = \quad \mathcal{H}_{\text{system}} - \mathcal{I}x + \mathcal{H}_{\text{pointer}}(x,q)$$

In such setup the current induces (so to say) a "translation" of a Von-Neumann pointer. After time t the position of the pointer is measured.

Detailed answer:

One can measure the quasi distribution $\mathrm{P}(Q;x)$

$$\rho_t(q,x) = \int \mathrm{P}(q-q';x)\rho_0(q',x)dq'$$

where

$$\mathrm{P}(Q;x{=}0) = \frac{1}{2\pi}\int \left\langle \left[\mathcal{T}\mathrm{e}^{-i(r/2)\mathcal{Q}}\right]^\dagger \left[\mathcal{T}\mathrm{e}^{+i(r/2)\mathcal{Q}}\right] \right\rangle \mathrm{e}^{-iQr}dr$$

If we ignore time ordering we get:

$$\mathrm{P}_{\text{naive}}(Q) \quad = \quad \frac{1}{2\pi}\int \langle \mathrm{e}^{+ir\mathcal{Q}}\rangle\, \mathrm{e}^{-iQr}dr \quad = \quad \langle \delta(Q-\mathcal{Q})\rangle$$

H. Everett, Rev. Mod. Phys. 29, 454 (1957).
L.S. Levitov and G.B. Lesovik, JETP Letters (1992).
L.S. Levitov and G.B. Lesovik, JETP Letters (1993).
Y.V. Nazarov and M. Kindermann, EPJ B (2003).

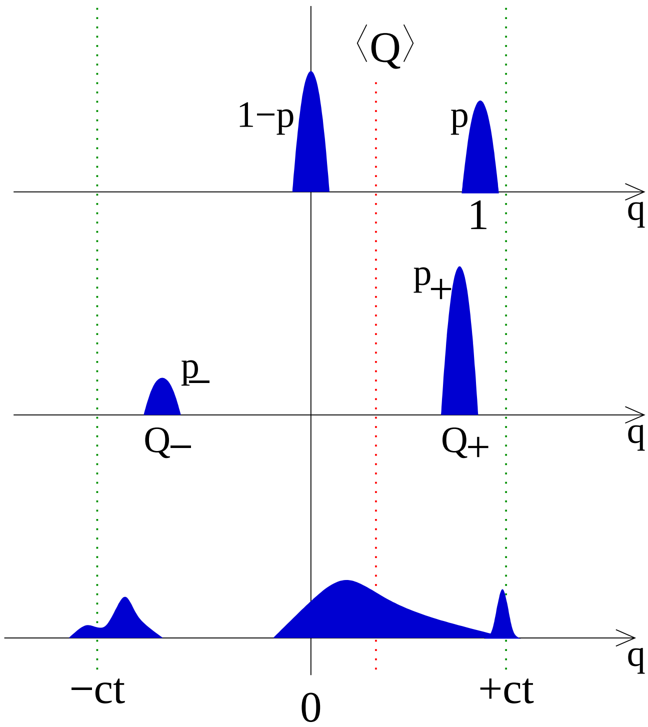
FCS for a Bloch transition

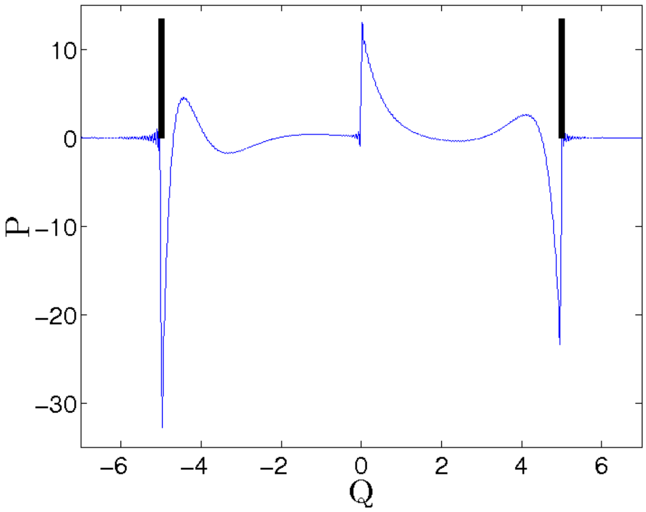
$$\rho_t(q,x) = \int \mathrm{P}(q-q';x)\rho_0(q',x)dq'$$

