# The non-equilibrium steady state and induced current in mesoscopically glassy systems with non trivial topology: Interplay of resistor-network theory and Sinai physics

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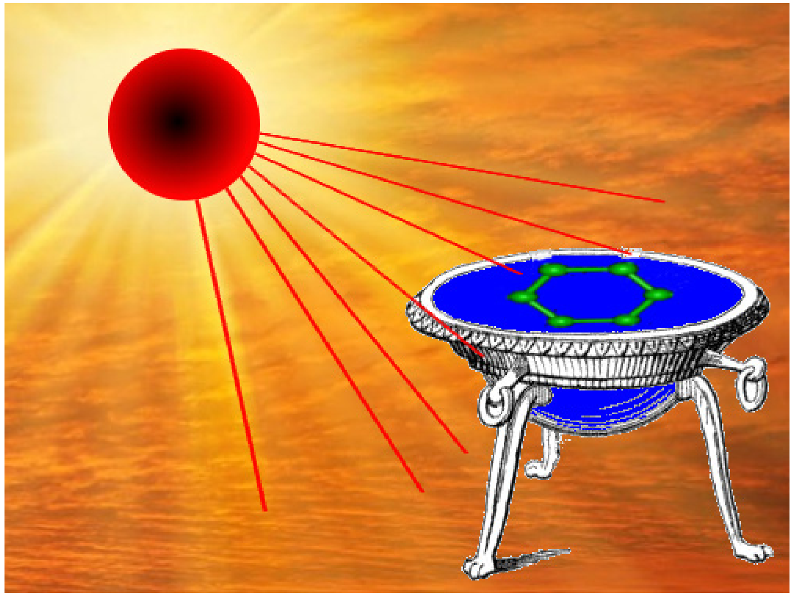
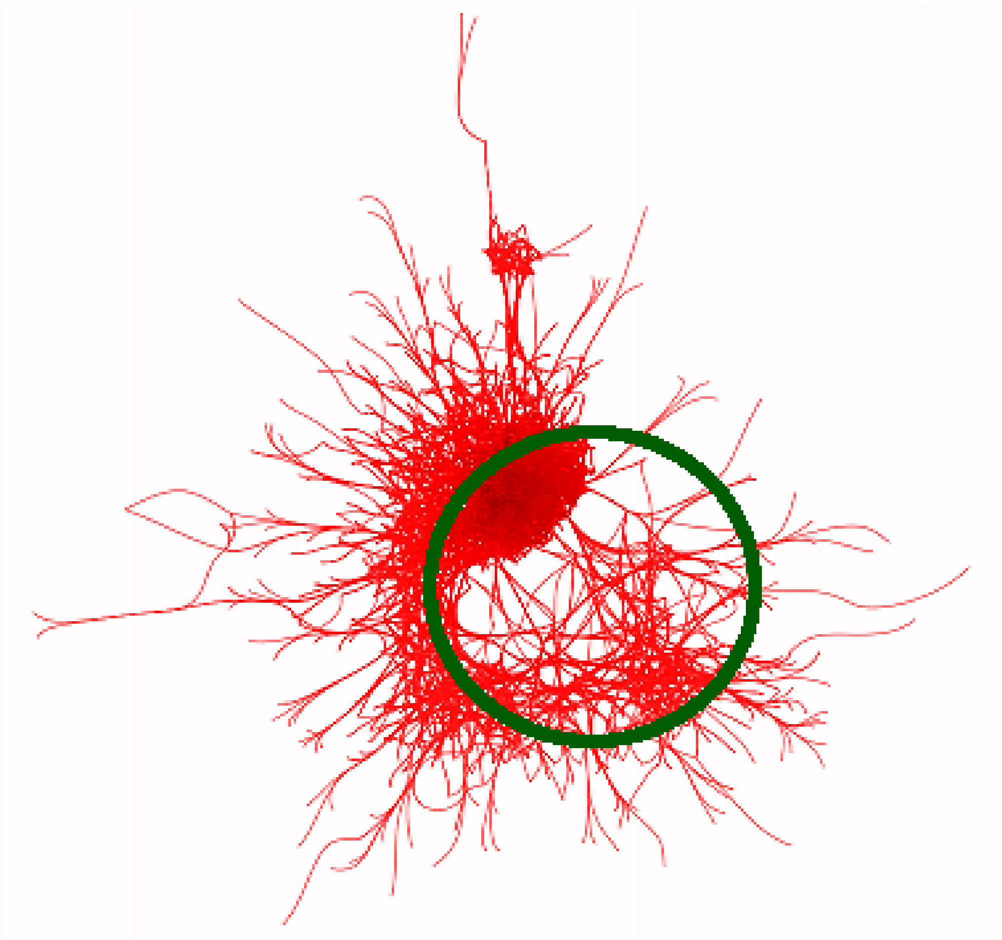
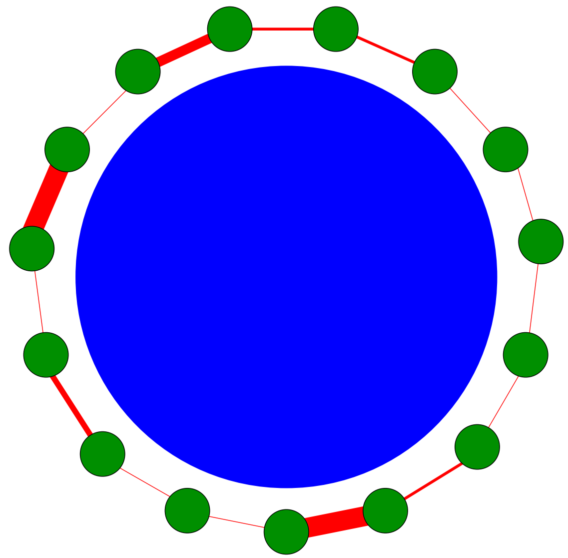


[1] D. Hurowitz and D. Cohen, Europhysics Letters 93, 60002 (2011)
[2] D. Hurowitz, S. Rahav and D. Cohen, Europhysics Letters 98, 20002 (2012)
**[3] D. Hurowitz, S. Rahav and D. Cohen, arXiv (2013)**

## Sparse systems



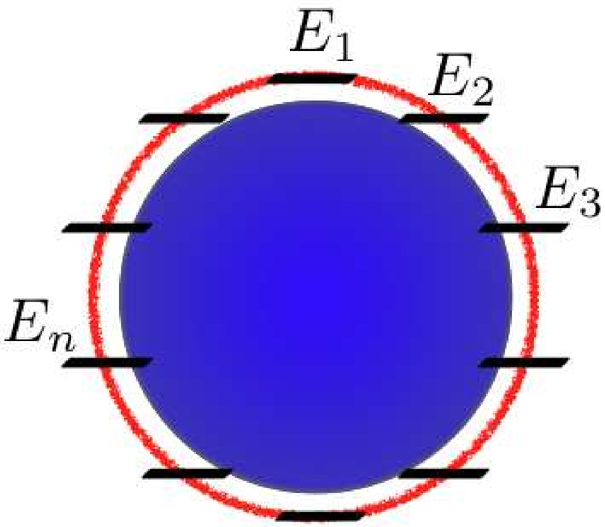
$$w_{\vec{n}} = w^{\beta}_{\vec{n}} + \nu g_n$$



In our study we consider systems that are "sparse" or "glassy", meaning that many time scales are involved.
Standard thermodynamics does not apply to such systems.