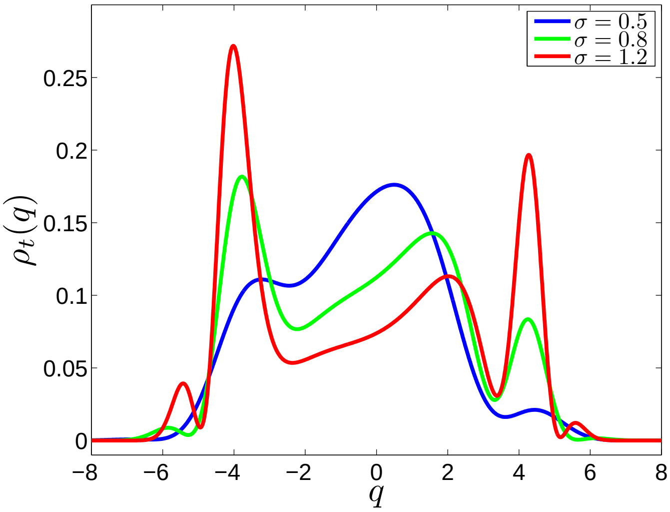
$$\mathrm{P}_{cl}(Q) = \begin{cases} 1-p & \text{for } Q=0 \\ p & \text{for } Q=1 \end{cases}$$

$$\mathrm{P}_{naive}(Q) = \begin{cases} p_- & \text{for } Q=Q_- \\ p_+ & \text{for } Q=Q_+ \end{cases}$$

$$\mathrm{P}_{qm}(Q;x=0)$$

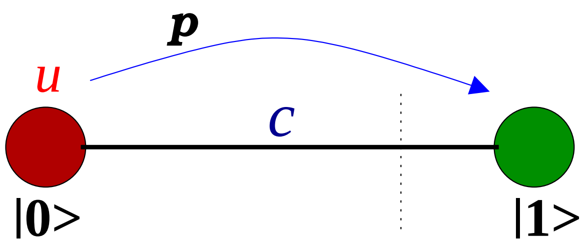






Counting statistics for a coherent transition

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & E_1 \end{pmatrix}$$



Naive expectation:

Given the probability p to make the transition

$$\mathrm{P}(Q) = \begin{cases} 1-p & \text{for } Q = 0 \\ p & \text{for } Q = 1 \end{cases}$$

$$\langle \mathcal{Q}^k \rangle \quad = \quad \mathrm{P}(1) \cdot 1^k + \mathrm{P}(0) \cdot 0^k \quad = \quad p$$

$$\mathrm{Var}(\mathcal{Q}) \quad = \quad (1-p)p$$

Quantum result:

$$\mathrm{P}(Q) = \begin{cases} p_- & \text{for } Q = Q_- \\ p_+ & \text{for } Q = Q_+ \end{cases}$$

where

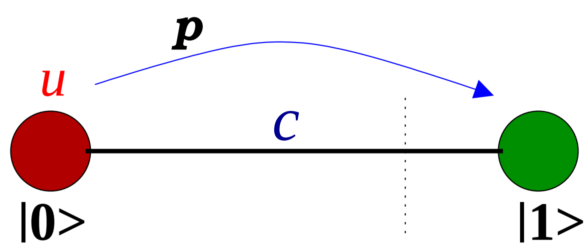
$$Q_\pm = \pm\sqrt{p}\ , \qquad\qquad p_\pm = \frac{1}{2}(1 \pm \sqrt{p})$$

hence

$$\langle \mathcal{Q}^k \rangle = p_+ Q_+^k + p_- Q_-^k = p^{\left\lfloor \frac{k+1}{2} \right\rfloor}$$

$\mathrm{Var}(\mathcal{Q})$ for a LZ transition

$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix},$$



leading order adiabatic approximation:

$$U(t) \approx \sum_n \Big|n(t)\Big\rangle \exp\left[-i\int_{t_0}^{t} E_n(t')dt'\right]\Big\langle n(t_0)\Big|$$

$$\mathcal{I}(t)_{nm} = \langle n|U(t)^\dagger \mathcal{I} U(t)|m\rangle$$

$$\approx \langle n(t)|\mathcal{I}|m(t)\rangle \exp\left[i\int_{t_0}^{t} E_{nm}(t')dt'\right]$$

$$\mathcal{Q} \equiv \begin{pmatrix} +Q_{\parallel} & iQ_{\perp} \\ -iQ_{\perp}{}^* & -Q_{\parallel} \end{pmatrix}$$

$$\mathrm{Var}(\mathcal{Q}) = |Q_{\perp}|^2 \approx \left| c\int_{-\infty}^{\infty} \mathrm{e}^{i\Phi(t)}dt \right|^2$$

$$\Phi(t) \equiv \int_0^t \sqrt{(\dot{u}t')^2 + (2c)^2}dt'$$

Var($\mathcal{Q}$) for a LZ transition (cont.)