# The model

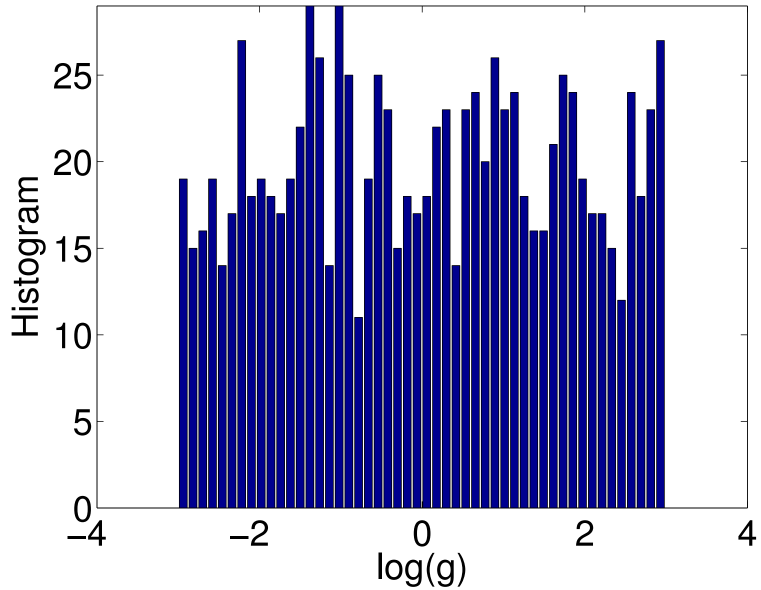
## System + Bath + Driving




$$w_{\overrightarrow{n}} = w^{\beta}_{\overrightarrow{n}} + \nu g_n$$

$$g_n = \text{couplings}$$

## Histogram of couplings



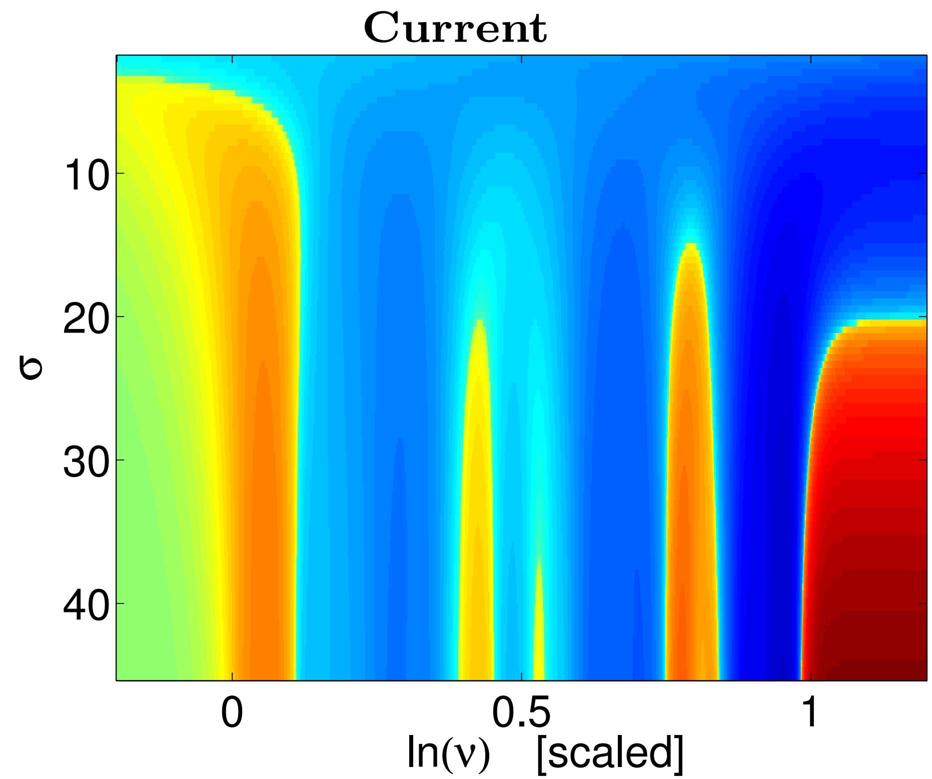

$\longleftarrow \sigma =$few decades $\longrightarrow$

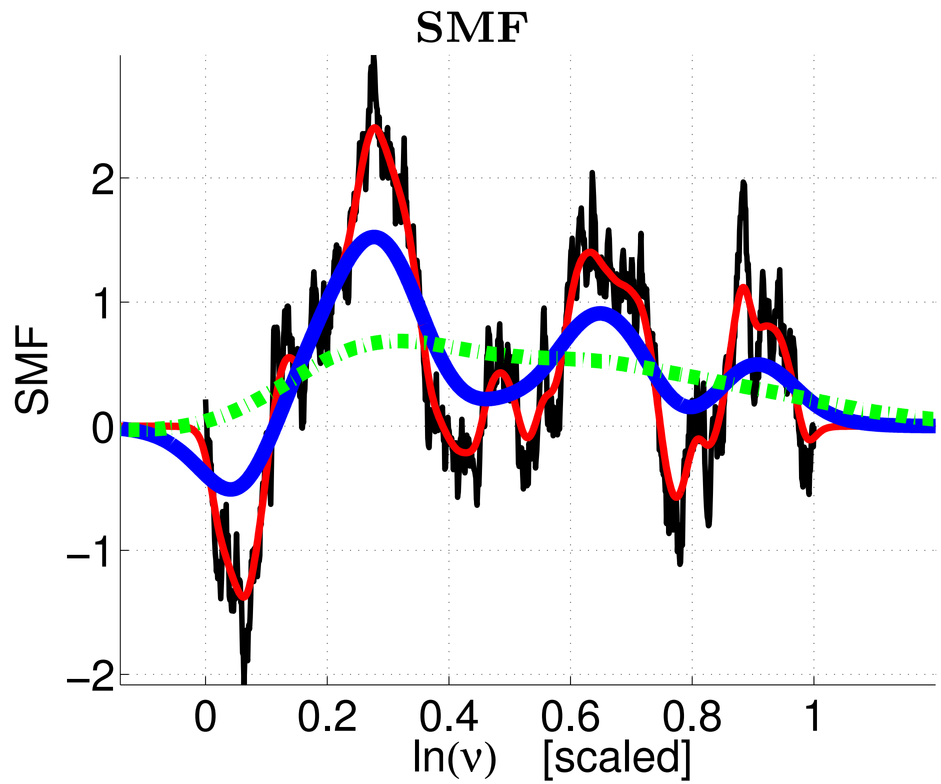
"sparsity" = log wide distribution of couplings

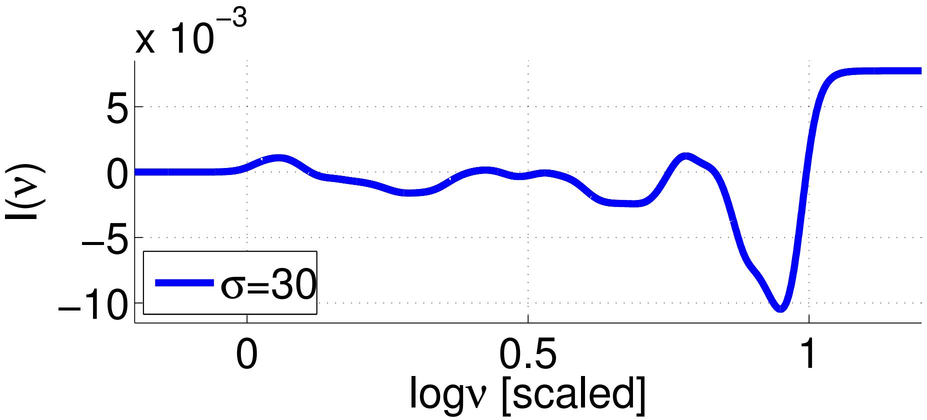
$$w^{\nu}_n = \nu g_n \qquad \text{corresponds to } T_A = \infty$$

$$\frac{w^{\beta}_{\overrightarrow{n}}}{w^{\beta}_{\overleftarrow{n}}} = \exp\left[-\frac{E_n - E_{n-1}}{T_B}\right] \qquad \text{corresponds to } T_B = \text{finite}$$