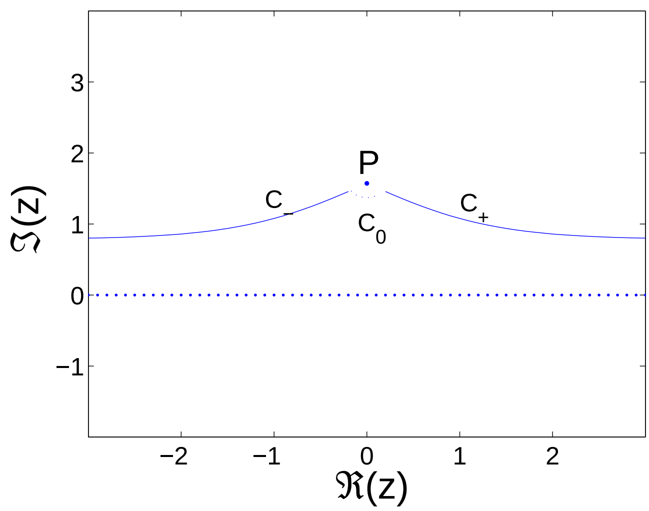
$$P_{\rm LZ} \approx \left| c\int_{-\infty}^{\infty} \frac{\dot{u}}{(\dot{u}t)^2+(2c)^2}\, e^{i\Phi(t)} \quad dt \right|^2$$

$$= \left|\frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{\cosh(z)}\, e^{i\Phi(z)} dz \right|^2$$

$$\mathrm{Var}(\mathcal{Q}) \approx \left| c\int_{-\infty}^{\infty} e^{i\Phi(t)} dt \right|^2$$

$$= \left|\frac{2c^2}{\dot{u}}\int_{-\infty}^{\infty} \cosh(z)\, e^{i\Phi(z)} dz \right|^2$$

??? $\mathrm{Var}(\mathcal{Q}) = (1-P_{\rm LZ})P_{\rm LZ}$???



$$P_{\rm LZ} \sim \left(\frac{\pi}{3}\right)^2 \exp\left[-\frac{\pi c^2}{\dot{u}}\right]$$

$$\mathrm{Var}(\mathcal{Q}) \sim \left(\frac{2c^2}{\dot{u}}\right)^{2/3} \exp\left[-\frac{\pi c^2}{\dot{u}}\right]$$

Restricted quantum-classical correspondence

$\mathcal{N}$ = occupation operator (eigenvalues $= 0, 1$)

$\mathcal{I}$ = current operator

Heisenberg equation of motion:

$$\frac{d}{dt}\mathcal{N}(t) \quad = \quad \mathcal{I}(t)$$

leads to

$$\mathcal{N}(t) - \mathcal{N}(0) = \mathcal{Q}$$

hence we find a relation between
counting statistics and occupation statistics:

$$\langle \mathcal{Q}^k \rangle \quad = \quad \left\langle (\mathcal{N}(t) - \mathcal{N}(0))^k \right\rangle \quad \stackrel{?}{=} \quad \langle \mathcal{N}^k \rangle_t \quad = \quad p$$

for $k = 1, 2$ only

$$\left\langle 0 \middle| \, (\mathcal{N}(t) - \mathcal{N}(0)) \, (\mathcal{N}(t) - \mathcal{N}(0)) \, (\mathcal{N}(t) - \mathcal{N}(0)) \, \middle| 0 \right\rangle \; \neq \; \left\langle 0 \middle| \mathcal{N}(t)^3 \middle| 0 \right\rangle$$