# Current sign reversals in the Sinai regime



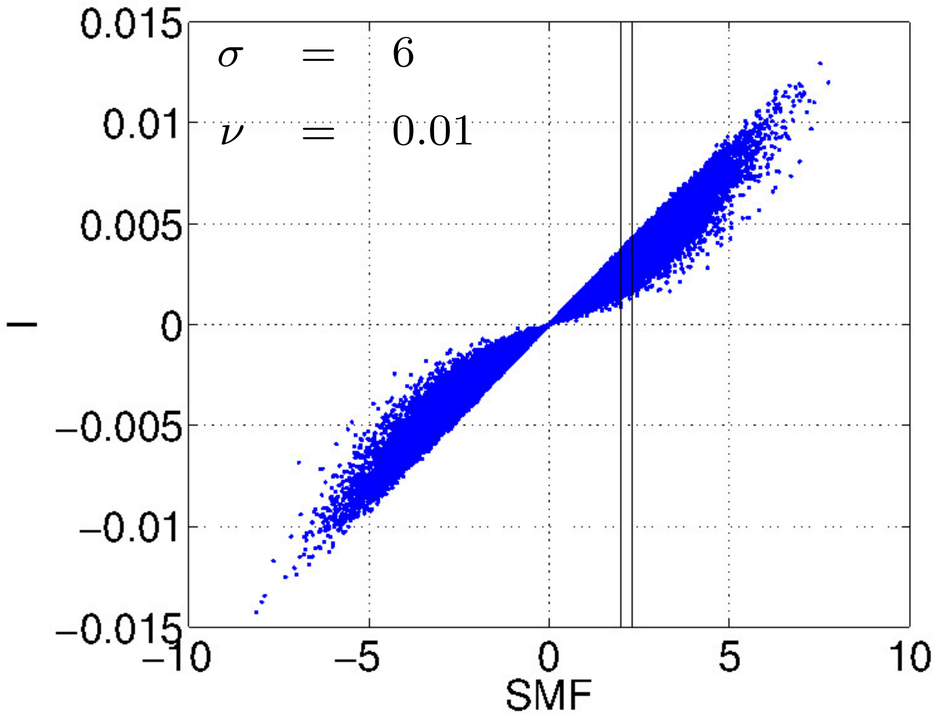

$$I(\nu) \quad \sim \quad \frac{1}{N}\, w_{\varepsilon}\, \mathrm{e}^{-B}\, 2\sinh\left(\frac{\mathcal{E}_{\circlearrowleft}}{2}\right)$$

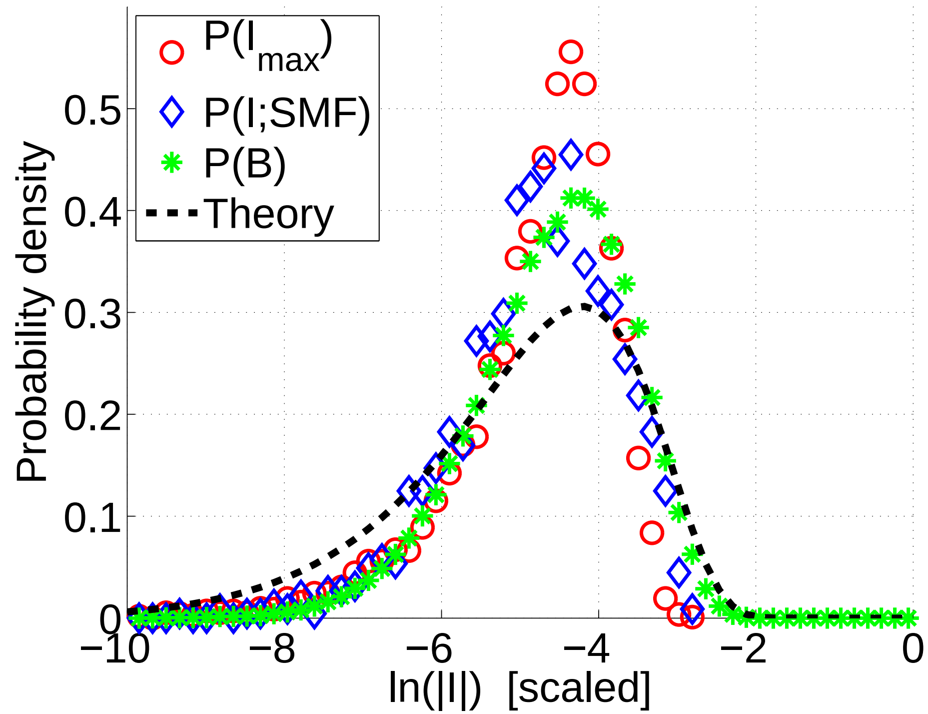
$\mathcal{E}_{\circlearrowleft}$ - Stochastic Motive Force

$B$ - Effective Activation Barrier

**The number of sign changes depends on the sparsity $\approx \sqrt{\pi\sigma}$**

# Statistics of the Current in the Sinai Regime




Single barrier approximation for the current

$$I(\nu) \quad \sim \quad \frac{1}{N}\, w_\varepsilon \, \mathrm{e}^{-B}\, 2\sinh\left(\frac{\mathcal{E}_{\circlearrowleft}}{2}\right)$$

$\mathcal{E}_{\circlearrowleft}$ - Stochastic Motive Force

$B$ - Effective Activation Barrier

Barrier distribution

$$\mathrm{Prob}\,\{\text{barrier} < B\} \sim \exp\left[-\frac{1}{2}\left(\frac{\pi\sigma_{\mathrm{B}}}{2B}\right)^2\right]$$

$$\sigma_{\mathrm{B}}^2 = 2\Delta^2 N\,\frac{\ln(g_{\max}\nu)}{\sigma}$$

# The stochastic potential and SMF

Steady state rate equations:

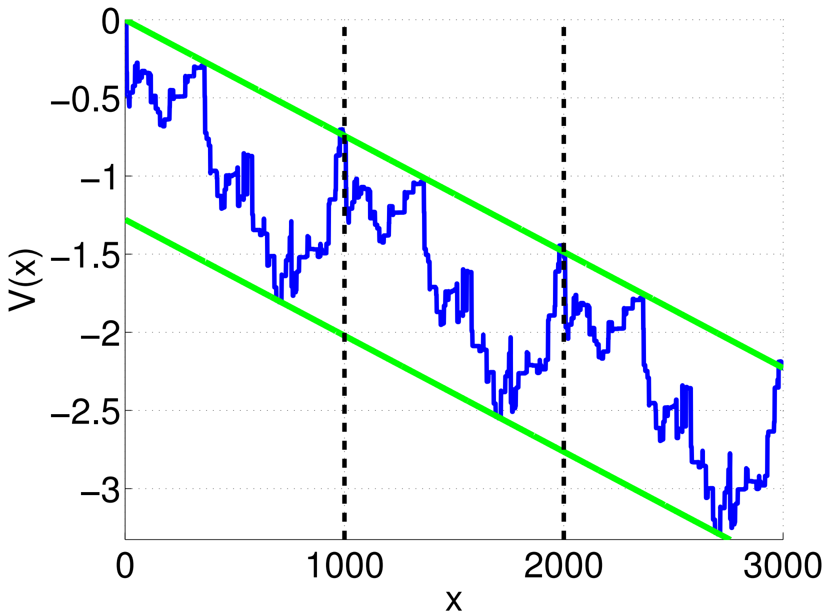
$$I = w_{\overrightarrow{n}} p_n - w_{\overleftarrow{n}} p_{n+1}$$

Stochastic field:

$$\mathcal{E}(x_n) \equiv \ln \left[ \frac{w_{\overrightarrow{n}}}{w_{\overleftarrow{n}}} \right] \approx - \left[ \frac{1}{1 + g_n \nu} \right] \frac{E_n - E_{n-1}}{T_B}$$

Stochastic potential:

$$V(x) = - \int^{x} \mathcal{E}(x')dx' \approx \sum_n \left[ \frac{1}{1 + g_n \nu} \right] \frac{E_n - E_{n-1}}{T_B}$$




Stochastic Motive Force:

$$\mathcal{E}_{\circlearrowleft} \equiv \ln \left[ \frac{\prod_n w_{\overrightarrow{n}}}{\prod_n w_{\overleftarrow{n}}} \right] = \oint \mathcal{E}(x)\, dx \text{ if no driving} = 0$$

Our model:

Telescopic correlations:

$\mathcal{E}(x_n) \sim \Delta_n \equiv (E_n - E_{n+1})$

**Yet... we have sparsely distributed couplings**

Sinai diffusion [1982]:

Random, Uncorrelated & non symmetric transition rates

$\rightsquigarrow$ Buildup of activation barrier $B \sim \sqrt{N}$

$\rightsquigarrow$ Exponentially low current $I \sim e^{-\sqrt{N}}$

# SMF vs. Driving intensity

Stochastic Motive Force

$$\mathcal{E}_{\circlearrowleft}(\nu) \;\approx\; -\sum_{n=1}^{N} \left[\frac{1}{1+g_n\nu}\right] \frac{E_n - E_{n-1}}{T_B}$$

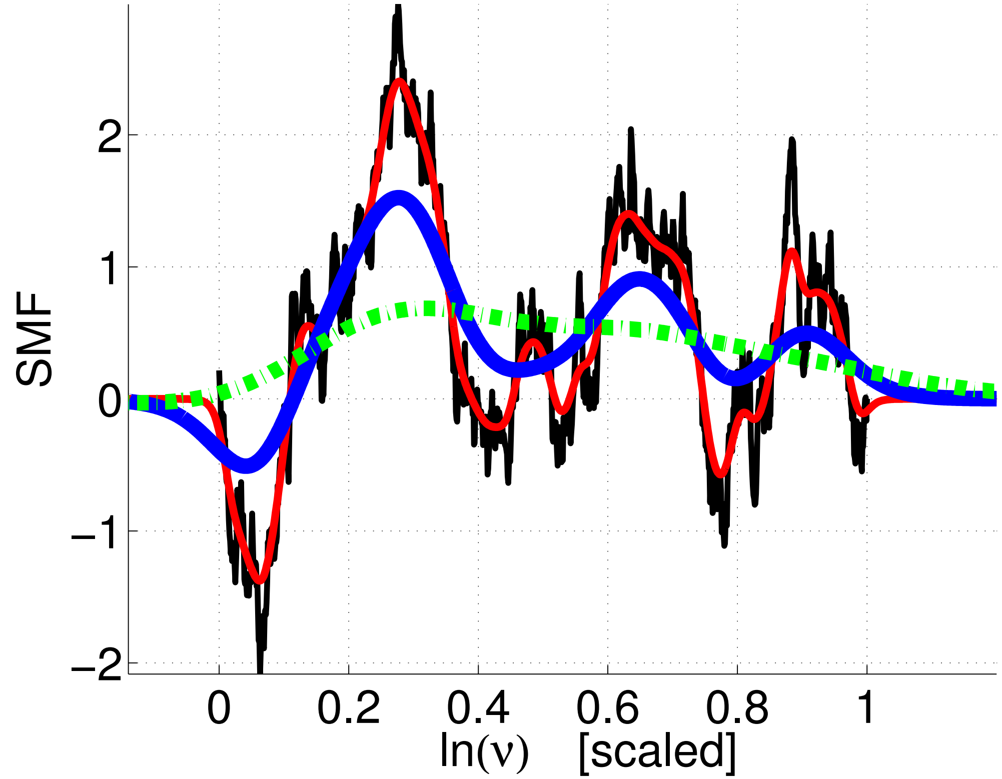
$$\tau \equiv \frac{1}{\sigma}\ln(g_{\text{max}}\nu), \quad \tau_n = \frac{1}{\sigma}\ln\left(\frac{g_{\text{max}}}{g_n}\right)$$

$$\sigma \;=\; \ln\frac{g_{\text{max}}}{g_{\text{min}}}, \qquad \text{[log-width of distribution]}$$

Coarse grained random walk:

$$\mathcal{E}_{\circlearrowleft}(\tau) = -\sum_{n=1}^{N} f_{\sigma}(\tau-\tau_n)\frac{E_n - E_{n-1}}{T_B}$$

$$f_{\sigma}(t) \;\equiv\; [1+\mathrm{e}^{\sigma t}]^{-1} \qquad \text{["step" function]}$$




$$\text{Sinai regime:} \quad \frac{1}{g_{\text{max}}} < \nu < \frac{1}{g_{\text{min}}}$$

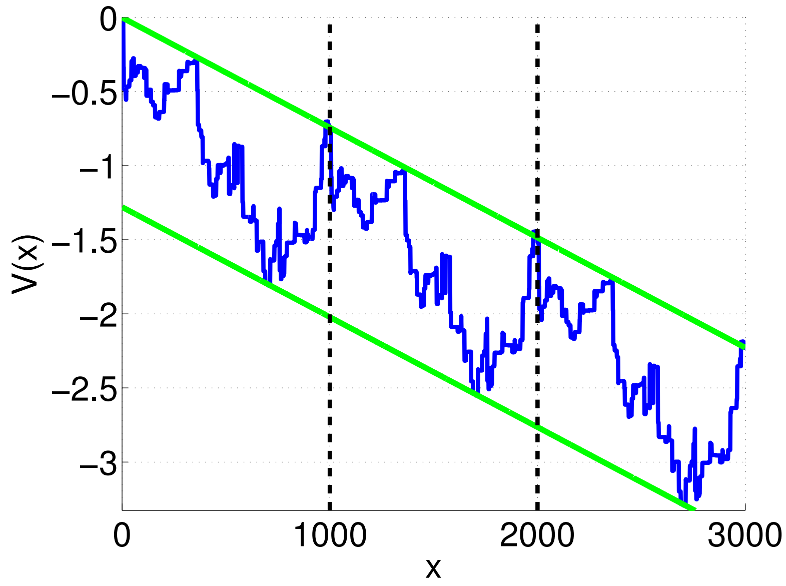
$$0 < \tau < 1$$

Expected number of sign changes $\approx \sqrt{\pi\sigma}$

## Barrier Statistics

Activation Barrier ≡ Occupation range of a random walk.