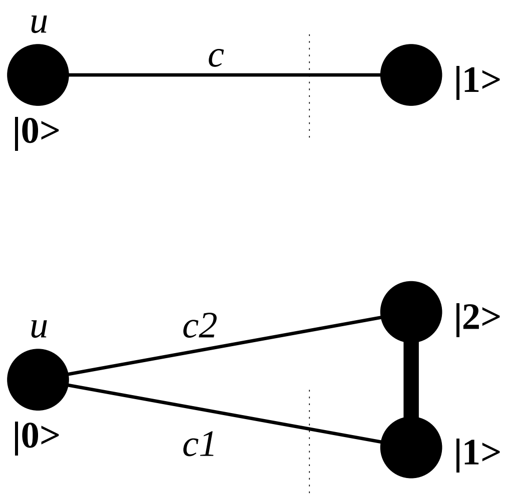
Restricted QCC is robust

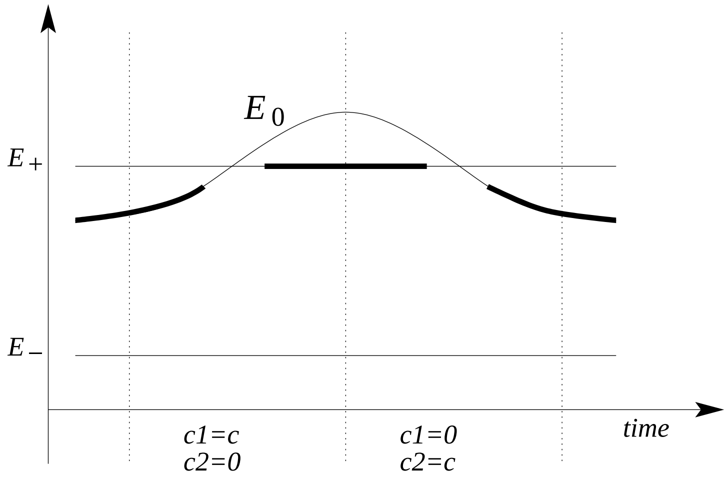
Detailed QCC is fragile

Restricted QCC survives even in the presence of diffraction:

A. Stotland and D. Cohen, J. Phys. A 39, 10703 (2006).

Hamiltonians for 2 and 3 site systems





$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & \frac{(c_1+c_2)}{\sqrt{2}} \\ \frac{(c_1+c_2)}{\sqrt{2}} & 1 \end{pmatrix}, \qquad \mathcal{I} = \frac{c_1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Double path adiabatic passage

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & 1 \end{pmatrix}, \qquad \mathcal{I} = \lambda \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

with effective coupling and splitting ratio

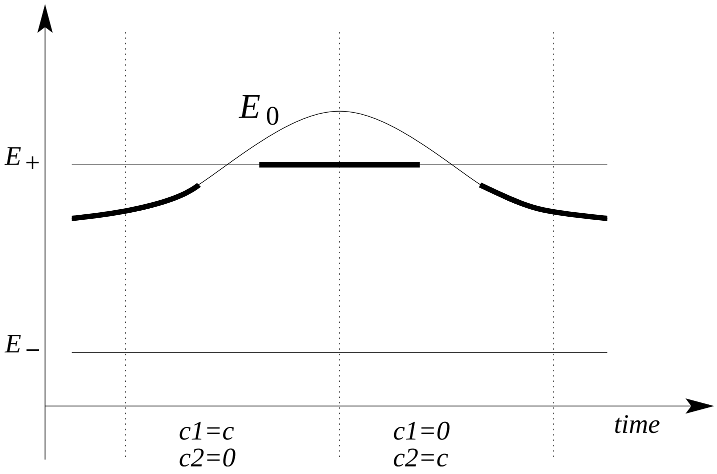
$$c \equiv \frac{(c_1+c_2)}{\sqrt{2}}, \qquad \lambda \equiv \frac{c_1}{c_1+c_2}$$

Accordingly:

$$\begin{aligned} \langle \mathcal{Q} \rangle &= \lambda p \\ \mathrm{Var}(Q) &= \lambda^2 \underset{\sim\sim\sim}{(1-p)p} \quad \neq \quad (1-\lambda p)\lambda p \end{aligned}$$

If c_1 and c_2 have opposite signs then (say) λ becomes larger than unity, while $(1-\lambda)$ is negative. This reflects that the driving induces a circulating current within the ring, and illuminates the fallacy of the classical peristaltic point of view.

Full stirring cycle



a sequence of two
Landau Zener crossings

$$
\begin{aligned}
\langle \mathcal{Q} \rangle \quad & \approx \quad \lambda_{\circlearrowleft} - \lambda_{\circlearrowright} \\
\mathrm{Var}(\mathcal{Q}) \quad & = \quad \left| \ \lambda \ c \int_{-\infty}^{\infty} \mathrm{e}^{i\Phi(t)} dt \ \right|^2 \\
& \approx \quad \left| \lambda_{\circlearrowleft} \mathrm{e}^{i\varphi_1} + \lambda_{\circlearrowright} \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}
\end{aligned}
$$

Due to interference the counting statistics becomes in general unrelated to the occupation statistics:

$$
\begin{aligned}
p \quad & = \quad \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{(\dot{u}/2c)\ \mathrm{e}^{i\Phi(t)}}{1 + (u/2c)^2} dt \right|^2 \\
& \approx \quad \left| \mathrm{e}^{i\varphi_1} - \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}
\end{aligned}
$$

Long time Counting Statistics

Naive expectation:

Probabilistic point of view implies

$$\delta Q \quad \propto \quad \sqrt{t}$$

Quantum result:

The eigenvalues $Q_{\pm}$ of the $\mathcal{Q}$ operator grow linearly with the number of cycles

$$\delta Q \quad \propto \quad t$$

If we have good control over the preparation we can select it to be a Floque state of the quantum evolution operator. For such preparation the linear growth of δQ is avoided, and it oscillates around a residual value.