$$B \approx \frac{1}{2}\left[\max\{U\} - \min\{U\}\right] \equiv 2R$$




- Joint probability that a RW occupies the interval $[x_a, x_b]$:

$$P_t(x_a, x_b) \equiv \text{Prob}(x_a < x(t') < x_b), \quad t' \in [0, t]$$

$$f(x_a, x_b) = -\frac{d}{dx_a}\frac{d}{dx_b}P_t(x_a, x_b)$$

- Make the transformation $X = \dfrac{x_a + x_b}{2}, \quad R = x_b - x_a$
- A random walk process occupies range $R$:

$$f(R) = \partial_R^2\left[R\ P_t(R)\right]$$

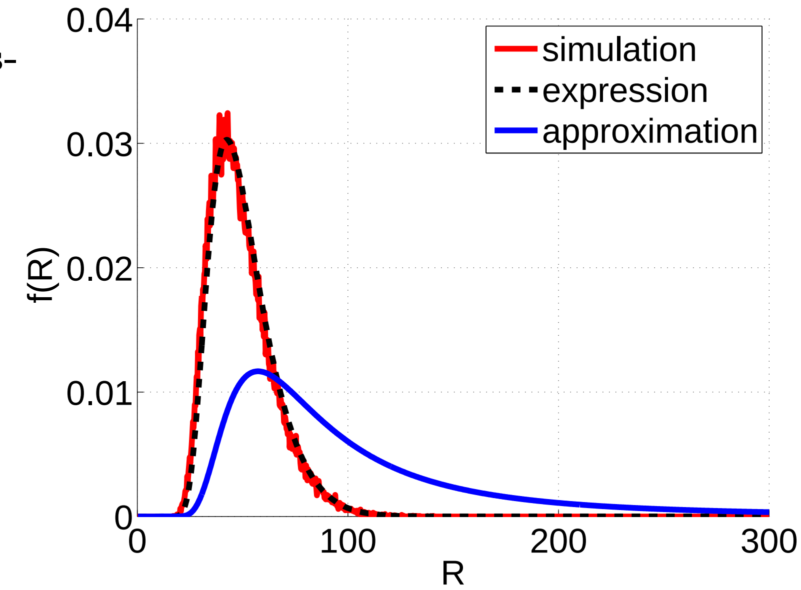
Survival probability of a diffusion process with initial uniform distribution: $P_t(R)$

- Solution to diffusion equation

$$\rho_t(x) = \sum_{n=1,3,5,\ldots}^{\infty} \exp\left[-D\left(\frac{\pi n}{R}\right)^2 t\right]\frac{4}{\pi n R}\sin\left(\frac{\pi n}{R}x\right)$$

$$P_t(R) = \int_0^R \rho_t(x)\ dx = \sum_{n=1,3,5,\ldots}^{\infty}\frac{8}{\pi^2 n^2}\exp\left[-D\left(\frac{\pi n}{R}\right)^2 t\right]$$

$$P_t(r) \approx \exp\left(-\frac{1}{2}\left(\frac{\pi\sigma}{R}\right)^2\right)$$



# Summary of main results

1. Number of current sign changes is determined by log-width of coupling distribution,
   Expected number of sign changes $\approx \sqrt{\pi\sigma}$.

2. The current in the Sinai regime may be estimate by a single barrier approximation,
   $I(\nu) \sim \frac{1}{N}\, w_{\varepsilon}\, \mathrm{e}^{-B}\, 2\sinh\left(\frac{\mathcal{E}_{\circlearrowleft}}{2}\right)$.

3. Exact expression for (non-canonical!) NESS occupation probability
   $p_n \;\propto\; \left(\frac{1}{w(x_n)}\right)_{\varepsilon} \mathrm{e}^{-(U(n)-U_{\varepsilon}(n))}$
   reflects crossover from Sinai spreading to resistor network picture.

4. Distribution of currents reflects underlying Barrier ,
   random walk occupation range statistics,
   $\mathrm{Prob}\left\{\text{barrier} < B\right\} \;\sim\; \exp\left[-\frac{1}{2}\left(\frac{\pi\sigma_{\mathrm{B}}}{2B}\right)^2\right]$, with $\sigma_B^2 \;=\; 2\Delta^2 N\,\frac{\ln(g_{\max}\nu)}{\sigma}$.